

Large-Scale $1/f$ Noise and High-Order Structure Functions in the Heliosphere: Target for PUNCH Science

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- 2 Large-scale Magnetic Correlation at 1 au
- 3 Low-frequency Magnetic Spectrum at 1 au
- 4 Preliminary results of high-order statistics at 1 au
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Solar wind beyond a simple steady-state turbulent system

- Solar wind turbulence is often modeled under assumption of statistically steady, ergodic plasma.
- Large-scale structures and long-range correlations say otherwise.

Coronal mass ejections (CME)

irregular eruptions perturbing solar wind



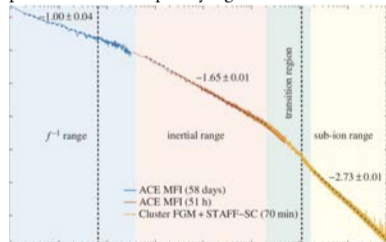
Heliospheric current sheet (HCS)

dynamic interface between magnetic polarities



Fluctuation $1/f$ noise

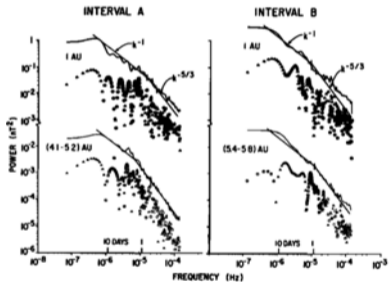
persistent low-frequency signal correlation



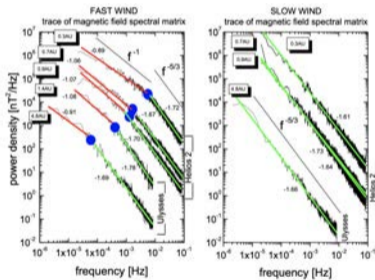
- Long-memory signatures indicate indirect, statistical relationships possibly rooted in solar dynamo.
- Wide-field polarized imaging from PUNCH enables continuous tracking of large-scale coronal structures and their evolution.

Heliospheric $1/f$ spectrum

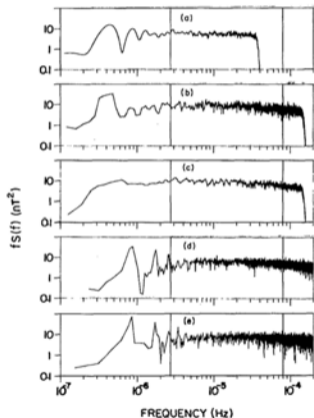
Examples of $1/f$ spectrum in interplanetary magnetic field:



Magnetic spectra at 1 au from IMP 8 and ISEE 3, and near 4 to 5 au from Voyager 1, Burlaga & Goldstein 1984.



$1/f$ in B trace spectrum in fast wind at various heliocentric distances from Ulysses and Helios 2, Bruno & Carbone 2013.



Magnetic spectra at 1 au from ISEE 3 (panels a, b), IMP (panel c), and OMNI (panels d, e), Matthaeus & Goldstein 1986.

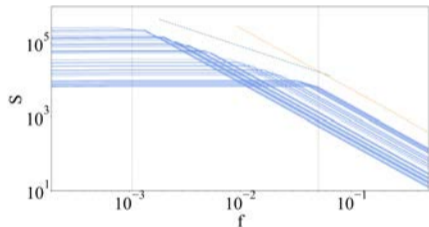
In long data record, 1 au $1/f$ magnetic spectrum spans from $\sim 2 \times 10^{-6}$ to $\sim 10^{-3}$ Hz.

Superposition principle as generation mechanism for $1/f$ noise

Machlup 1981 - Superposition of random processes (steady state, Gaussian, & Markov $\rightarrow e^{-\tau/\tau_c}$ autocorrelation) with characteristic correlation times scale-invariantly distributed ($\rho(\tau_c)d\tau_c \propto d\tau_c/\tau_c$) produces $1/f$ ensemble spectrum:

$$S(\tau_c, \omega) \propto \int_{-\infty}^{\infty} e^{-i\omega\tau} e^{-\tau/\tau_c} d\tau \propto \frac{\tau_c}{1 + \omega^2\tau_c^2},$$

$$\bar{S}(\omega) = \int_{\tau_1}^{\tau_2} S(\tau_c, \omega) \rho(\tau_c) d\tau_c \propto \frac{\arctan(\tau_c\omega)}{\omega} \Big|_{\tau_1}^{\tau_2}.$$



