
# Evaluation of polar motion excitation using C21 and S21 coefficients from hybrid SLR+DORIS solutions and hydrological models

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IAG 2025 Scientific Assembly, 1–5 September 2025, Rimini, Italy

# Introduction

- The excitation of polar motion (PM) by hydrospheric mass redistribution is described by Hydrological Angular Momentum (HAM), derived from hydrological models, gravity field variations, or climate models. GRACE/GRACE-FO provide essential gravimetric data, while low-degree coefficients (e.g., C20, sometimes $C_{21}$, $S_{21}$) are more reliably obtained from Satellite Laser Ranging (SLR).
- Previous studies (Nastula et al., 2025) showed that **HAM from hybrid SLR solutions (SLR combined with GRACE spatial EOF patterns)** aligns well with the **hydrological signal in geodetic PM** (hereafter referred to as **GAO**), often outperforming GRACE-only series for non-seasonal variations. Correlations with GAO reach ~0.6 for short-term and up to 0.9 for long-term variations, while seasonal signals are well captured in all datasets.
- In this study, we extend HAM analysis using updated hybrid solutions from six SLR and ten DORIS satellites. These series are compared with global hydrological models (LSDM, WaterGAP, GLWS2.0) and validated against GAO estimates of hydrological and cryospheric contributions to geodetic angular momentum (GAM).

*Nastula, J., Śliwińska-Bronowicz, J., & Wińska, M. (2025). Harnessing C21 and S21 coefficients from GRACE and hybrid solutions for comprehensive analysis of polar motion excitation. Journal of Geodesy, 99(48), 1–33. https://doi.org/https://doi.org/10.1007/s00190-025-01969-z*

# HAM computation from $\Delta C_{21}$, $\Delta S_{21}$ and TWS

## 2 methods of computing gravimetric excitation of polar motion (HAM)

### From variations of $C_{21}$, $S_{21}$ coefficients of geopotential (GSM)

$$\chi_1 = -\sqrt{\frac{5}{3}} \cdot \frac{1.608 \cdot R_e^2 \cdot M}{C - A'} \Delta C_{21}\ ; \quad \chi_2 = -\sqrt{\frac{5}{3}} \cdot \frac{1.608 \cdot R_e^2 \cdot M}{C - A'} \Delta S_{21}$$

$R_e$ and M are the Earth's mean Earth's radius and mass, respectively; A, B, and C are the principal moments of inertia for Earth; A' = (A + B)/2 is an average of the equatorial principal moments of inertia; and $\Delta C_{21}$ and $\Delta S_{21}$ are the normalized spherical harmonics coefficients of the gravity field

### From terrestrial water storage (TWS) anomalies

$$\chi_1 = -\frac{1.0966 R_e^2}{C - A} \iint TWS(\varphi, \lambda, t) \sin\varphi \cos\varphi \cos\lambda dS$$

$$\chi_2 = -\frac{1.0966 R_e^2}{C - A} \iint TWS(\varphi, \lambda, t) \sin\varphi \cos\varphi \sin\lambda dS$$

$R_e$ is Earth's mean radius, dS is the surface area and C and A are Earth's principal moments of inertia. The factor 1.0966 accounts for the yielding of the solid Earth to surface load, rotational deformation and core-mantle decoupling (*Eubanks, 1993*)

**HAM from GSM and TWS are not identical in all cases.**

**The reason is not the difference between the methods, but the difference in the input data — for example, whether TWS refers only to land areas, with residual ocean signals being masked.**

# Data – $\Delta C_{21}, \Delta S_{21}$

| Solution type | Solution name | Acronym used here | Provider |
|---|---|---|---|
| GRACE/GRACE-FO only *One year gap in data* | CSR RL06 | GGSM | Center for Space Research, University of Texas at Austin (CSR) |
| | CSR RL06M | GGSMA | Center for Space Research, University of Texas at Austin (CSR) computed from the spherical harmonic representation of the mascon solution |
| Hybrid SLR+DORIS | IGG-SLR+DORIS HYBRID | S02E, S04E,…….S520E | Institute of Geodesy and Geoinformation, University of Bonn (IGG) |
| | IGG-SLR+DORIS HYBRID ensemble mean | SLRDORM | |
| Hybrid SLR | IGG-SLR-HYBRID ensemble mean | SLRM | Institute of Geodesy and Geoinformation, University of Bonn (IGG) |

The solutions are provided as monthly sets of spherical harmonic coefficients.

# Data – TWS

| Solution type | Solution name | Acronym used here | Provider |
|---|---|---|---|
| GRACE/GRACE-FO only *One year gap in data* | CSR RL06M | GTMAGL | Center for Space Research, University of Texas at Austin (CSR) TWS from the mascon solution |
| | CSR RL06M | GTMALN | Center for Space Research, University of Texas at Austin (CSR) TWS from the mascon solution ocean areas masked out |
| | CSR RL06M | GTGSLN | Center for Space Research, University of Texas at Austin (CSR) TWS from GSM coefficients ocean areas masked out |
| Hydrological model | Water – Global Assessment and Prognosis (WaterGAP 2.2e) | WGHM | University of Frankfurt |
| TWS data from assimilation of GRACE and WGHM | Global Land Water Storage Release 2 (GLWS2) | GLWS2 | Institute of Geodesy and Geoinformation, University of Bonn (IGG) |
| TWS data from assimilation of GRACE and WGHM | Global Land Water Storage Release 3 (GLWS3) | GLWS3 | Institute of Geodesy and Geoinformation, University of Bonn (IGG) |
| Hydrological model | Land Surface Discharge Model (LSDM) | LSDM | GeoForschungsZentrum (GFZ) |

# $\Delta C_{21}, \Delta S_{21}$ data – SLR + DORIS Hybrid Solutions

**IGG-SLR-DORIS: Monthly Gravity Field Solution** fields from satellite laser ranging (SLR) and DORIS satellites in which empirical orthogonal functions derived from GRACE data are fitted to the observations.

**SLR satellites:** Lageos-1, Lageos-2, Ajisai, Starlette, Stella (since Nov 1993), Lares (since Dec 2012)

**DORIS satellites:** Spot-2, Spot-3, Spot-4, Spot-5, Envisat, Cryosat-2, HY-2A, Saral, Sentinel-3A, Sentinel-3B (when available)

Löcher, A., Kusche, J., Nie, Y. (2025). *A 40-year record of the Earth's time-variable gravity field from SLR and DORIS. Advances in Space Research*. https://doi.org/10.1016/j.asr.2025.05.089

Löcher, A., Kusche, J,*A hybrid approach for recovering high-resolution temporal gravity fields from satellite laser ranging Journal of Geodesy (2021) 95:6.* https://doi.org/10.1007/s00190-020-01460-x

| File | SH degrees estimated | Number of EOFs estimated |
|---|---|---|
| IGG-SLR-DORIS_Sx+xE | 0 - 5 | 2-20, with step 2 |
| IGG-SLR+DORIS_EnsMean – weighted mean of the 15 solutions (SH degree ≤ 5 + 10–14 EOFs) | | |
| IGG-SLR_EnsMean – weighted mean of the 5 SLR only solutions ( S0+6E, S2+6E, S3+6E, S4+6E, S5+4E) | | |

Sx – number of SH
xE – number of EOF

# Hydrological signal in PM – GAO

Hydrological signal in **PM** excitation – **GAO** is computed as a difference between geodetic angular momentum **(GAM)** and a sum of atmospheric angular momentum **(AAM)** and oceanic angular momentum **(OAM)**:

$$\mathbf{GAO} = \mathbf{GAM} - \mathbf{AAM} - \mathbf{OAM}$$

We use the following datasets ($\chi_1$ and $\chi_2$ components):

- **GAM** based on Earth Orientation Parameters **EOP 14 C04** solution

Two sets of AAM and OAM data are subtracted from GAM, creating two estimates of GAO:

**GAO1**

- **AAM** based on **ECMWF** (European Center for Medium–Range Weather Forecasts) model
- **OAM** based on **MPIOM** (Max Planck Institute Ocean Model) model

**GAO2**

- **AAM** based on **NCEP/NCAR** (National Centre for Environmental Prediction/National Centre for Atmospheric Research) reanalysis model
- **OAM** based on **ECCO** (Estimating the Circulation and Climate of the Ocean) model

**The final GAO is the arithmetic mean of GAO1 and GAO2**

# Analyses

**The emphasis is placed on non-seasonal variations.**

- Non-seasonal variations are obtained by subtracting the seasonal and trend components from the time series.
- Seasonal oscillations are derived by fitting a combination of annual, semi-annual, and terannual signals to the time series using the least squares method.

