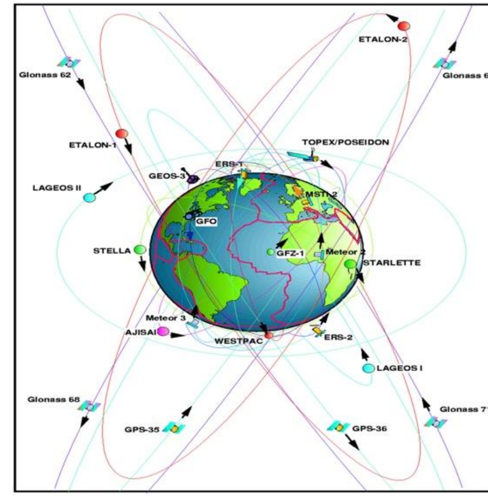
Variations in the Earth's Figure Axis from SLR and GRACE



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- **Mean pole model and Earth's Figure Axis**
- **Seasonal and Decadal Variations in C21/S21 from SLR and GRACE/GRACE-FO**

Reference:
Wahr (1987): The Earth's C21 and S21 gravity coefficients and the rotation of the core, Geophys. J . R. Astr. SOC.(1987) 88,265-276
Cheng, Ries and Tapley (2011):'Variations of the Earth's figure axis from satellite laser ranging and GRACE', *JGR*, 2011, doi:10.1029/2010JB000850

Figure Axis determined by C21/S21

– The figure axis of the Earth is the axis of maximum inertia for the deformed (oblate) Earth, is described by the geopotential coefficients C21 and S21.

- Any difference between the mean figure and the mean rotation pole can be attributed to the long-period fluid motion in the atmosphere, ocean, or Earth's core [Wahr, 1987, 1990].
- C_{21}/S_{21} from the mean pole $(\bar{x}_p, \bar{y}_p)$ (IERS 2010) $(C_{21}^{mp}, S_{21}^{mp})$:
- $C_{21}^{mp} = \sqrt{3} C_{20} \bar{x}_p - C_{22} \bar{x}_p + S_{22} \bar{y}_p$, $S_{21}^{mp} = -\sqrt{3} C_{20} \bar{y}_p - C_{22} \bar{y}_p - S_{22} \bar{x}_p$
- The correction of the solid Earth (C_{21}^{E}, S_{21}^{E}) and Ocean pole tide (C_{21}^{o}, S_{21}^{o})
- $\Delta C_{21} = C_{21}^{mf} - C_{21}^{mp} - C_{21}^{E} - \Delta C_{21}^{t}$, $\Delta S_{21} = S_{21}^{mf} - S_{22}^{mp} - S_{21}^{E} - \Delta S_{21}^{t}$
- The $\Delta C_{21}/\Delta S_{21}$ represents the variations in the mean Earth's figure axis $(C_{21}^{mf}, S_{21}^{mf})$(which is related the mean rotation axis) are determined from Satellite Laser Ranging (SLR), and GRACE/GRACE-FO data. The AOD RI06 were applied for both GRACE and SLR anlysis.

Data used

- **GRACE/GRACE-FO monthly solution: CSR GSM**
- **SLR monthly solution from TN11E, and over the period from 1980 to Aug. 2025, Signal from GRACE and SLR is expected to be due to hydrological mass changes**
- **IERS EOP, and Atmospheric and oceanic ex-**
- **citations of PM from atmospheric and oceanic angular momentum from https://hpiers.obspm.fr/eop-pc**