If $\tau_1 \ll \tau_2$ (large span of correlation times), then within frequencies $\tau_1 \ll 1/\omega \ll \tau_2$,

$$\bar{S}(\omega) \propto \frac{\pi/2 + \mathcal{O}(\tau_2\omega)^{-1} + \mathcal{O}(\tau_1\omega)^1}{\omega}. \quad (I)$$

Other models

- Wang et al. 2026: Superposition of Kolmogorov-like $f^{-5/3}$ spectra leads to $1/f$.
- Dmitruk & Matthaeus 2007: Inverse cascade & relaxation generate long timescales and $1/f$ in frequency space.
- Bak, Tang, & Wiesenfeld 1987: Self-organized criticality creates universal $1/f$ -type spectrum.

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Abstract

We present a broad review of $1/f$ noise observations in the heliosphere, and discuss and complement the theoretical background of generic $1/f$ models as relevant to NASA's Polarimeter to UNify the Corona and Heliosphere (PUNCH) mission. First observed in the voltage fluctuations of vacuum tubes, the scale-invariant $1/f$ spectrum has since been

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The Polarimeter to UNify the Corona and Heliosphere (PUNCH) Mission: An Overview

Sections Figures References

Abstract

Introduction

2D NS turbulence, $Re=14,000$, visualization of vorticity [Montgomery et al., 1998](#)

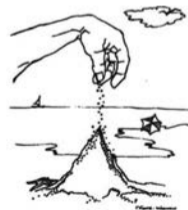
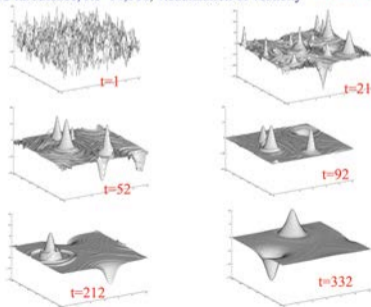


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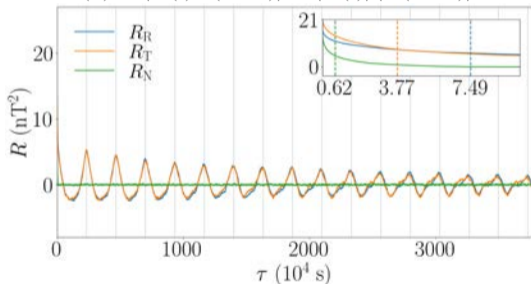
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1 au magnetic field autocorrelation, from 1 minute to 1.2 years

12 years (Feb 1998 to Dec 2009) of magnetic field data with 1-minute cadence from the Advanced Composition Explorer (ACE) mission.

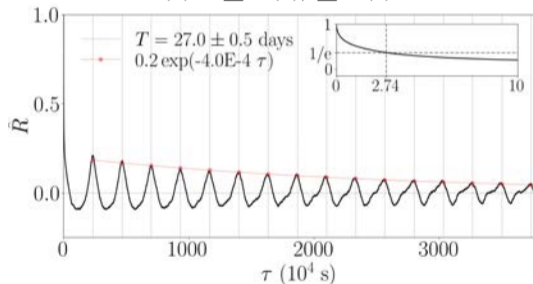
Component autocorrelation:

$$R_i(\tau) = \langle B_i(t)B_i(t + \tau) \rangle - \langle B_i(t) \rangle \langle B_i(t + \tau) \rangle.$$



Normalized correlation trace:

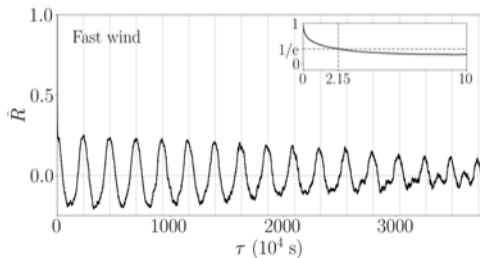
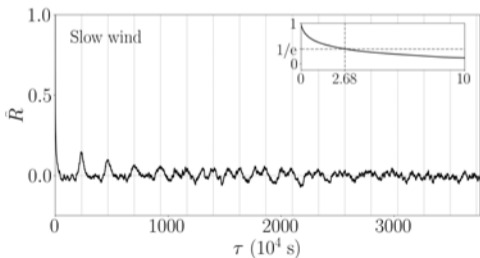
$$\hat{R}(\tau) = \sum R_i(\tau) / \sum R_i(0).$$



- R_R and R_T have ~ 27 d periodicity (synodic period of equatorial solar rotation); R_N has e -folding time ~ 1.7 hr with no clear periodic behavior.
- Trace \hat{R} has ~ 27 d periodicity with local maxima decaying roughly exponentially.

