Purely Data-Driven Sub-Weekly Mass Change Models from GRACE Residuals

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GRACE-FO 2025 Science Team Meeting

TU Delft

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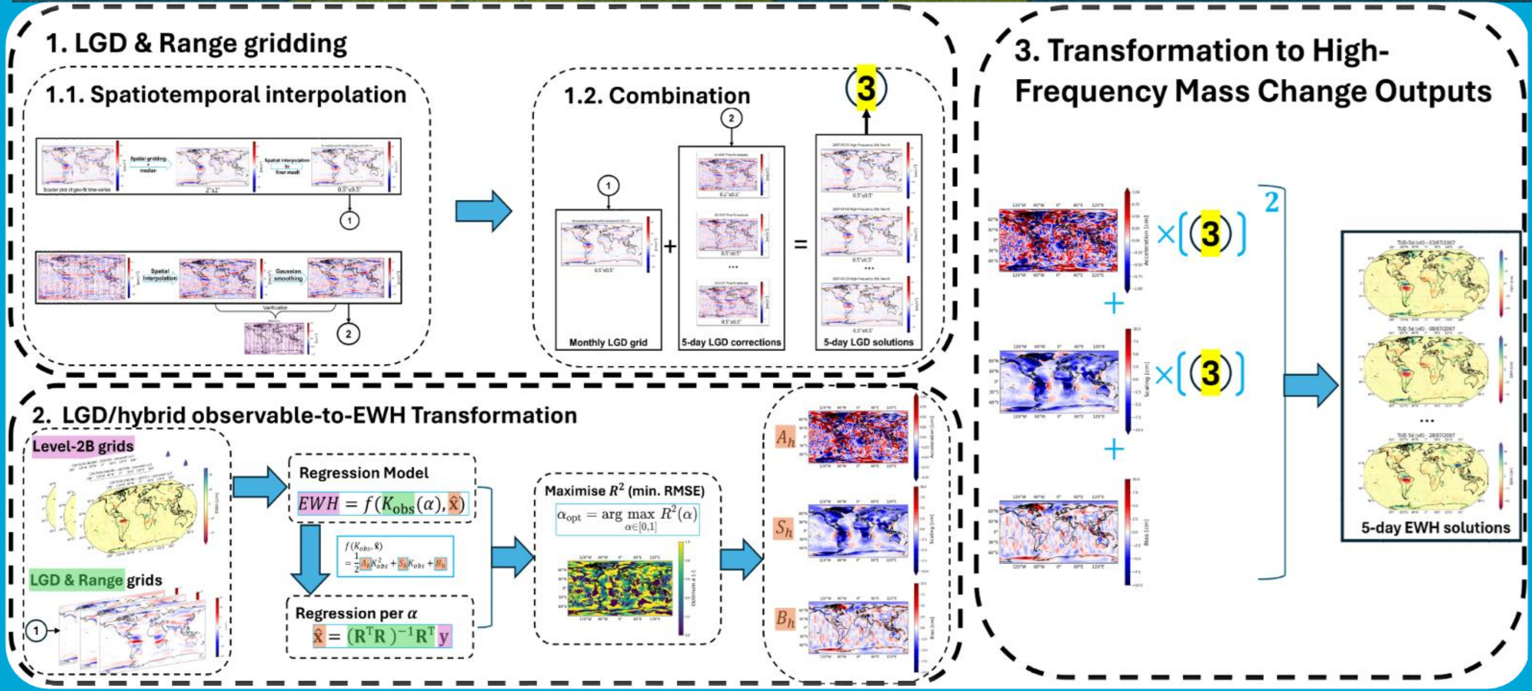
JWG C10. Tailored Parameterization Strategies for ClimateApplications of Satellite Gravimetry



Introduction

Overall research goal

Design high-frequency mass change models and constrain them with GRACE residual Level-1B data.

Current goals

1. Construct *GRACE-only* sub-weekly mass change models using *purely* post-fit range-rate data.
2. Estimate associated uncertainties.

Presentation outlook

- Along-orbit residual Line-of-sight Gravity Difference
- Novel High-Frequency Methodology
- 5-day solutions results
- Preliminary uncertainty estimations

LGDs from L1B post-fit data

"Post-fit" range rate vector:

$\hat{\boldsymbol{l}} = \boldsymbol{A}\hat{\boldsymbol{x}}$

$\hat{\boldsymbol{l}}$: post-fit residual range rate,
$\boldsymbol{A}$: design matrix,
$\hat{\boldsymbol{x}}$: estimated parameter vector.

Residual Level-1B (post-fit) data (RL06 CSR*):

- Geo-fit range-rate:
  → part of the post-fit range rate ***only*** containing the monthly gravity signal.

- Post-fit residuals: $\delta\boldsymbol{l} = \boldsymbol{l} - \boldsymbol{A}\,\hat{\boldsymbol{x}}.$
  → Contain sub-monthly gravity signals (w.r.t monthly mean) and background model errors.