**Additionally, the non-seasonal series are divided into two categories:**

- variations with periods shorter than 900 days, referred to as *non-seasonal short-term variations*,
- variations with periods longer than 900 days, referred to as *non-seasonal long-term variations*.

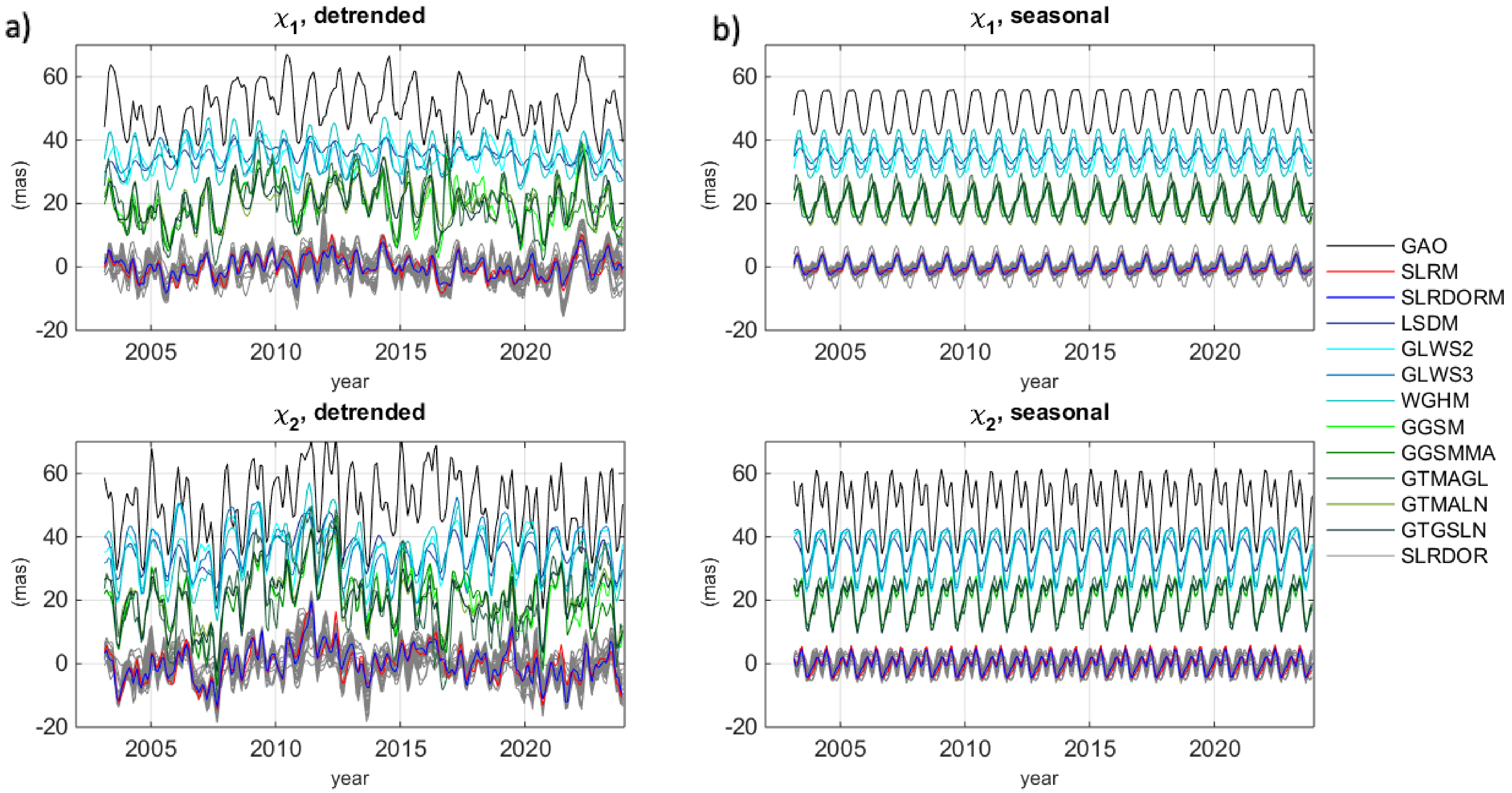
**Our analysis is based on the comparison of:**

- mostly correlation coefficients between model-based excitation functions and GAO (R),
- as well as the percentage of GAO variance explained by model-based excitations (PVE).

**Additional details:**

- The data gap between GRACE and GRACE-FO was filled using the Autoregressive Integrated Moving Average (ARIMA) method.

# Time series comparison



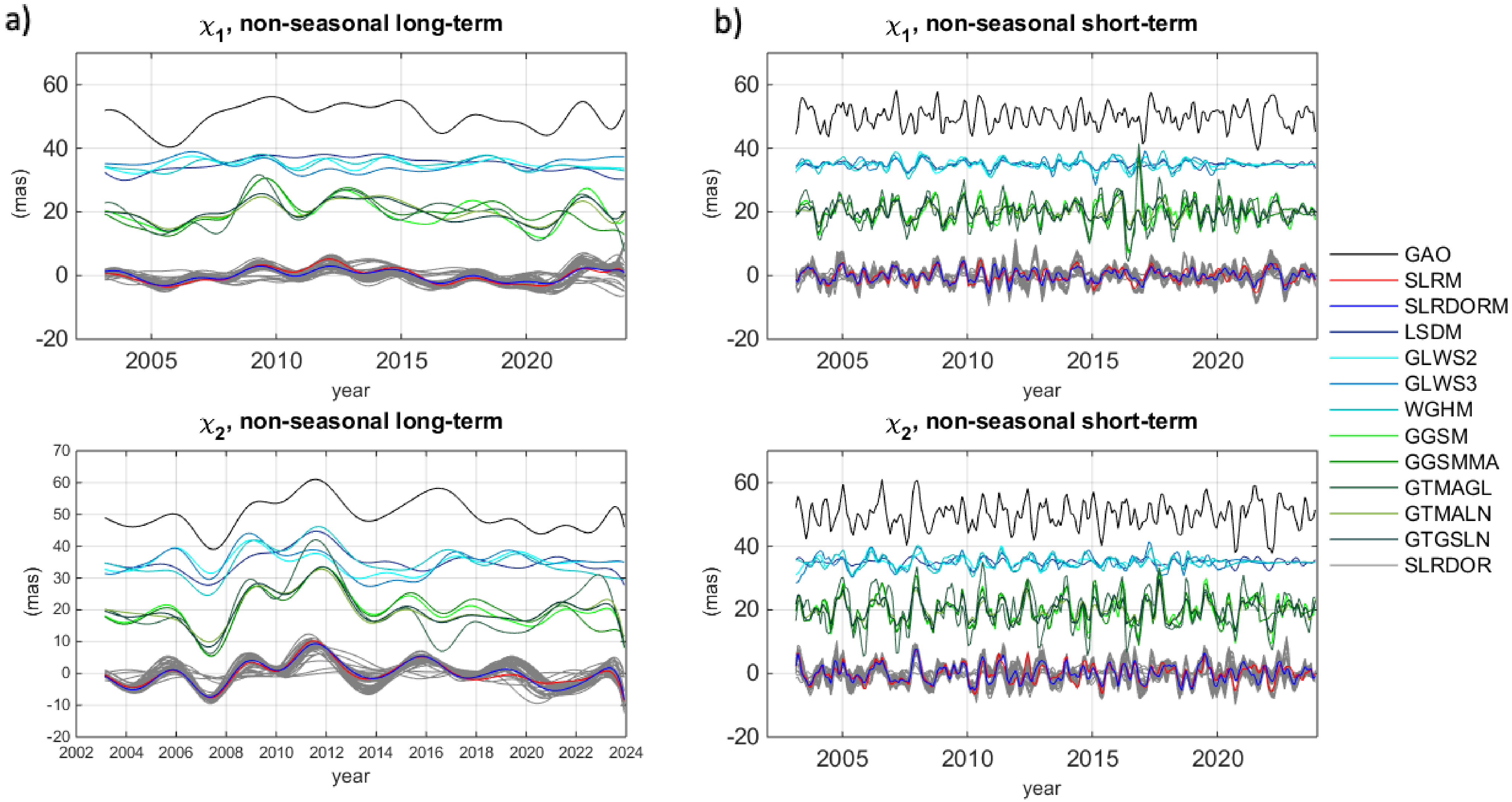

**Acronyms**

**GAO –geodetic residua**

**SLRM** – SLR ensemble mean
**SLRDORM** – SLR + DORIS ensemble mean
**LSDM** – Land Surface Discharge Model
**GLWS2** – Global Land Water Storage, Release 2
**GLWS3** – Global Land Water Storage, Release 3
**GGSM** – GRACE ΔC21, ΔS21 global
**GGSMMA** – GRACE ΔC21, ΔS21 mascon global
**GTMAGL** – GRACE TWS mascon global
**GTMALN** – GRACE TWS mascon land
**GTGSLN** – GRACE TWS GSM coefficients land
**SLRDOR -S02E, S04E, …, S520E** – SLR + DORIS individual solutions