Drastically different correlation trace of slow vs. fast wind

- Slow wind (≤ 400 km/s): 27d correlation enhancement disappears after ~ 4 solar rotations, overtaken by fluctuations at finer scales in lag \Rightarrow many periodicities (higher-order harmonics) from the coronal source.
- Fast wind (≥ 500 km/s): predominantly 27-day periodicity; 13.5-day secondary structures after ~ 12 solar rotations.



Correlation trace of solar min vs. max

- Solar min (2006-2009): 13.5d periodicity emerges (!); also reported in geomagnetic index, solar wind parameters, and chromospheric fluxes (Mursula & Zieger 1996).
- Solar max (1999-2002): no distinct secondary structures but shifts out of phase with 27d periodicity after ~ 5 solar rotations (towards ~ 26 d) \Rightarrow Possible existence of several closely separated periodicities.

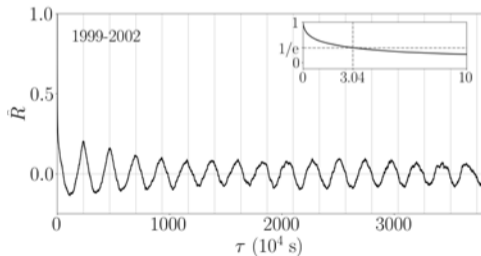
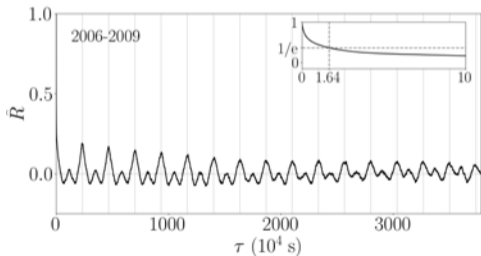


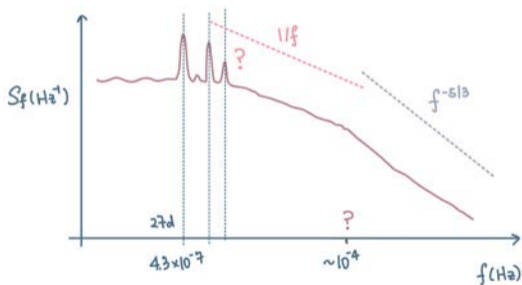
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An impression of 1 au broadband interplanetary spectrum

Power spectral density as discrete Fourier transform of symmetrized correlation:

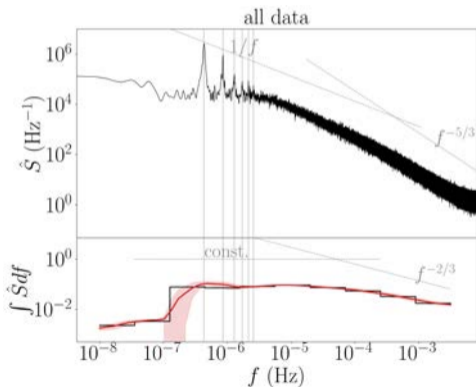
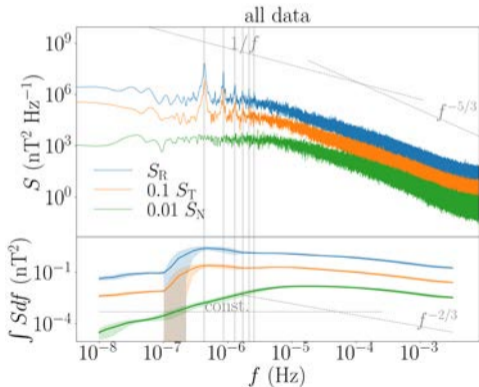
$$S(f) = \int_{-\infty}^{\infty} R(\tau) e^{-i2\pi f\tau} d\tau. \quad (2)$$



- How does $1/f$ connect with $f^{-5/3}$? Differently in slow vs. fast or min vs. max wind?
- How does $1/f$ interact with point spectral features of solar rotation?