Obtaining residual Line-of-sight-Gravity Difference (LGD):

- Step 1: apply numerical differentiation → range-accelerations $\delta\ddot{\rho}$,
- Step 2: apply band-pass filter to remove KBR noise and long wavelengths $[0.9, 11]$ mHz,
- Step 3: apply frequency-based Transfer Function by *Ghobadi-Far, K. et al 2018*, denoted as $Z(f)$, to remove residual centrifugal acceleration.

$$\delta g_{\mathrm{AB}}^{\mathrm{LoS}}(t) \approx \mathrm{F}^{-1}\{Z(f)\mathrm{F}\{\delta\ddot{\rho}(t)\}\}$$

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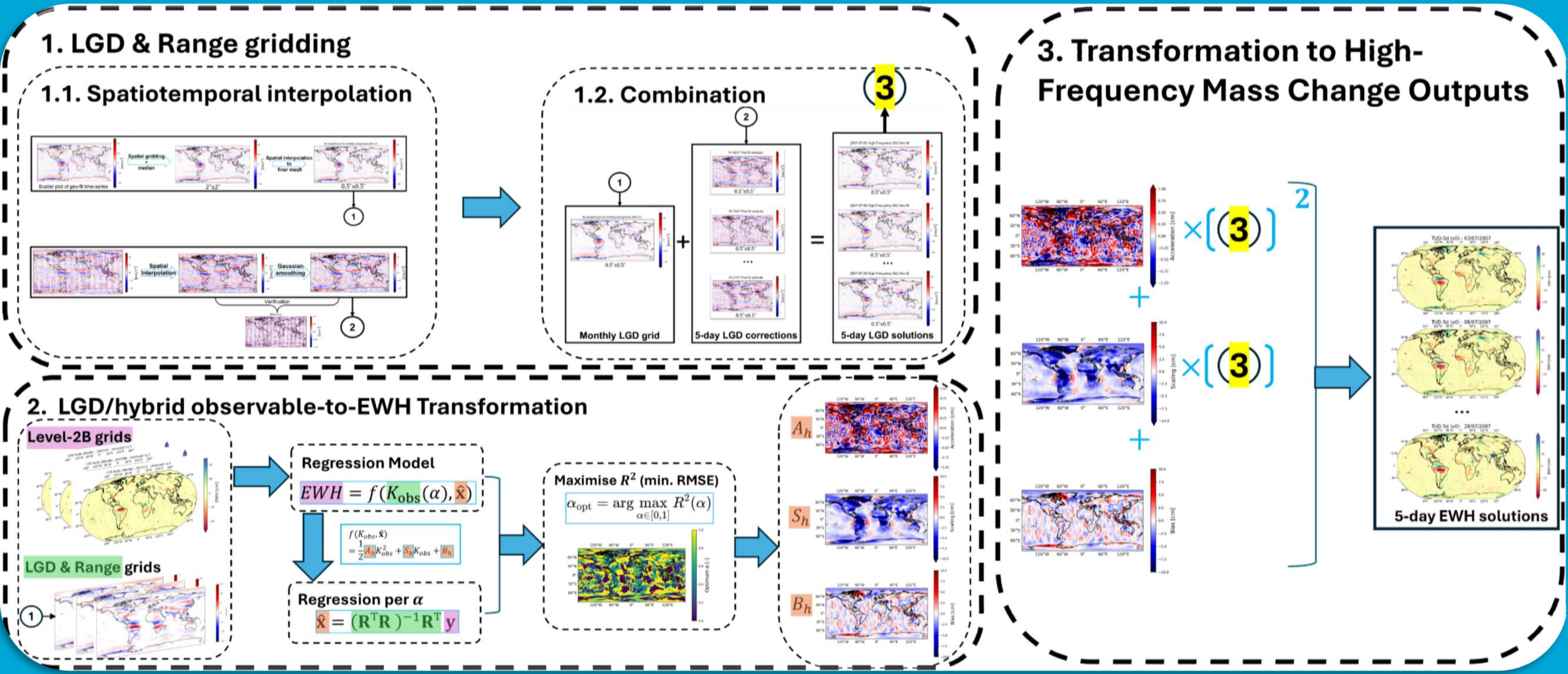
** We thank CSR for providing us their pre-fit/post-fit data from RL06.*



Novel Framework (core method)

Uncertainty estimation

$$\sigma^2_{\widehat{EWH}} = \left(\frac{\partial EWH}{\partial K_{\text{obs}}}\right)^2 \sigma^2_{K_{\text{obs}}} + \boldsymbol{p}^T \boldsymbol{\Sigma}_{\hat{x}} \boldsymbol{p} + \sigma^2_{EWH,L2}$$

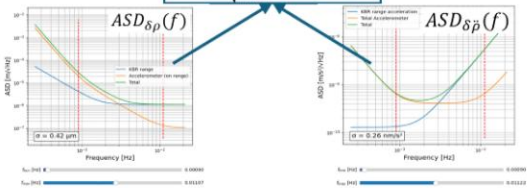
Empirical model

$$EWH = \frac{1}{2} A\, K^2_{obs} + S K_{obs} + B$$

1. LGD/hybrid observable uncertainty

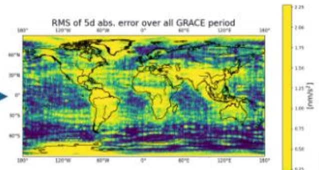
Instrument error (analytical spectra)

$$\sigma_{inst} = \sqrt{\int_{f_{min}}^{f_{max}} ASD^2_{\text{tot}}(f)\, df}$$



Interpolation error (5-day tracks)