**Fig. 1**. (a) Overall detrended series of $\chi_1$ (top) and $\chi_2$ (bottom), (b) seasonal series of $\chi_1$ (top) and $\chi_2$ (bottom) components of GAO and HAM

# Time series comparison

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**SLRDOR -S02E, S04E, …, S520E** – SLR + DORIS individual solutions

**Fig. 2** (a) Non-seasonal long-term series of $\chi_1$ (top) and $\chi_2$ (bottom), (b) non-seasonal short-term series of $\chi_1$ (top) and $\chi_2$ (bottom) components of GAO and HAM

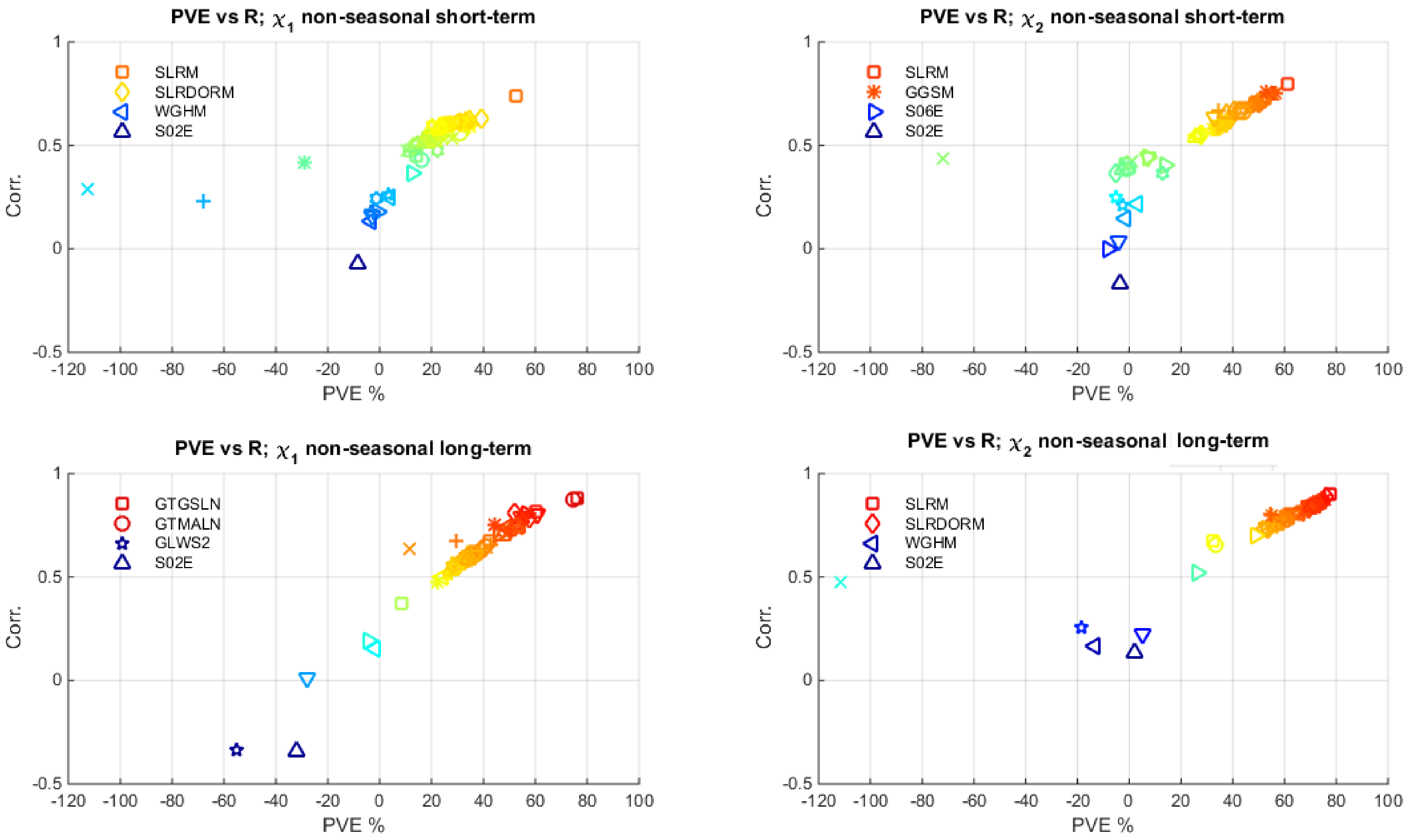
# PVE vs. R

**Highest agreement**

- **SLRM** – SLR ensemble mean
- **SLRDORM** – SLR + DORIS ensemble mean
- **GGSM** – GRACE $\Delta C_{21}$, $\Delta S_{21}$
- **GTMALN** – GRACE TWS mascon lands
- **GTGSLN** – GRACE TWS from GSM coefficients lands

**Low agrrement**

- **S02E, S06E,** SLR + DORIS individual solutions
- **WGHM** – WaterGAP 2.2d
- **GLWS2** – Global Land Water Storage, Release




The models in the upper-right corner of the plot exhibit the highest agreement with GAO.

Models with the highest R values also show the highest PVE values.

**Fig. 3** Percent of variance of GAO explained by HAM vs. correlation between HAM and GAO for: non-seasonal short-term $\chi_1$, non-seasonal short-term $\chi_2$, non-seasonal long-term $\chi_1$, non-seasonal long-term $\chi_2$