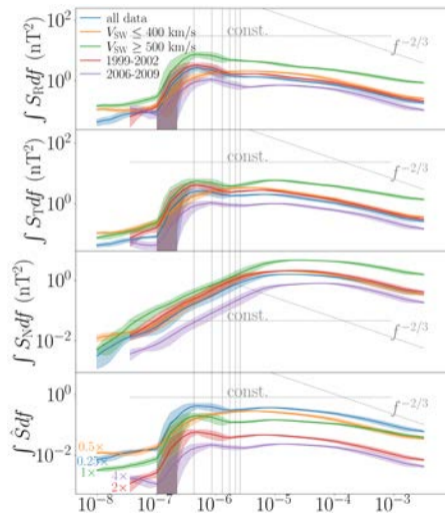
1 au magnetic spectrum

- $1/f$ scaling in trace spectrum is a combined effort from all three components.
- $1/f$ band exists from $\sim 10^{-6}$ Hz (first superharmonic of solar rotation) to $\sim 10^{-4}$ Hz.



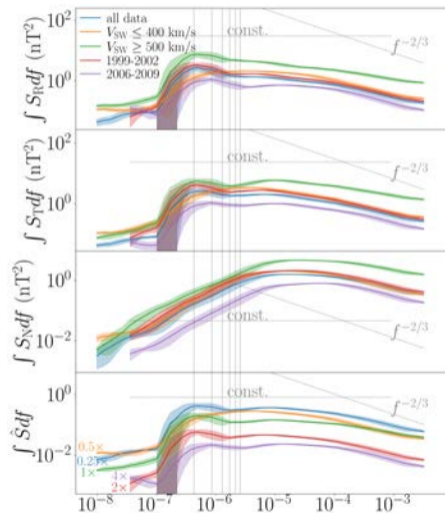
Estimated Integrated Magnetic Spectrum

- $1/f$ upper bound at $\sim 10^{-5}$ Hz for solar max and slow wind, $\sim 10^{-4}$ Hz for min and fast wind.
 - agrees with Bruno+ 2019: turbulence structures have more time to develop in slow wind.



Estimated Integrated Magnetic Spectrum

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- In all cases, $1/f$ extends below 2×10^{-6} Hz (Matthaeus & Goldstein 1986) and underlies rotation harmonics. For entire 12yr and solar min, $1/f$ persists down to fundamental 27d frequency.
 - Solar rotation structures do not disrupt $1/f$ signatures.
 - Range of influence of MHD is limited, so $1/f$ must originate below the solar corona, possibly in the solar dynamo.



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 - Solar rotation structures do not disrupt $1/f$ signatures.
 - Range of influence of MHD is limited, so $1/f$ must originate below the solar corona, possibly in the solar dynamo.
- **Solar wind at 1 au retains solar-origin long-memory signatures that render departure from Gaussianity across scales.**

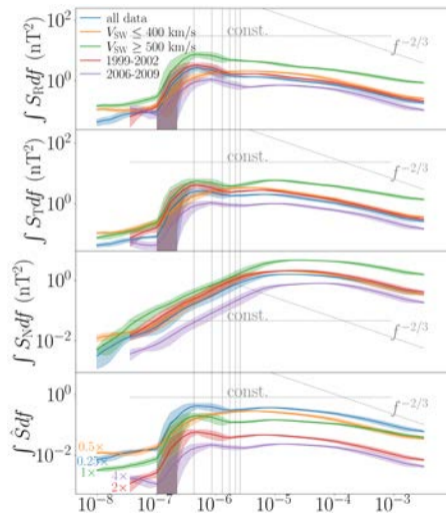


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Kurtosis - non-Gaussianity in magnetic field fluctuations

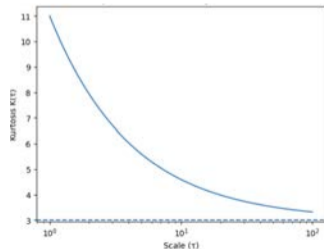
- Second-order statistics (e.g. variance, correlation) describe energy distribution.
- Higher-order statistics capture intermittency, rare events, extreme fluctuations, etc. (PUNCH sees these!)