Simulate 5-day observables



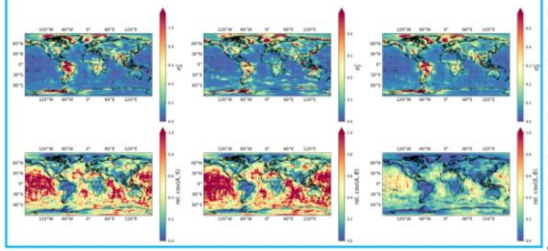
2. Regression uncertainty

Estimates

$$\hat{\boldsymbol{x}} = \begin{bmatrix} \hat{A} \\ \hat{S} \\ \hat{B} \end{bmatrix} = (\mathbf{R}^T\mathbf{R})^{-1}\mathbf{R}\boldsymbol{y}$$

Estimate Covariance matrix

$$\boldsymbol{\Sigma}_{\hat{x}} \approx \hat{\sigma}^2 (\mathbf{R}^T\mathbf{R})^{-1}$$

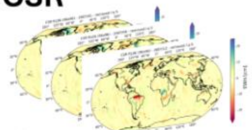


Compute Model Sensitivities

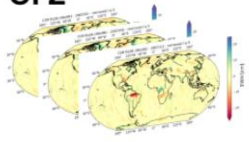
$$\boldsymbol{p} = \begin{bmatrix} \partial EWH/\partial A \\ \partial EWH/\partial S \\ \partial EWH/\partial B \end{bmatrix}$$

3. Level-2 EWH uncertainty

CSR



GFZ



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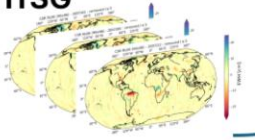
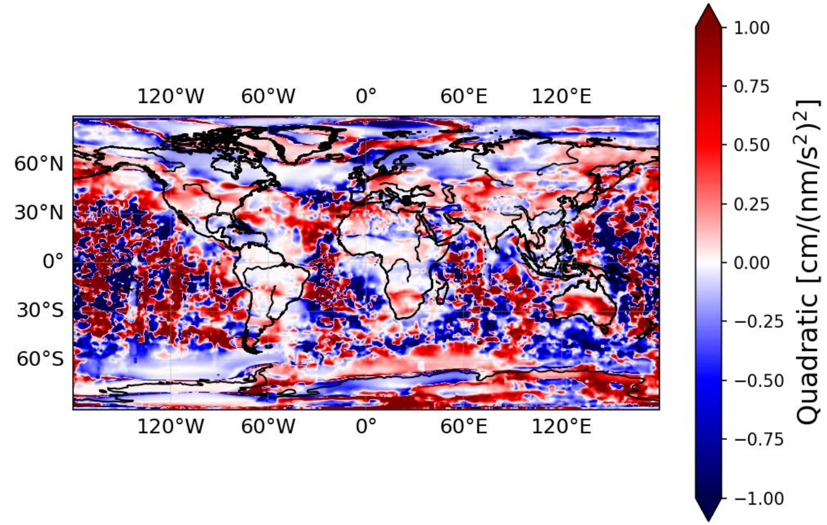
ITSG



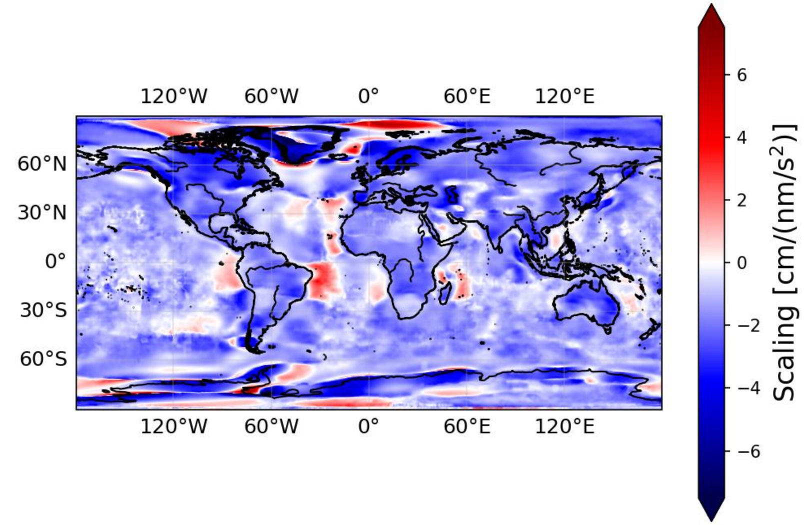
Spread between different analysis centers/ solutions