# Correlations GAO vs HAM

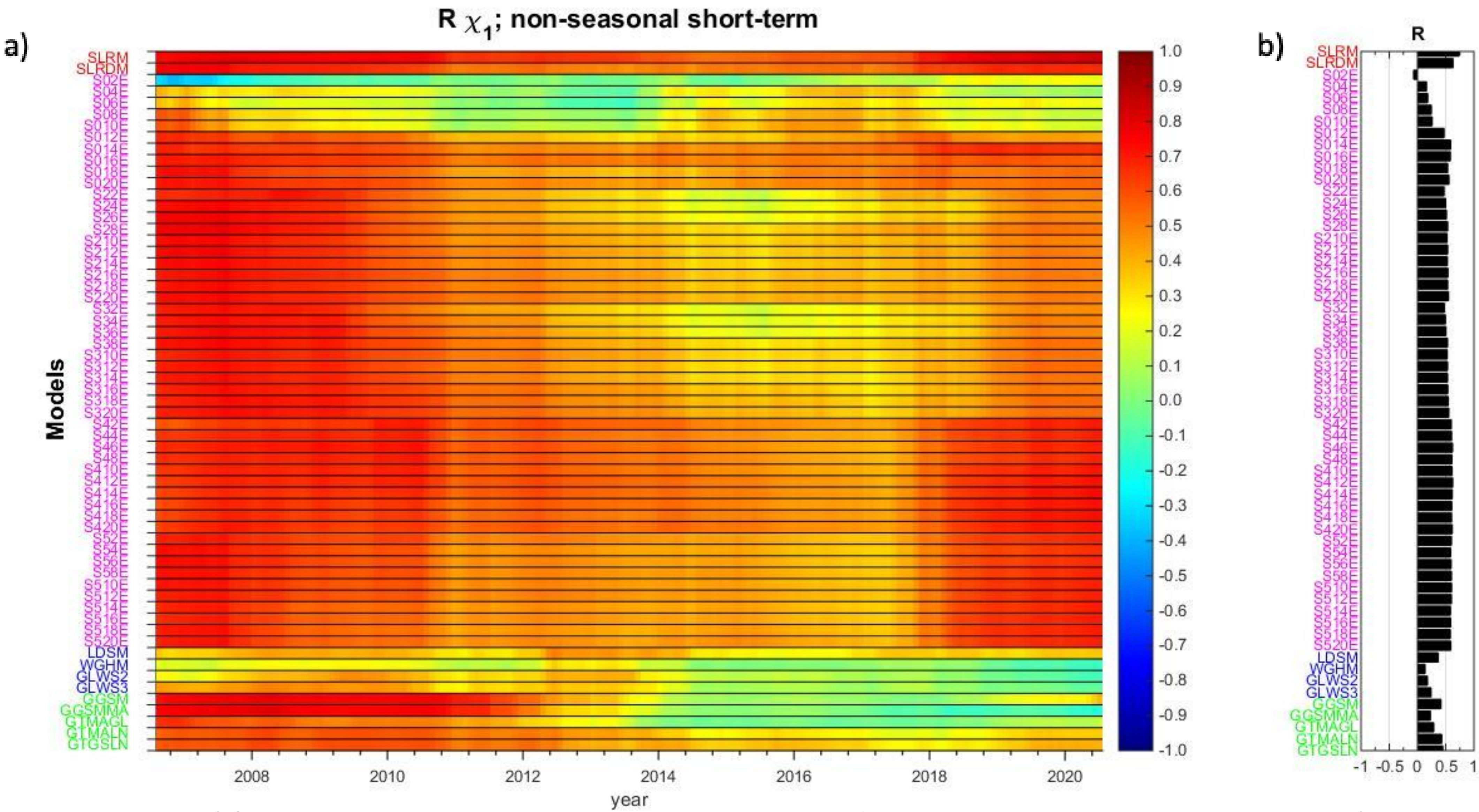



**Fig. 4.** (a) Time-variable correlation between GAO and HAM for non-seasonal short-term $\chi_1$ variations (7-year window, shifted by 30 days). (b) Corresponding correlation coefficients

Max values

SLRM (Mean SLR)

R=0.74 ± 0.16

SLRDORM (Mean SLR+DORIS)

R=0.63 ± 0.16

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- **LSDM** – Land Surface Discharge Model
- **GGSM** – GRACE $\Delta C_{21}$, $\Delta S_{21}$ global
- **GGSMA** – GRACE $\Delta C_{21}$, $\Delta S_{21}$ mascon global
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- **GTMALN** – GRACE TWS mascon land
- **GTGSLN** – GRACE TWS GSM coefficients land

# Correlations GAO vs HAM

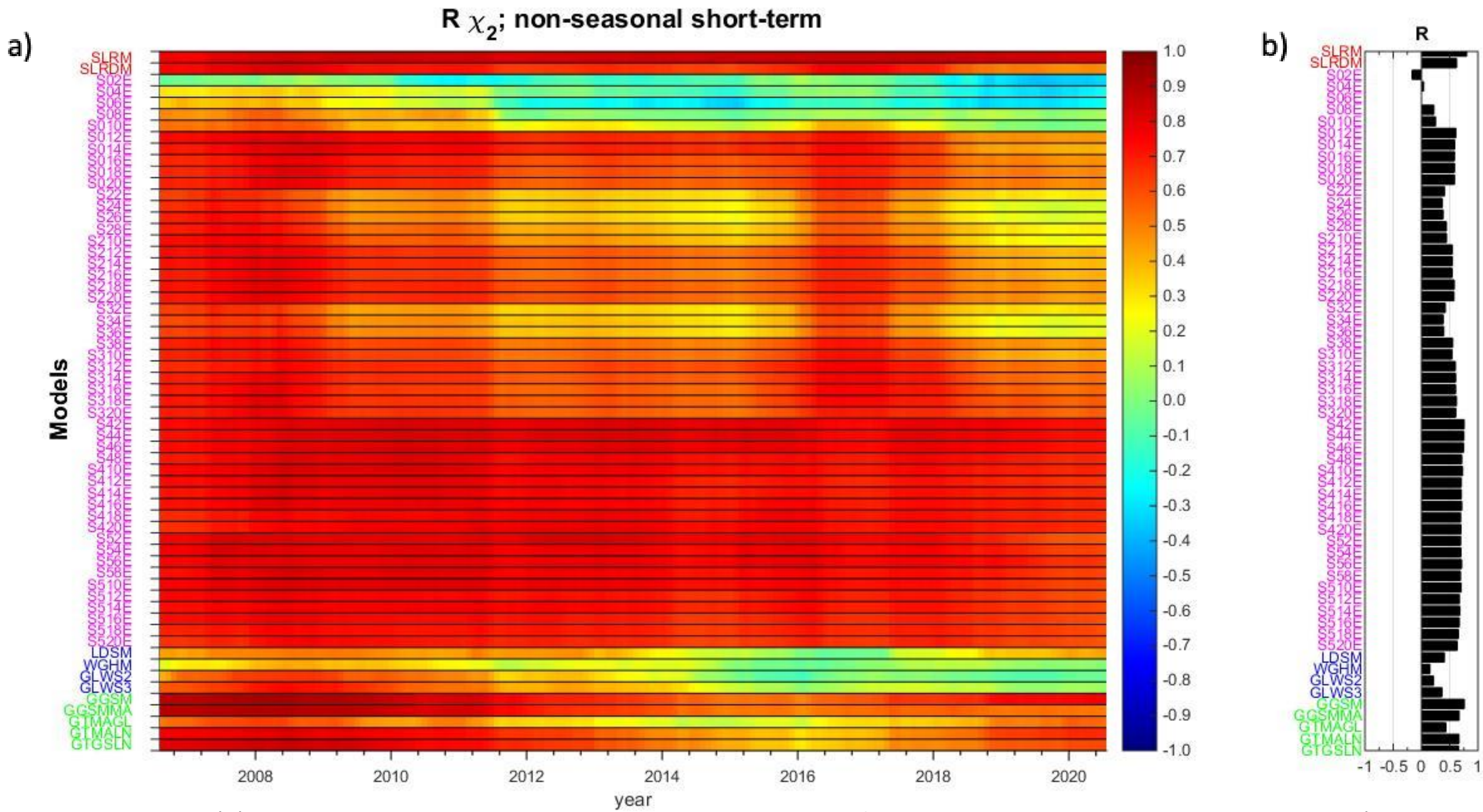


**Fig. 5.** (a) Time-variable correlation between GAO and HAM for non-seasonal short-term $\chi_2$ variations (7-year window, shifted by 30 days). (b) Corresponding correlation coefficients

<u>Max values</u>

SLRM (Mean SLR)

R=0.80 ±0.20

GGSM

(GRACE $\Delta C_{21}$, $\Delta S_{21}$)