Kurtosis (4th moment) measures the “flatness” or “tailedness” of a distribution:

$$K_i(\tau) = \langle \delta B_i^4 \rangle / \langle \delta B_i^2 \rangle^2 \quad (3)$$

A classical impression of solar wind Kurtosis:

- MHD turbulence scale: $K \gg 3 \Rightarrow$ strong intermittency, abundance of coherent structures (e.g. current sheet, flux tubes).
- Large scale: $K \approx 3 \Rightarrow$ Gaussian fluctuations from central limit theorem???



Scale-dependent kurtosis at 1 au

Magnetic fluctuation distribution with 4-day mean and 12yr statistics (left):

$$K_i = \langle (B_i - \langle B_i \rangle)^4 \rangle / \langle (B_i - \langle B_i \rangle)^2 \rangle^2$$

- Leptokurtic distribution with $K > 3$.
- Results similar to Padhye et al. (2001)

Scale-dependent kurtosis observed by ACE (right):

$$K_i = \langle [B_i(t + \tau) - B_i(t)]^4 \rangle / [B_i(t + \tau) - B_i(t)]^2^2$$

- Distinct solar rotation signatures in R and T components.
- All components have $K > 3$ across lags from day to year.

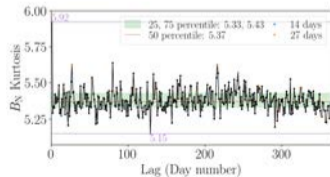
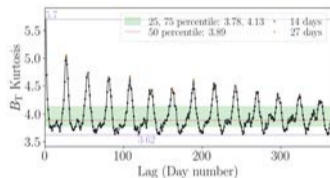
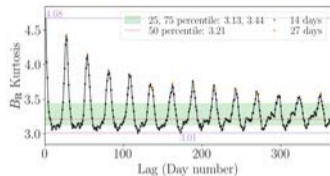
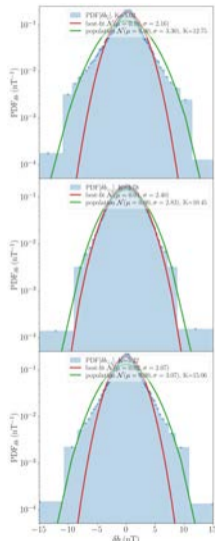


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Summary

- Long-duration (12yr) magnetic field measurements at 1 au reveal persistent large-scale correlations extending far beyond typical MHD correlation scales.
- Spectral analysis show a broad $1/f$ continuum, spanning from solar rotation scales (~ 27 days) down to $\sim 10^{-4}$ Hz, coexisting with discrete rotational harmonics.
- Correlation reveals multi-periodic behavior, including a strong 13.5-day signature during solar minimum and beat-like patterns, suggesting contributions from closely spaced rotational frequencies rather than a single periodic driver.
- Scale-dependent kurtosis ($K > 3$ across lags) indicates non-Gaussian, intermittent fluctuations at large scales, consistent with correlated low-frequency $1/f$ -type signals.
- PUNCH enables direct imaging of large-scale coronal structures, providing a critical test of whether these long-memory signatures are imprinted near the Sun.

Thank you – questions welcome.

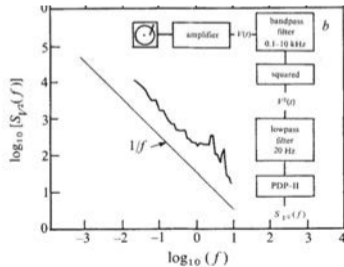
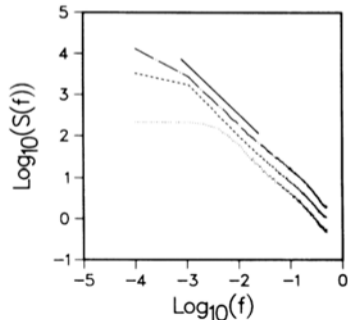
Appendix

1/f noise is frequently observed in various systems

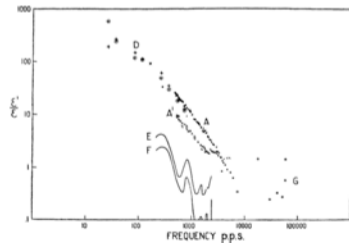
1/f signal is uniquely described as being scale-invariant, i.e., the integration of the spectrum over frequency range $[f_1, f_2]$ depends solely on the ratio f_2/f_1 :

$$\int_{f_1}^{f_2} df/f = \log(f_2/f_1). \tag{4}$$

Examples of 1/f signals exist in vacuum tube noise, music, self-organized systems, etc.:



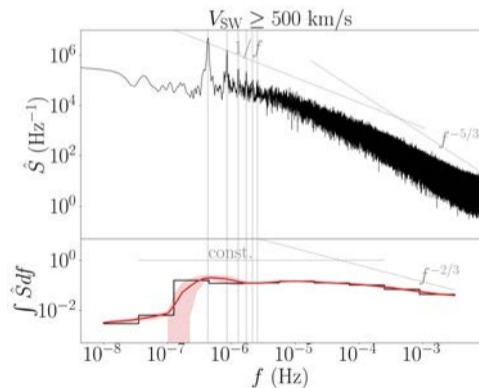
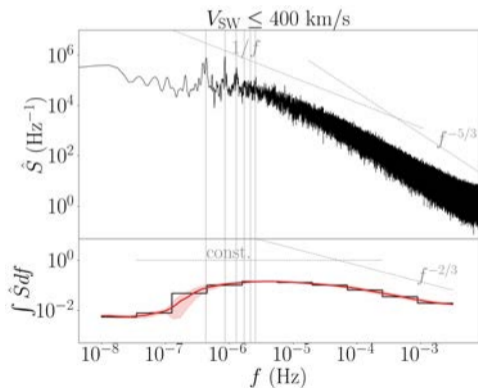
Voltage fluctuation in vacuum tubes, Johnson 1925.



Particle number on simulated 2D lattice models following simplified diffusive model, Jensen 1990.

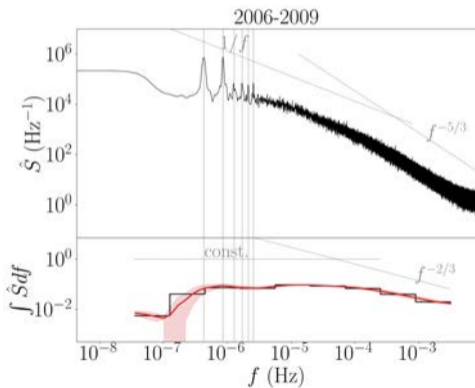
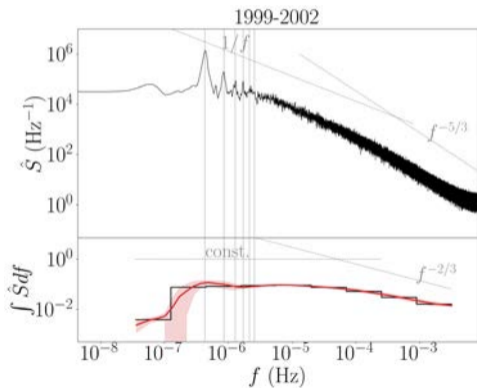
More spectra - slow vs. fast

- Slow wind has two closely separated peaks around 27 and 30 days, possible source of beat-like interference.
- Fast wind $1/f$ extends to \sim decade higher in frequency.



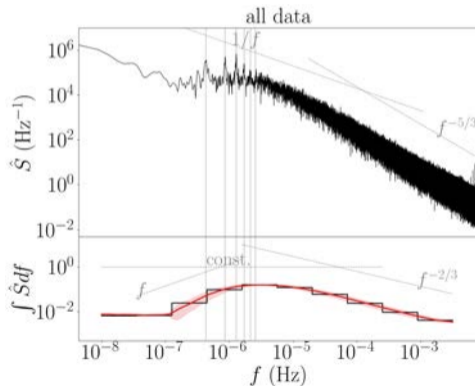
More spectra - max vs. min

- Solar min first harmonic has greater power than the fundamental, corresponding to 13.5d periodicity evident in correlation.



More spectra - density

- Solar rotation spectral peaks superposed on top of large-scale uncorrelated signals, in contrast to magnetic field spectra.