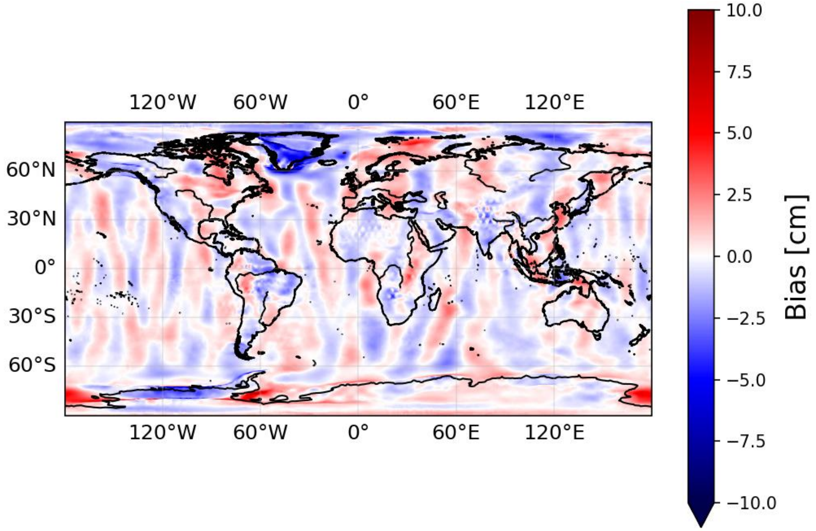
Novel Framework (uncertainties)

Estimated parameters



$$EWH \approx \frac{1}{2} \mathbf{A}\, LGD^2 + \mathbf{S}\ LGD + \mathbf{B}$$





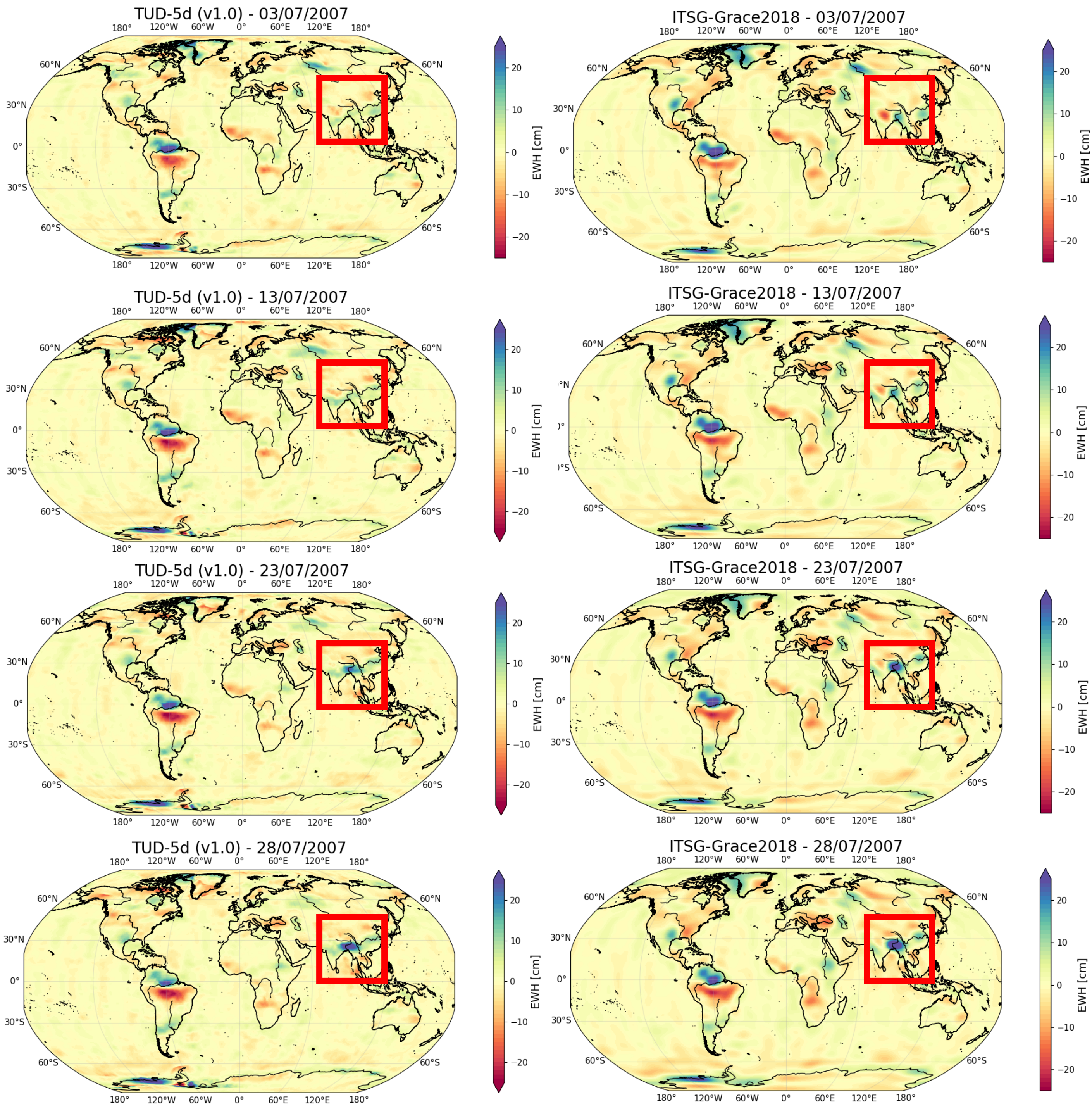
<u>Period:</u> 2003-2016.