R=0.76 ±0.20

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# Correlations GAO vs HAM

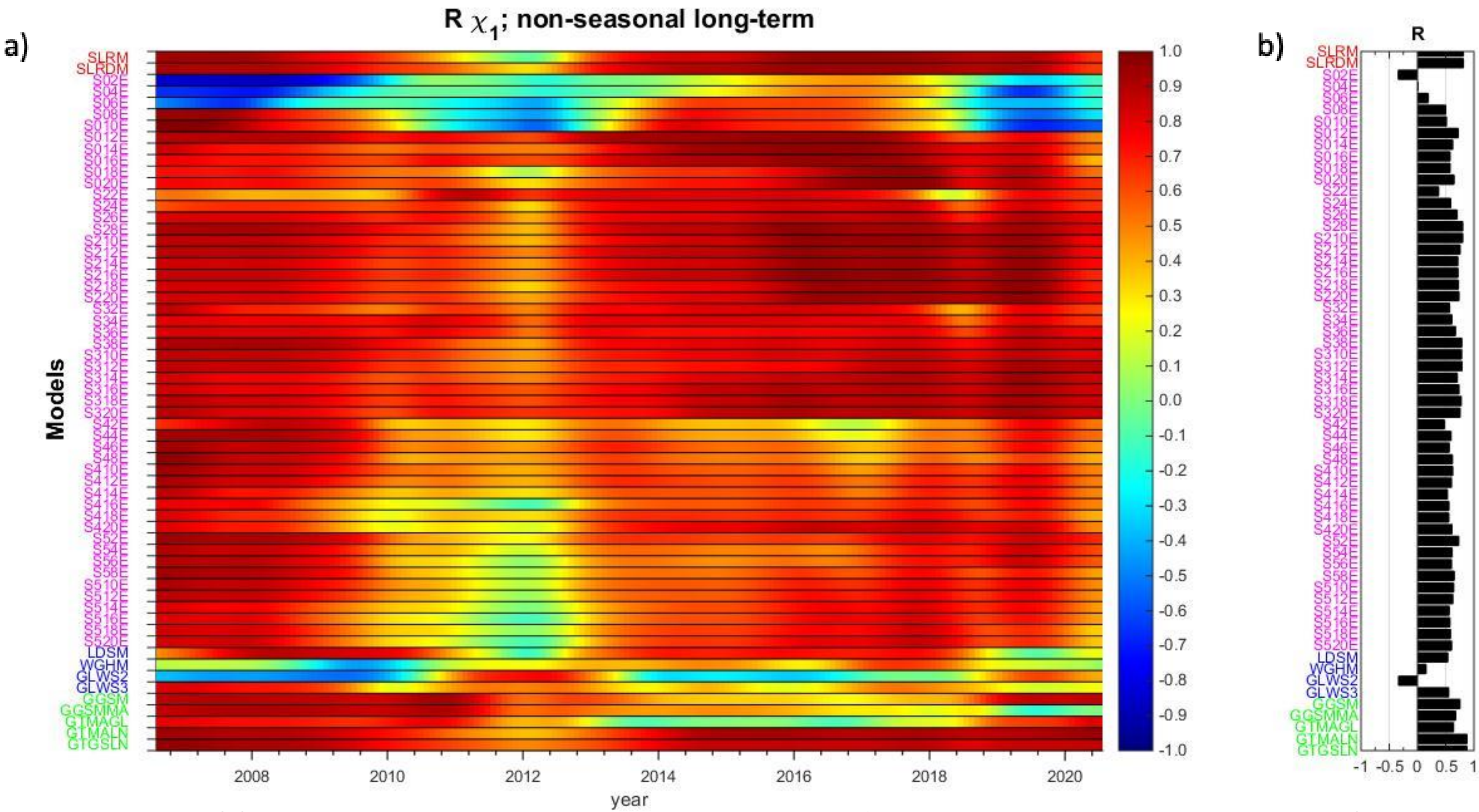


**Fig. 6.** (a) Time-variable correlation between GAO and HAM for non-seasonal long-term $\chi_1$ variations (7-year window, shifted by 30 days). (b) Corresponding correlation coefficients

Max values

GTGSLN (GRACE TWS)

R=0.88 ±0.23

GTMALN (GRACE TWS MASCONS)

R=0.88 ±0.23

**Acronyms**

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# Correlations GAO vs HAM

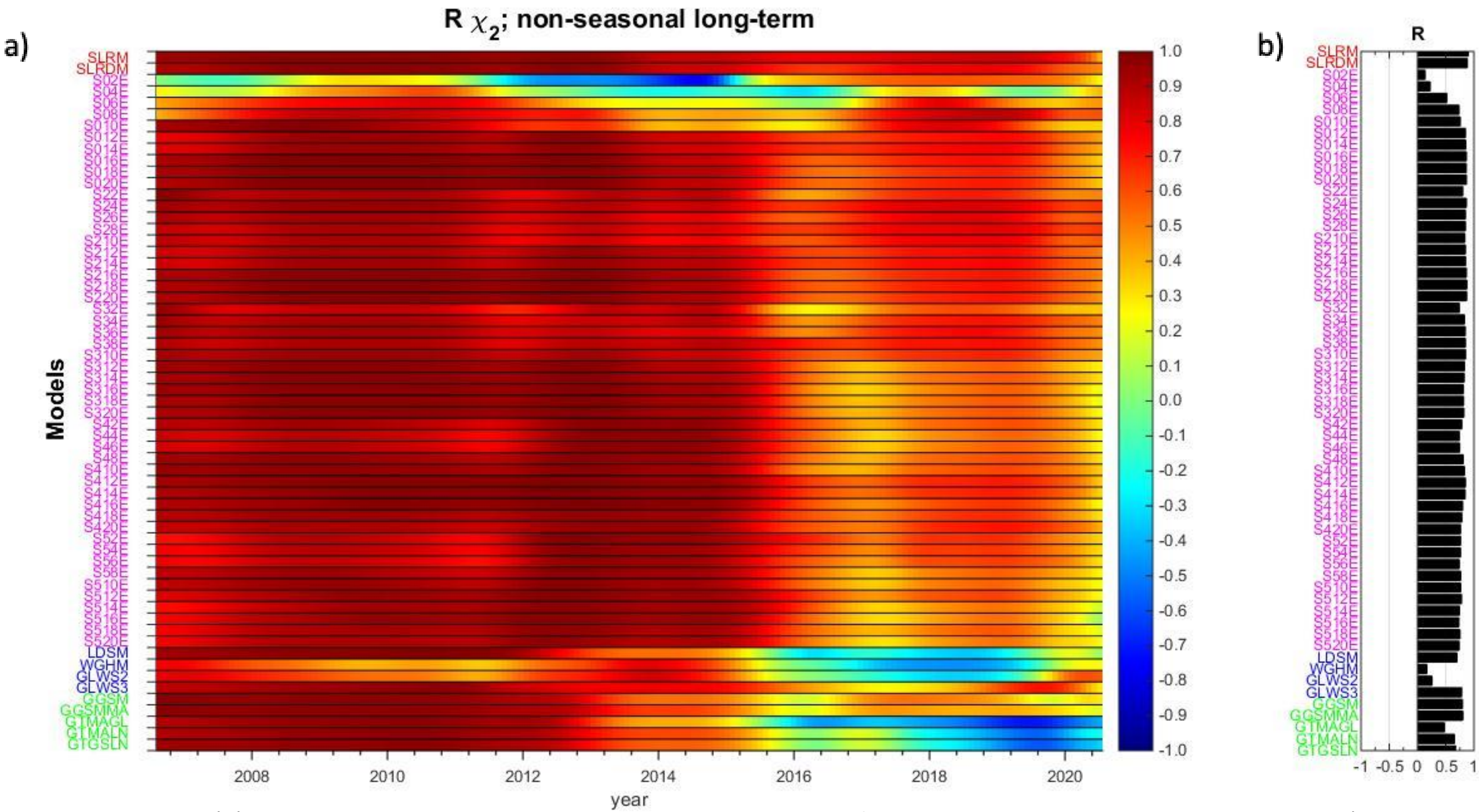


**Fig. 7** (a) Time-variable correlation between GAO and HAM for non-seasonal long-term $\chi_2$ variations (7-year window, shifted by 30 days). (b) Corresponding correlation coefficients

<ins>Max values</ins>

SLRM (Mean SLR)

R=0.90 ±0.17

SLRDORM (Mean SLR+DORIS)

R=0.88+0.17

**Acronyms**

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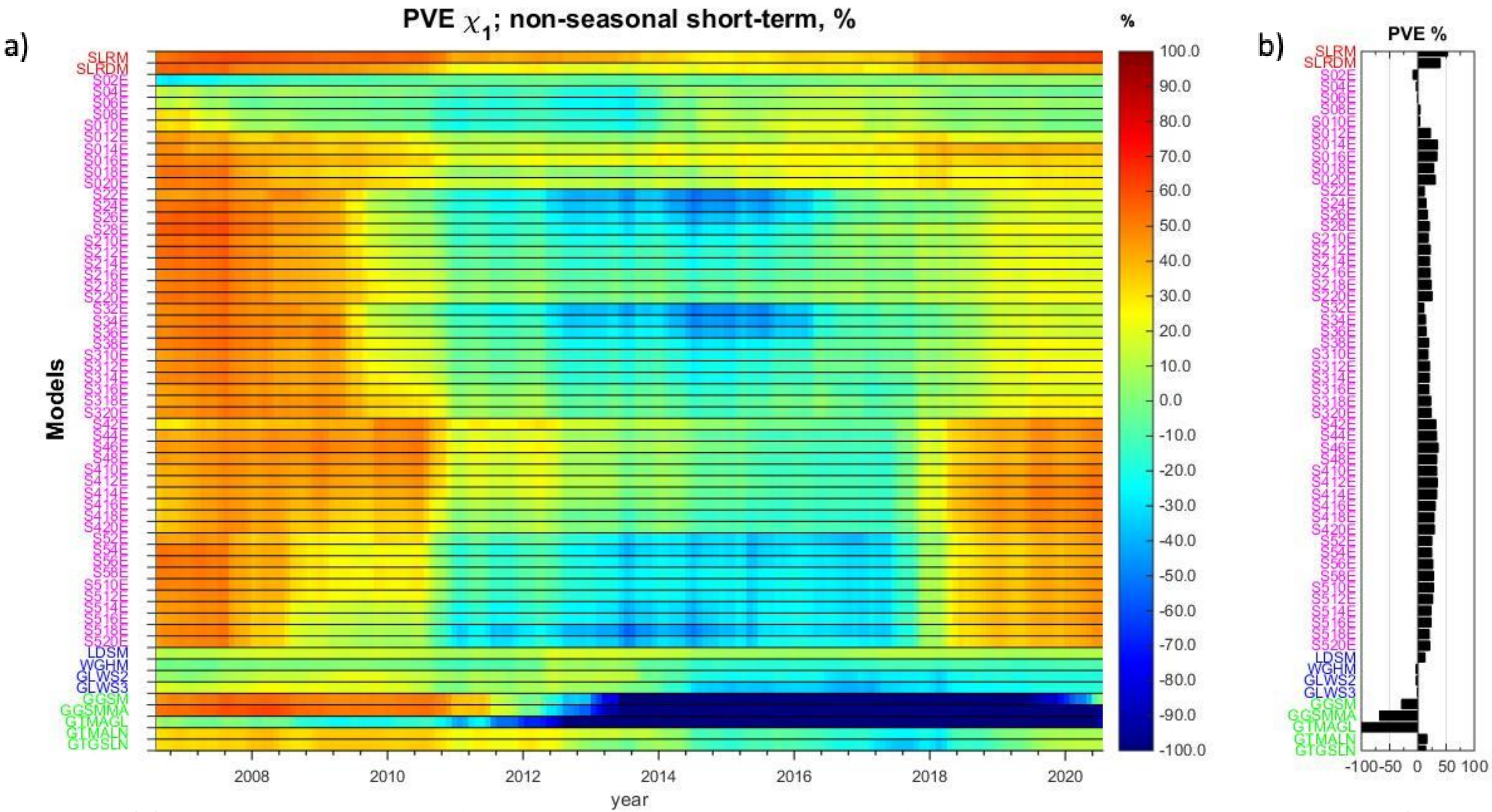
# Time variable percent of variance explained