Similarity between lognormal and scale-invariant distribution

A log-normal distribution is defined for variable $X = e^{\mu + \sigma Z}$ as

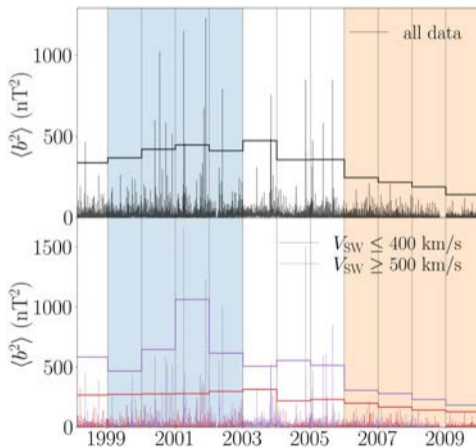
$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{(\log x - \mu)^2}{2\sigma^2}\right],$$

where μ and σ is the mean and standard deviation of the variable $\log X$, respectively. Define $\bar{x} \equiv e^{\mu}$, then $f(x) \propto 1/x$ when

$$\frac{[\log(x/\bar{x})]^2}{2\sigma^2} \ll 1. \quad (5)$$

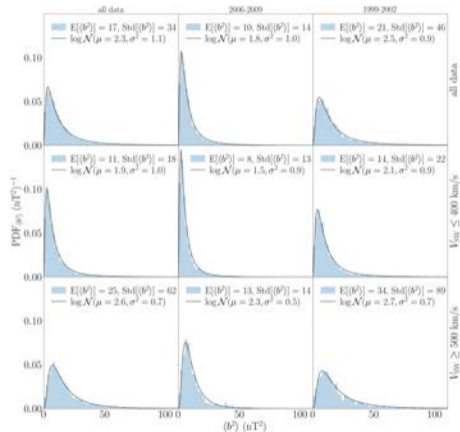
Magnetic field fluctuation variance reflecting solar phases

- Solar max (1999 - 2002) and min (2006 - 2009) phases are defined based on sunspot number, and can be identified here based on the temporal variation of magnetic field fluctuation variance, $\langle b^2 \rangle$. (See Zhao+ 2018 for a detailed study.)
- Component-wise fluctuation b_i is first calculated using a boxcar sliding window of 6hr, \sim correlation time at 1 au.
- Fluctuation variance is $\langle b^2 \rangle = \langle b_R^2 \rangle + \langle b_T^2 \rangle + \langle b_N^2 \rangle, \langle \dots \rangle$ denotes averaging over consecutive 6hr intervals.



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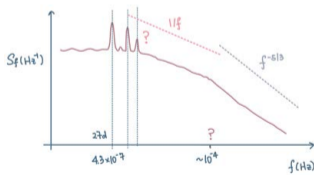
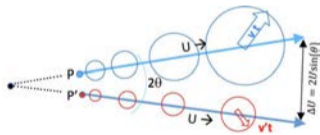
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- Both expectation and variance of $\langle b^2 \rangle$ are always highest during solar max and lowest during solar min, highest for fast wind and lowest for slow wind.



Range of influence - where are $1/f$ signals generated in the solar wind?

Range of influence - the scale over which MHD process exerts influence during transit to 1 au.

If $1/f$ signals are generated in the local wind, how does it extend below \sim correlation scales (6hr $\simeq 5 \times 10^{-5}$ Hz)?



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Abstract

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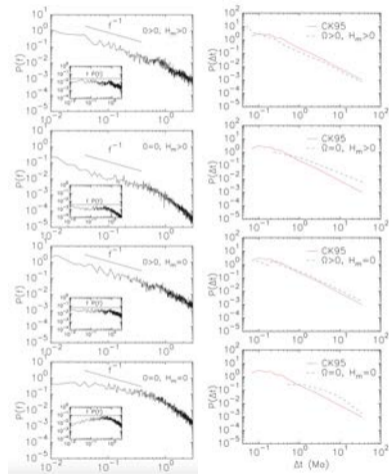
Abstract

Introduction

Possible Connection to Solar Dynamo and Inverse Cascade

Could $1/f$ noise originate from solar dynamo?

- MHD simulations indicate emergence of $1/f$ with rotation and/or magnetic helicity; dipole reversal waiting time distribution compatible with geodynamo.

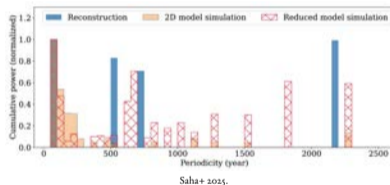


Left: Magnetic dipole spectra from ideal MHD simulation in spherical geometry. Right: waiting time distribution of dipole reversals, Dmitruk+ 2014.