- Quadratic factor shows high variability in oceans, and very low (near zero values) in major river basins.
- Scaling factor shows overall negative values (expected negative LGD peak due to positive mass anomaly).
- Bias shows the mean difference between LGDs and EWH:
    i. mean N-S stripe noise,
    ii. Larger difference in polar regions.

ITSG vs. TUD v1.0

- Evident hydrological sub-weekly signals observed in highly hydrologically active river basins.
- Differences:

  → Higher frequency content up to degree 57.
  → monthly model set as background,
  → background model errors in polar regions.



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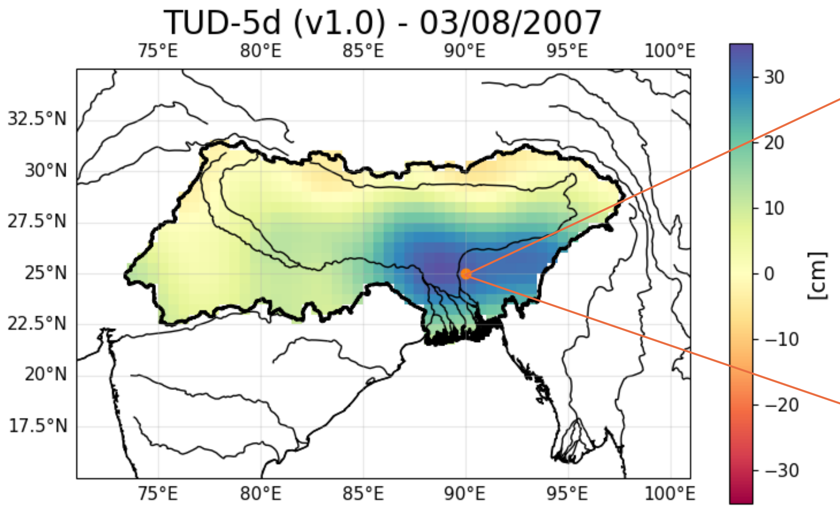
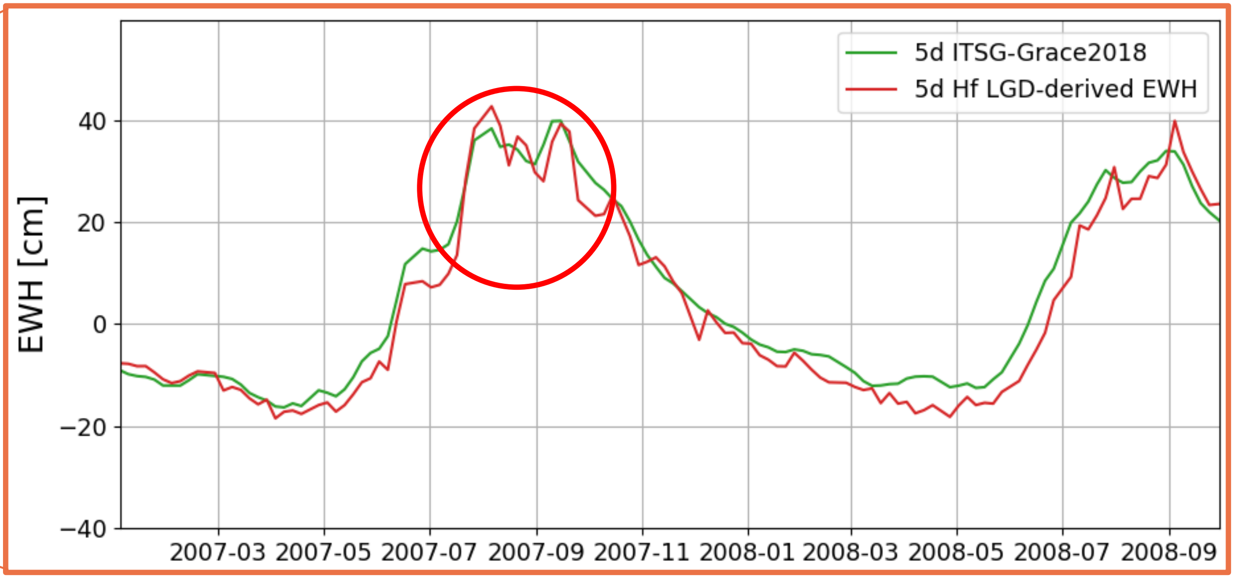
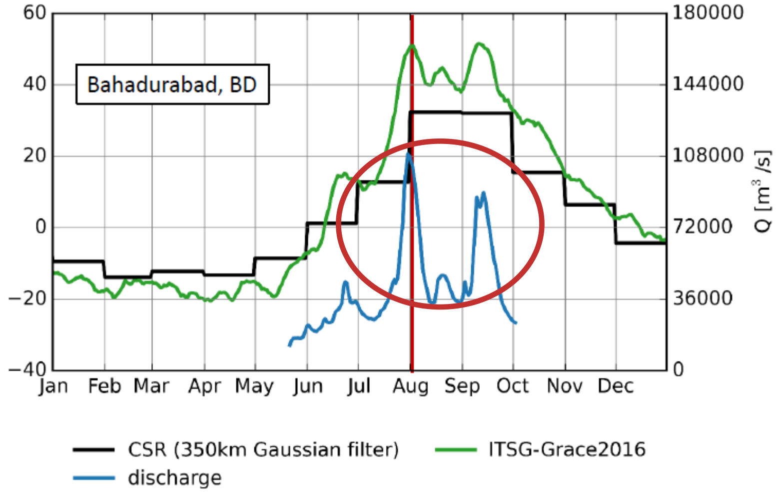
Bangladesh 2007 floods







Kvas, A., Kalman filtered daily GRACE gravity field solutions in near real-time - first steps **Poster,** 2016.