<u>Max values</u>

SLRM (Mean SLR)

PVE=52.6±24.2 %

SLRDORM (Mean SLR+DORIS)

PVE=39.4 ±24.2%

**Acronyms**

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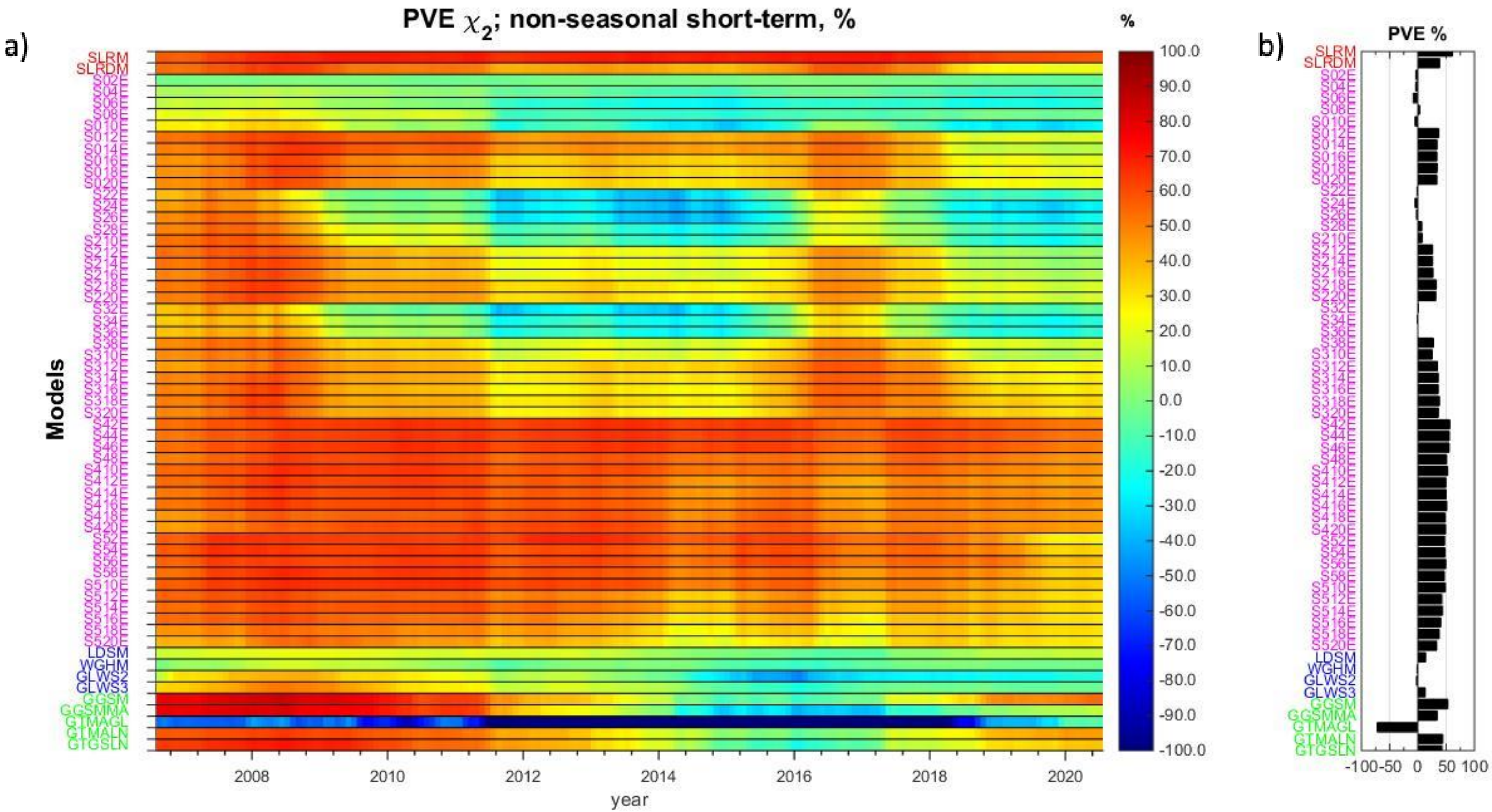
- **GTGSLN** – GRACE TWS GSM coefficients land

**Fig. 8** (a) Time-variable percent of variance of GAO explained by HAM for non-seasonal short-term $\chi_1$ variations (7-year window, shifted by 30 days). (b) Corresponding percent of variance explained

# Time variable percent of variance explained




Max values

SLRM (Mean SLR)

PVE=61.2 ±24.1%

SLRDORM (Mean SLR+DORIS)

PVE =53.1 ±24.1%

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- **GTMALN** – GRACE TWS mascon land
- **GTGSLN** – GRACE TWS GSM coefficients land

**Fig. 9** (a) Time-variable percent of variance of GAO explained by HAM for non-seasonal short-term $\chi_2$ variations (7-year window, shifted by 30 days). (b) Corresponding percent of variance explained

