# Assimilating GRACE/GRACE-FO Observations into JPL Terrestrial Reference Products.

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# 1. Outline

- Report on the progress made on the project aimed at assimilating GRACE and GRACE-FO (GGFO) data into JPL Terrestrial Reference Frame (JTRF) products (see e.g. JTRF2020).
- A few methodological insights on JTRF products.
- How to **assimilate** GGFO data into JTRF analyses.
- GGFO Data sets adopted in the analyses.
- From GGFO data to elastic displacements.
- Some preliminary results.

# The JPL Terrestrial Reference Frame.

# 2. JPL Terrestrial Reference Frame (JTRF) Products

- TRFs are, in essence, catalogues of observing stations whose Cartesian coordinates w.r.t an underlying reference system are accurately known.
- JTRF products, unlike standard products such as the ITRF, are quasi-instantaneous frames represented through time series of station positions, EOPs, and Helmert transformation parameters.
- We do that by adopting a SRI (square-root information) filter and DMC (Dyer-McReynolds) smoother algorithm. We called it SREF - **S**quare-root **R**eference frame **E**stimation **F**ilter.
- JTRF products are computed assimilating station position, EOP, and local tie observations from GNSS, VLBI, SLR, and DORIS (cf Slide [8] to get a sense of the station distribution of the space-geodetic global network).

# 3. The JTRF State Vector

- The frame state adopted in **SREF** consists of *station-related* and **network-related**, parameters such as the **EOPs** and the **Helmert transformation** parameters.
- If we focus on the *station-related* portion of the forward dynamic equations:

$$\begin{bmatrix} x_{k+1} \\ v_{k+1} \\ s_{k+1} \\ c_{k+1} \\ a_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \Delta t & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \omega\,\Delta t & 0 \\ 0 & 0 & -\omega\,\Delta t & 1 & 0 \\ 0 & 0 & 0 & 0 & \phi \end{bmatrix}}_{\mathbf{F}(\Delta t, \omega, \phi)} \begin{bmatrix} x_k \\ v_k \\ s_k \\ c_k \\ a_k \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ w_k \end{bmatrix} \qquad (1)$$

where $(x_k, v_k)$ are the trend parameters, $(s_k, c_k)$ the oscillators, and $a_k$ are the *(stochastic)* parameters of a first-order auto-regressive process -AR[1].

# How to assimilate GGFO Data into JTRF products

# 4. JTRF and GRACE/GRACE-FO Data

- SREF recursively estimates the frame state parameters reported in Eq [1] where the state oscillators $(s_k, c_k)$ represent the non-tidal seasonal oscillations.
- GGFO data can be directly used to inform and control the way in which the space-geodetic network deforms due to surface mass redistribution (i.e. non-tidal loading).
- Assuming non-tidal loading $u_k(x)$ can be reliably determined from GGFO data, then it could be used as control input driving the state-space dynamics:

$$\begin{bmatrix} x_{k+1} \\ v_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ v_k \end{bmatrix}}_{\bar{\mathbf{F}} \cdot \mathbf{x_k}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_k \\ 0 \end{bmatrix}}_{\mathbf{G} \cdot \mathbf{u}_k} + \begin{bmatrix} w_k \\ 0 \end{bmatrix} \tag{2}$$

- The control input $\mathbf{G}u_k$ -a proxy for the seasonal oscillators of Eq [1]- drives the load-responsive components of the TRF states without treating GGFO data as observational constraints.

# GRACE/GRACE-FO Data Sets

# 5. GGFO Data Sets

1. **JPL** *monthly* Mascon Solution RL06.1v3.
   - JPL-RL06M are **gridded** data types reporting surface mass changes in Equivalent Water Thickness with a **spatial sampling** of 0.50° in both latitude and longitude [6].
2. **CSR** *monthly* Mascon Solution RL06.02.
   - State-of-the-art CSR Mascon solution reporting surface mass changes in Equivalent Water Thickness on a 0.25° lon-lat grid.