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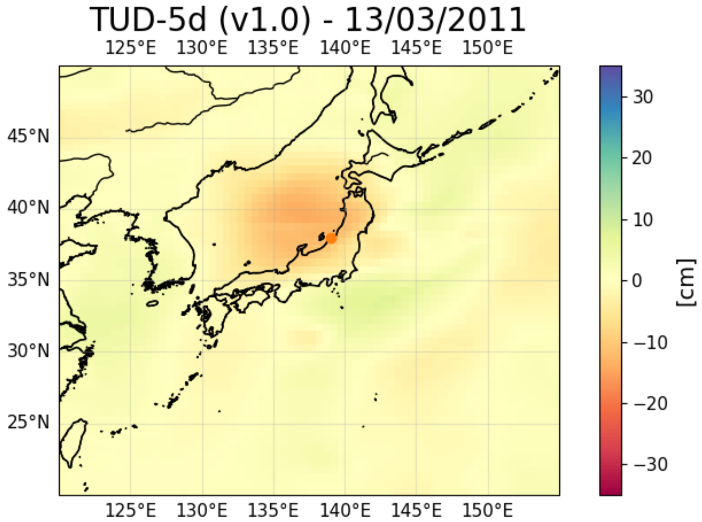
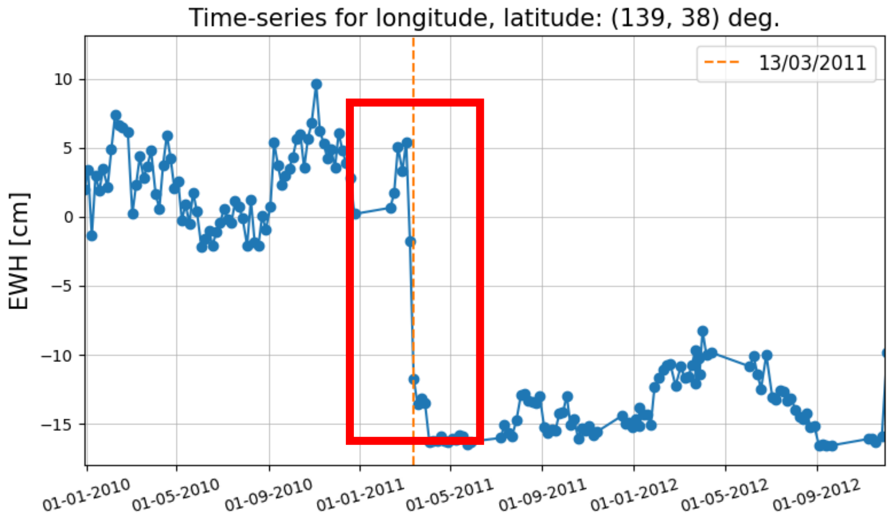
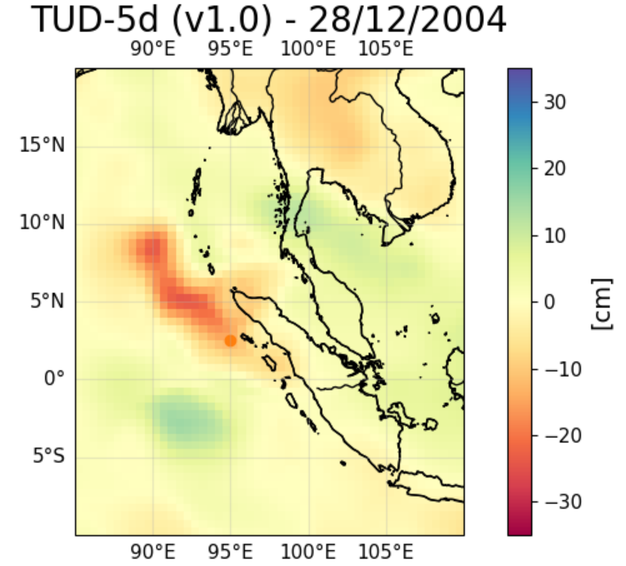
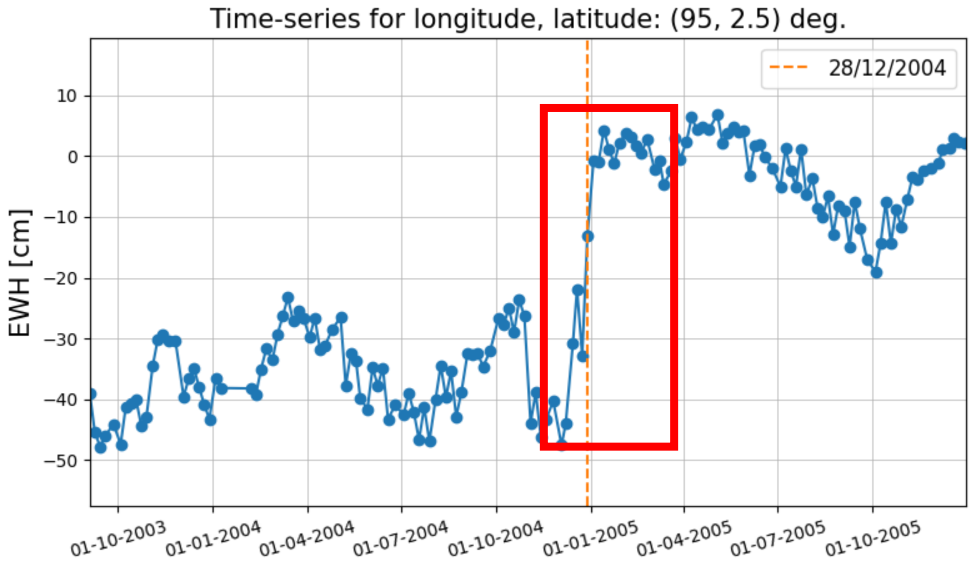
Co-seismic sub-monthly signals