# Time variable percent of variance explained

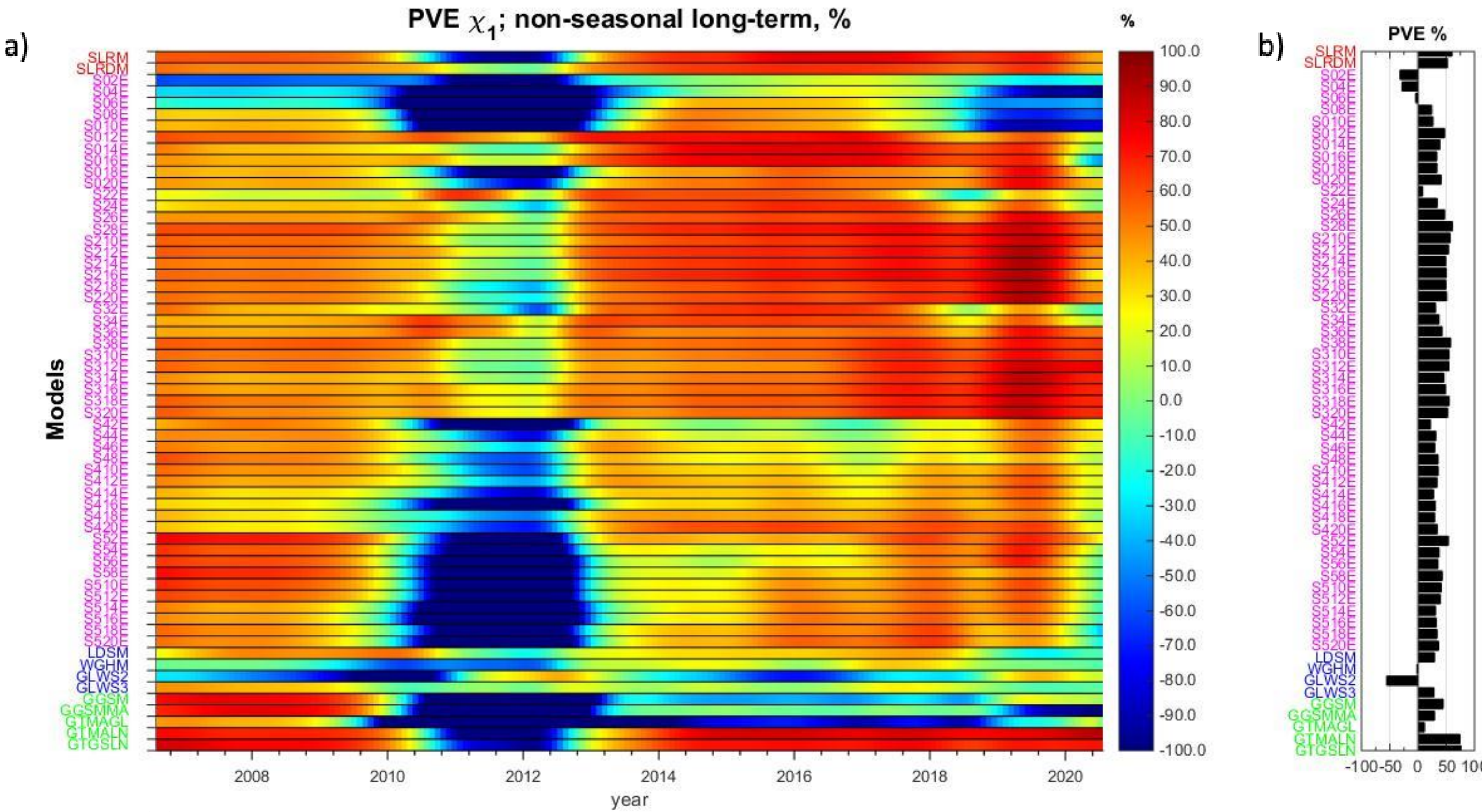


**Fig. 10** (a) Time-variable percent of variance of GAO explained by HAM for non-seasonal long-term $\chi_1$ variations (7-year window, shifted by 0.5 yr). (b) Corresponding percent of variance explained

Max values

GTGSLM (GRACE TWS)

PVE=75.4 ±22.8%

GTMALN (GRACE TWS MASCONS)

PVE=74.5 ±22.8%

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# Time variable percent of variance explained

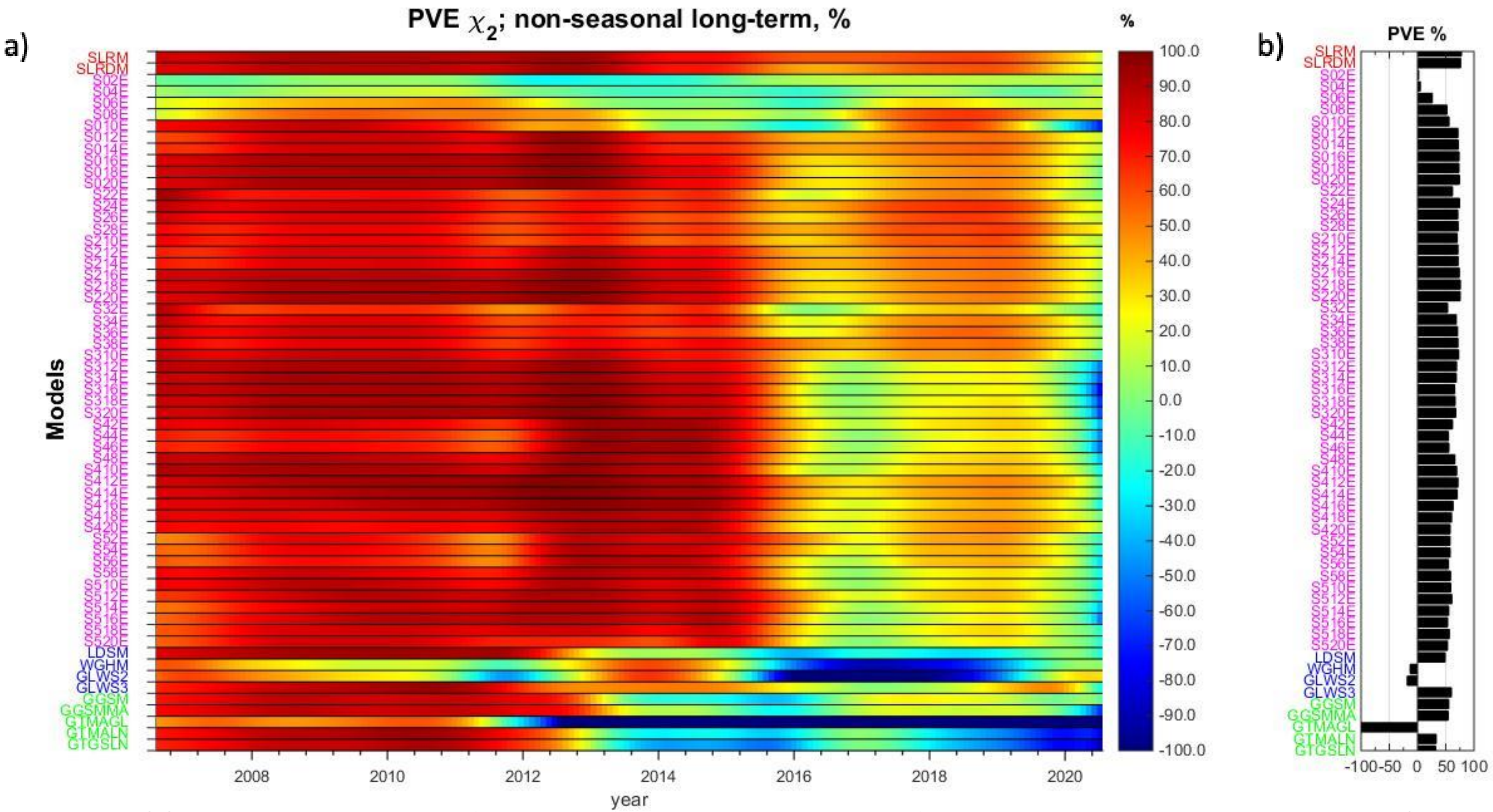


**Fig. 11** (a) Time-variable percent of variance of GAO explained by HAM for non-seasonal long-term $\chi_2$ variations (7-year window, shifted by 30 days). (b) Corresponding percent of variance explained

<u>Max values</u>

SLRM (Mean SLR)

PVE=77.9+29.8%

SLRDORM (Mean SLR+DORIS)