Possible Connection to Solar Dynamo and Inverse Cascade

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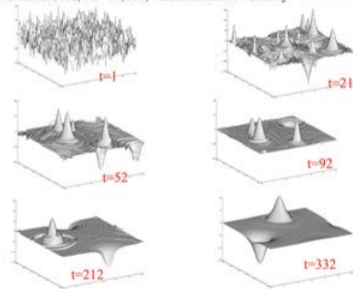


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- Solar dynamo is less chaotic and has 11-year quasi-periodicity. But long (hundred- and thousand-year) timescales exist.
- Inverse cascade may occur and are known to generate timescales much longer than the bandlimited scales (e.g., eddy turnover time) of the system.

2D NS turbulence, $Re=14,000$, visualization of vorticity Montgomery et al, 1999

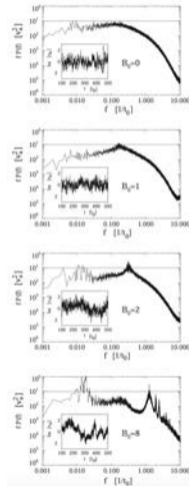


Montgomery+ 1991.

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Could $1/f$ noise originate from solar dynamo?

- MHD simulations indicate emergence of $1/f$ with rotation and/or magnetic helicity; dipole reversal waiting time distribution compatible with geodynamo.
- Solar dynamo is less chaotic and has 11-year quasi-periodicity. But long (hundred- and thousand-year) timescales exist.
- Inverse cascade may occur and are known to generate timescales much longer than the bandlimited scales (e.g., eddy turnover time) of the system.
- $1/f$ develops at lowest wavenumber mode in inverse cascade systems.

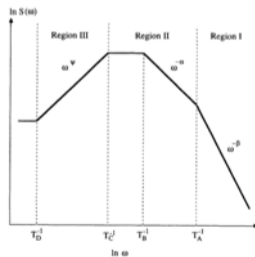
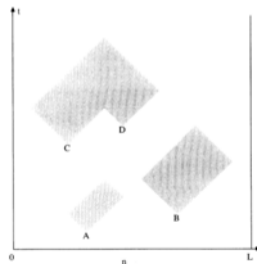
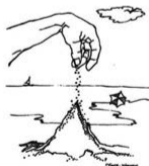


Magnetic spectra from compressible MHD simulations where structure becomes more 2D with increasing mean field. Inverse cascade occurs in 2D turbulence under pseudo-invariants (energy and mean-square potential), Dmitruk & Matthaes 2007.

Possible Connection to Self-organized Criticality

Inverse cascade may be associated with the framework of self-organized criticality.

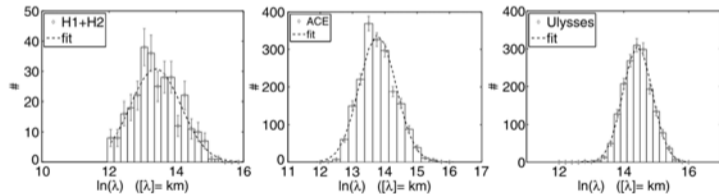
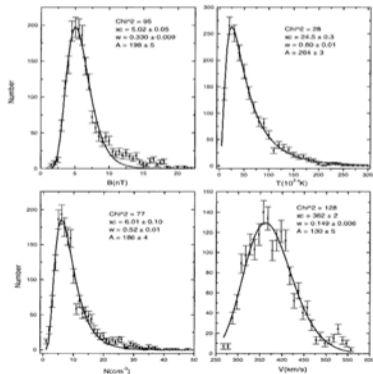
- Simplistic picture: unstable systems with many dof naturally evolves into minimally stable state, where clusters of stable structure develop with non-characteristic spatial scales. Perturbation to this state propagates at finite non-characteristic timescales. A $t^{-\alpha}$ distribution of noise lifetimes leads to a $\omega^{-2+\alpha}$ power-law frequency spectrum.
- Interaction of these dynamic clusters lead to $1/f$ noise.



Hwa & Kardar 1992

Log-normal distributions in the solar wind

Many solar wind variables, including the correlation times or lengths, are log-normal.
 The range of $1/f$ roughly corresponds to the range of the scale-invariant portion in a log-normal distribution (Montroll & Shlesinger 1982).



Distributions of magnetic correlation lengths at 1 au, observed by Helios, ACE, and Ulysses, Ruiz+ 2014.

Distributions of $|B|$, T_p , N , and V_{SW} at 1 au, Burlaga & Szabo 1999.