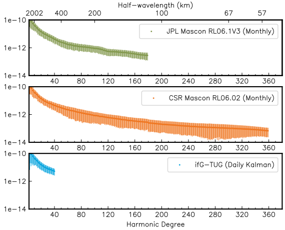
- **Why Mascons instead of Spherical Harmonic Solutions**?
  The Mascon formulations allow direct use of their gravity field products without signal attenuation from smoothing and de-striping.

3. **ifG-TUG** (Daily SHM fields) ITSG-Grace2018.
   - ifG-TUG are daily SHM solutions generated at TU Graz (Austria) and computed with a Kalman filter and smoother [3]. Although band-limited in the interval $2 \leq \ell \leq 40$, the daily TUG solution is well-suited for TRF analyses (JTRF also adopts a daily time step).

# 6. Spectral Properties of GGFO Solutions at a glance

- Power Spectrum as a function of the Harmonic Degree:

$$P_\ell = \sum_{m=-\ell}^{+\ell} |c_{lm}|^2 \quad (3)$$




- JPL-RL06M has been expanded in SH up to $\ell = 180$
- The higher resolution CSR-RL06 has also been expanded in SH up to $\ell = 360$
- In the Mascon products from JPL and CSR, the geophysical signal is approximately concentrated within the spectral band $2 \leq \ell \leq 70$.
- Although characterized by lower resolution, TUG solution in the band $2 \leq \ell \leq 40$ appears to be fully consistent with JPL's and CSR's.

Across-time Median amplitude (square root of Equation 3) spectra for JPL, CSR, and TUG solutions. De-aliasing products were restored in all solutions and linear trends removed from the Stokes coefficients.

# GGFO-derived non-tidal Elastic Displacements

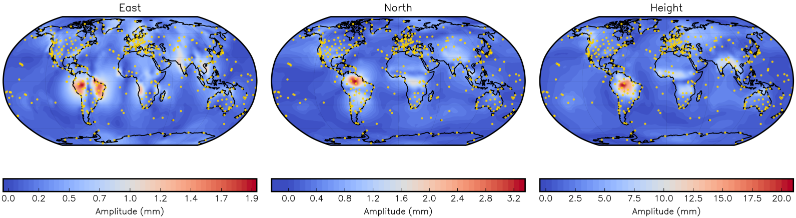
# Elastic Displacements $u(\mathbf{x})$ from GGFO Data

- If the surficial load admits a representation in **real spherical harmonics (rSH)** $\mathcal{Y}_{\ell m}$, then

$$u(\mathbf{x}) = \underbrace{\left(\sum_{lm} U_{lm}\mathcal{Y}_{lm}\right)\mathbf{e_r}}_{\text{Radial Component}} + \underbrace{\left(\sum_{lm} V_{lm}\mathcal{Y}_{lm}\right)\mathbf{e_t}}_{\text{Tangential Component}} \quad , \quad \mathbf{l \geq 2}$$

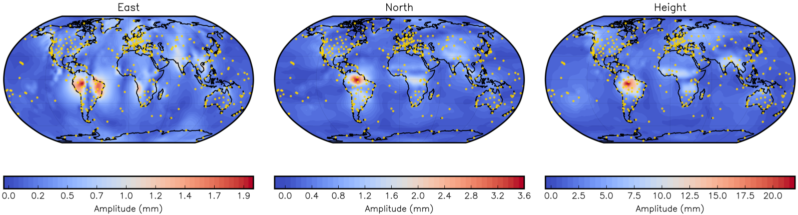
  where the rSH coefficients $(U_{lm}, V_{lm})$ encode Earth's elastic response and are functions of the load Love numbers $(h_l^{'}, l_l^{'})$.
- JPL and CSR Mascon solutions were expanded in band-limited $0 \leq \ell \leq 180(360)$ rSH coefficients, subsequently converted into Stokes, i.e. geopotential, coefficients [4].
- Non-tidal de-aliasing products (ocean and atmosphere) have been restored [1] and the Stokes coefficients were finally detrended.
- Elastic displacements $u(\mathbf{x})$ were computed by using load Love numbers derived from PREM [5].