- Earthquake events:
  - i. Sumatra 2004,
  - ii. Tohoku 2011.

- Tohoku 2011

Clear co-seismic sub-monthly signal in high-frequency models.

- Sumatra 2004

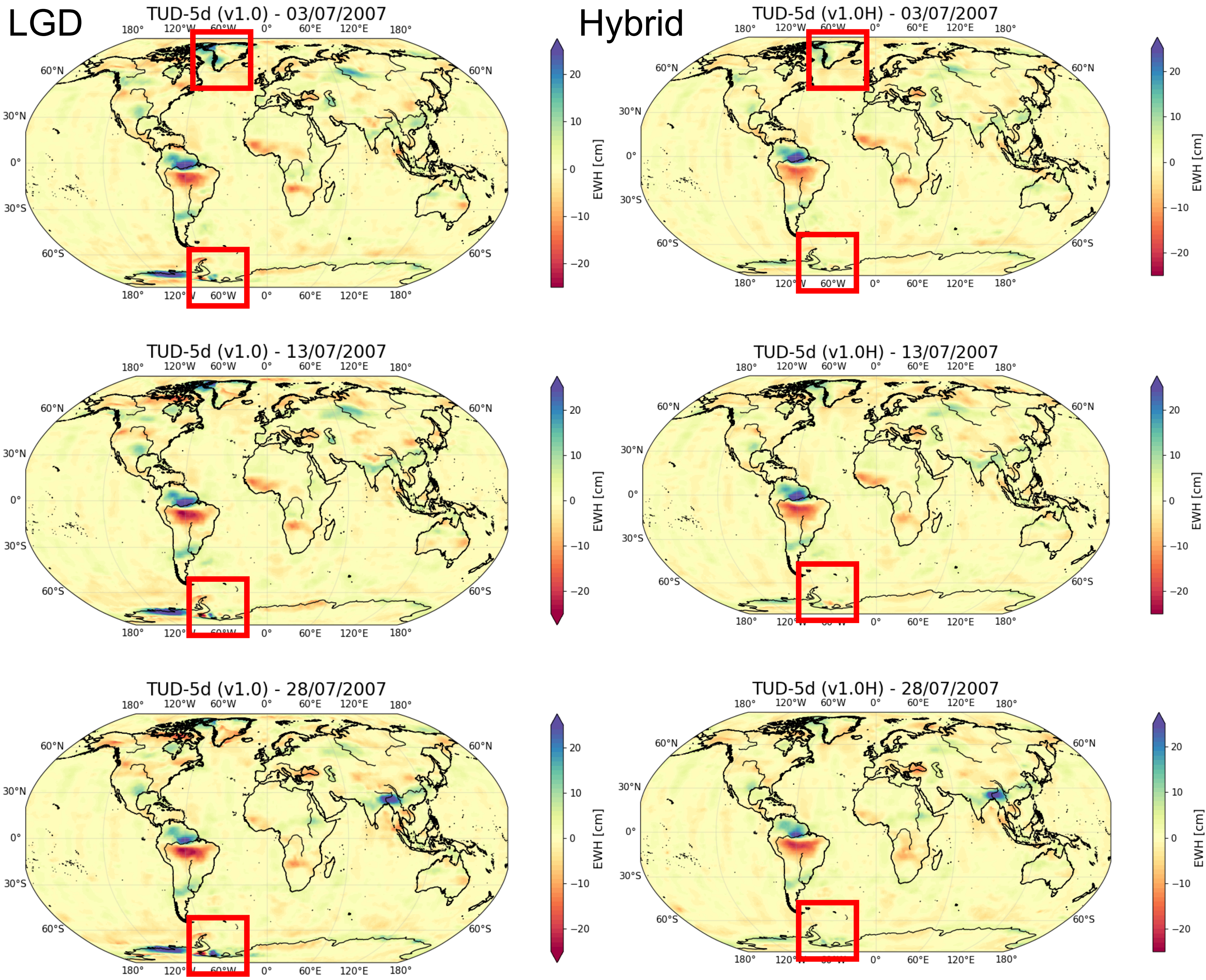
Less clear as the event happened near the beginning of January 2005.