PVE =76.4 ±29.8%

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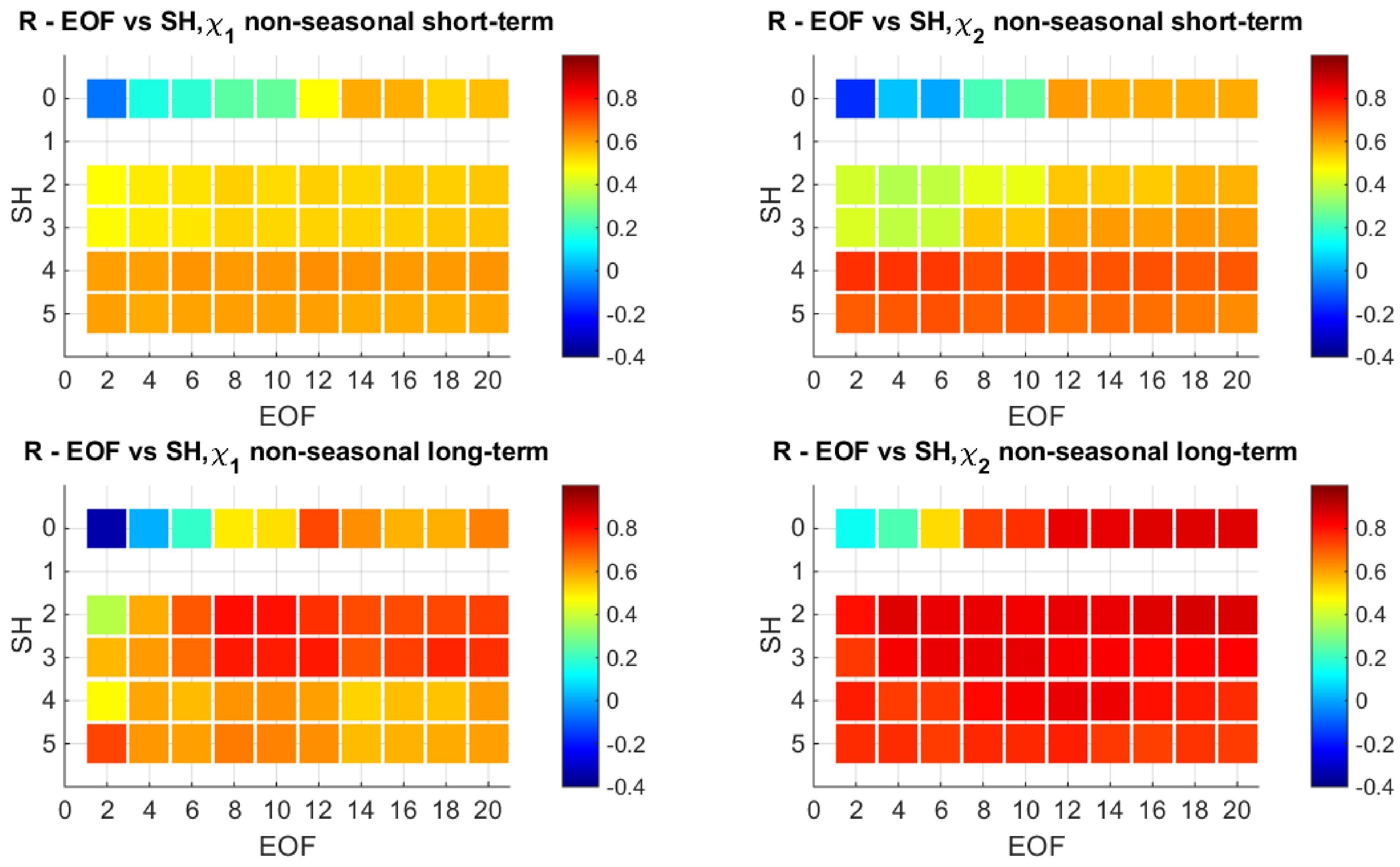
# R vs. SH expansions and EOFs (hybrid data only)




With a low number of SHs, the correlation (R) between GAO and model-derived HAM increases as more EOFs are added. However, with a higher number of SHs — especially for $\chi_2$ in the long term — the correlation tends to saturate.

**Fig. 12** Correlation coefficient GAO vs HAM (R) vs. number of SH expansions and EOFs in the IGG-SLR-DORIS Monthly Gravity Field Solution .

# PVE vs. SH expansions and EOFs (hybrid data only)

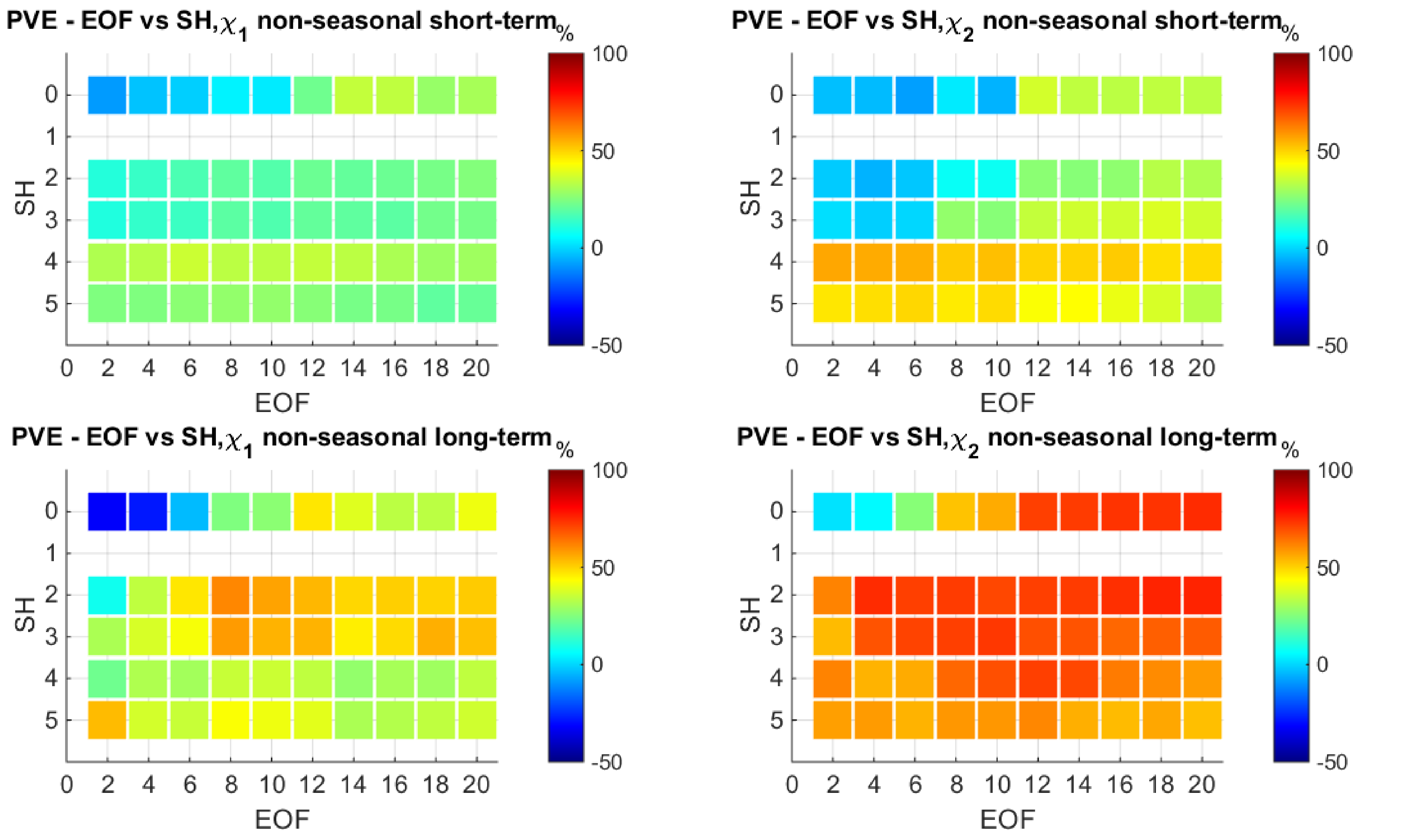



**Fig. 13** Percent of variance explained (PVE) vs. number of SH expansions and EOFs in the hybrid model

# Conclusions

- The results confirm our previous findings, highlighting the potential of hybrid solutions for analysing HAM, particularly in the non-seasonal spectral band.
- For HAM obtained from hybrid solutions, strong correlations with GAO were found: 0.7 (in $\chi_1$) and 0.8 (in $\chi_2$) for non-seasonal short-term variations, and up to 0.9 for non-seasonal long-term changes (in both $\chi_1$ and $\chi_2$). The highest correlations, especially for shorter periods, were observed in the series based on the weighted average of SLR data. These values exceeded those obtained from the weighted average of SLR+DORIS data.
- However, analysis of changes in correlation and in percent of variance explained over time indicates several periods of poorer agreement between HAM and GAO, for example around 2010/2011 in the case of non-seasonal long-term variations in $\chi_1$.
- For hybrid solutions, the level of consistency between HAM and GAO also depends on the number of spherical harmonic (SH) expansions and empirical orthogonal functions (EOFs) in the hybrid model.
- Further research is necessary, particularly focusing on the time intervals where both correlation and percent of variance explained between HAM and GAO were weakest, to better understand and improve the consistency of hybrid solutions in these periods.

# Thank you

# Acronyms

| Acronym | Description |
|---|---|
| SLRM | SLR ensemble mean |
| SLRDORM | SLR + DORIS ensemble mean |
| S02E, S04E, …, S520E | SLR + DORIS individual solutions |
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| GLWS3 | Global Land Water Storage, Release 3 |
| LSDM | Land Surface Discharge Model |
| GGSM | GRACE $\Delta C_{21}$, $\Delta S_{21}$ global |
| GGSMMA | GRACE $\Delta C_{21}$, $\Delta S_{21}$ mascon global |
| GTMAGL | GRACE TWS mascon global |
| GTMALN | GRACE TWS mascon land |
| GTGSLN | GRACE TWS GSM coefficients land |