# 8. Elastic Displacements u(x) from JPL-RL06M




Spatial variability of the **annual oscillation** of the elastic displacements from JPL Mascon-RL06 determined through the load Love numbers outlined in [5] and based upon PREM [2]. Non-tidal variability of Ocean and Atmosphere has been restored into the Mascons. The elastic displacements deliberately exclude $\ell = 1$ coefficients and hence do not reflect the degree-1 surface deformation. Yellow dots represent the location of space-geodetic stations used to determine JTRF.

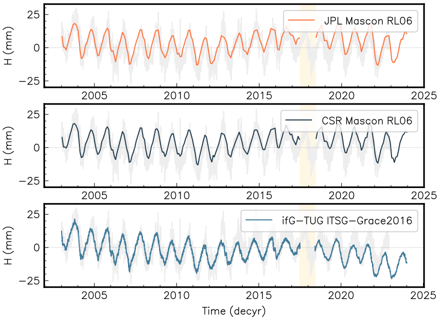
# 9. Elastic Displacements u(x) from CSR-RL06M




Spatial variability of the **annual oscillation** of the elastic displacements from CSR Mascon-RL06 determined through the load Love numbers outlined in [5] and based upon PREM [2]. Non-tidal variability of Ocean and Atmosphere has been restored into the Mascons. The elastic displacements deliberately exclude $\ell = 1$ coefficients and hence do not reflect the degree-1 surface deformation. Yellow dots represent the location of space-geodetic stations used to determine JTRF.

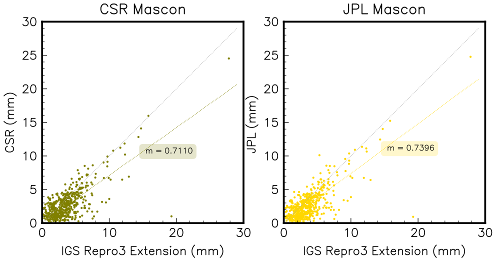
# Assessing the agreement between frame input data and GGFO-derived elastic displacements

# 10. The IGS GNSS Station at BRAZ, Brasilia, Brazil




Up component of the GGFO-derived elastic displacements determined at the IGS station BRAZ, Brazil. The site is characterized by markedly evident seasonal oscillations due to the intense hydrological cycle. The plot shows and compares $u_k$ determined from JPL (top), CSR (middle) Mascons, and TUG (bottom) daily solutions. Superimposed in gray is the up detrended component as measured by GNSS.

# 11. GGFO vs GNSS Annual Amplitudes.




The scattergrams visualize the correlation of the amplitude of the annual signals for all of the GNSS stations included in JTRF (roughly 530) and estimated on the GGFO-derived elastic displacements and the GNSS observations. The *m* values reported in the plots represent the correlation between the two data types.

# 12. Work in Progress

- GGFO -elastic displacements appear to agree, on average, reasonably well with the observed displacements as measured by GNSS. This suggests that GGFO assimilation in JTRF combination scheme is feasible.
- We're actively working on implementing the model of Eq [2] which will allow us to incorporate GGFO data into the JTRF assimilation scheme.
- For a discussion on the role of GGFO-elastic displacements in improving predictability of station positions in an analogous Kalman filter framework, the reader is invited to consult the paper by Chin et al (B.4 Solid Earth Sciences - Oct 9 2025)
- Plan to update JPL and CSR Mascon solutions with the latest solutions.

# 13. Acknowledgements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- For any comment, question or specific request, please feel free to contact us at jtrf@jpl.nasa.gov.

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