Comparison of LGD-only vs Hybrid

Observations:

1. Spatial patterns similar to EWH^{L2},
2. Dampening observed: $|\widehat{EWH}^{hyb}| \leq |\widehat{EWH}^{LGD}|$

➔Polar region and other lobe artifacts are dampened!

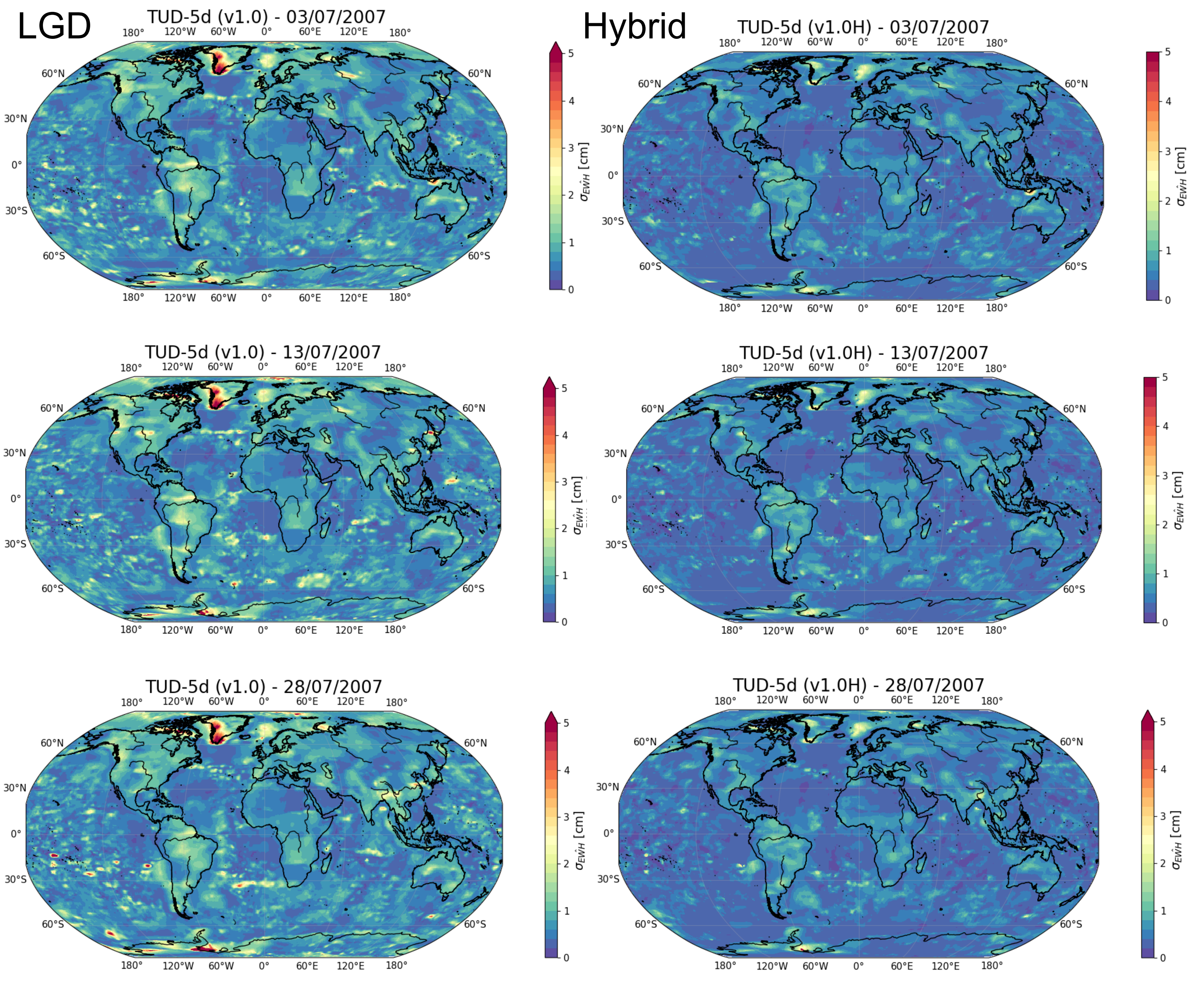


Uncertainty estimates (LGD vs Hybrid)

Observations:

1. Hybrid formulation unc. is approx. half of the LGD-only.
2. Hybrid unc. < 2.5 cm.
3. LGD-formulation ➔ overestimates.

** Currently only regression uncertainties, transfer function and instrument errors are included in the model uncertainties.*



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Summary and looking forward

Goal
Construct *GRACE-only* sub-weekly mass change models using purely post-fit range-rate data.

Novel Methodology (purely data-driven)
- Usage of along-track res. KBR LGDs and range.
- Derivation of an ***empirical and spatially distributed model*** between LGDs into EWH.
- Two different model formulations (LGD-only vs. KBR hybrid form).
- Error estimation based on analytical noise models and regression errors.

Comparison with ITSG-Grace2018 daily solutions

Further work: towards a complete version.
Finalising error estimation:
→ spatial interpolation error,
→ L2 product errors.

Questions? → Michal Cuadrat-Grzybowski: M.Cuadrat-Grzybowski-1@tudelft.nl

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References

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