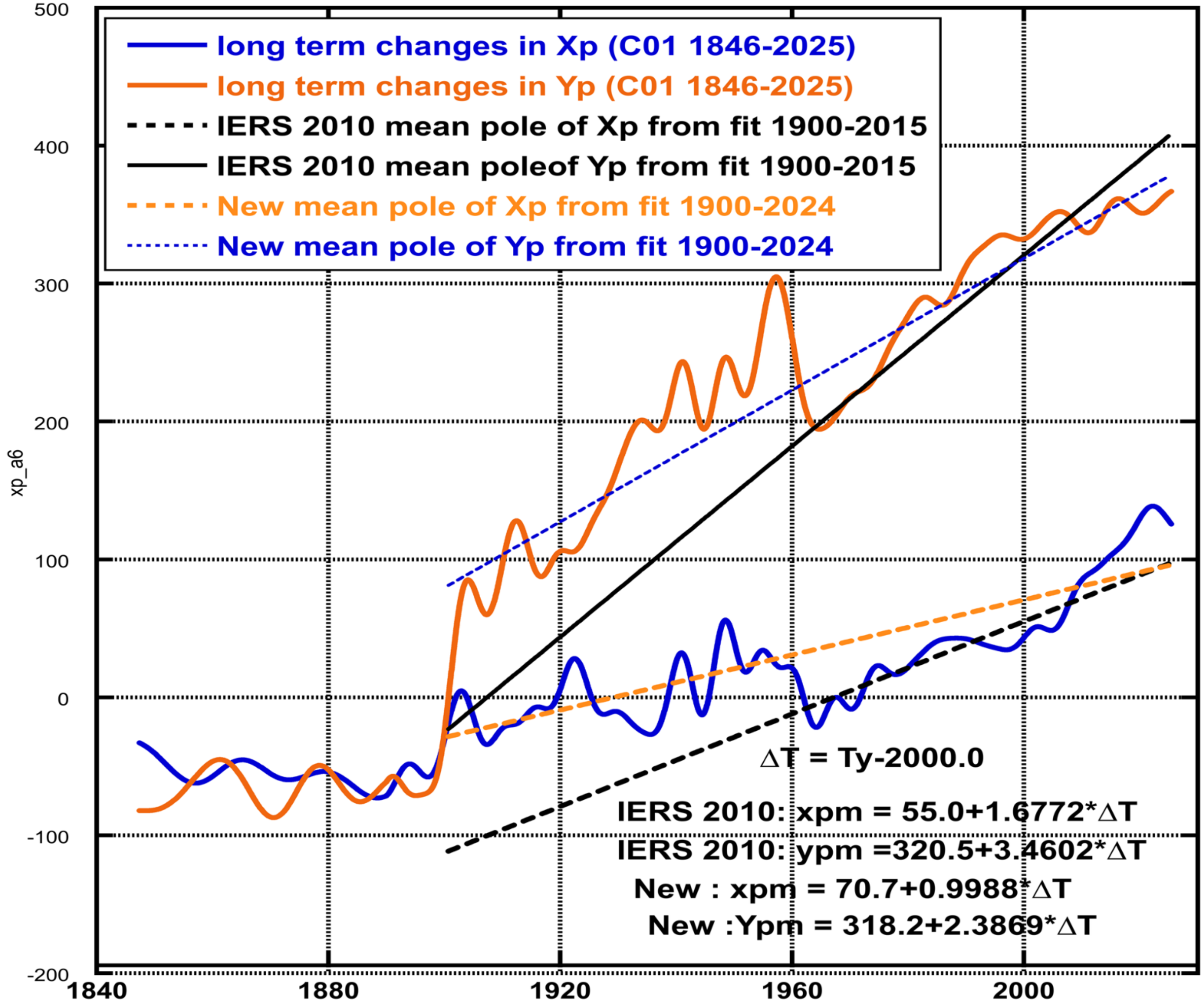
$$\begin{pmatrix}\Delta C_{21}\\ \Delta S_{21}\end{pmatrix} = -(1+k_2')\sqrt{\frac{3}{5}}\,\frac{(C-A)}{1.12MR^2}\begin{pmatrix}x_p-\chi_{xp}^{motion}\\ y_p-\chi_{yp}^{motion}\end{pmatrix}$$

- **Continental Water storage (LWS) from NOAA CPC model**

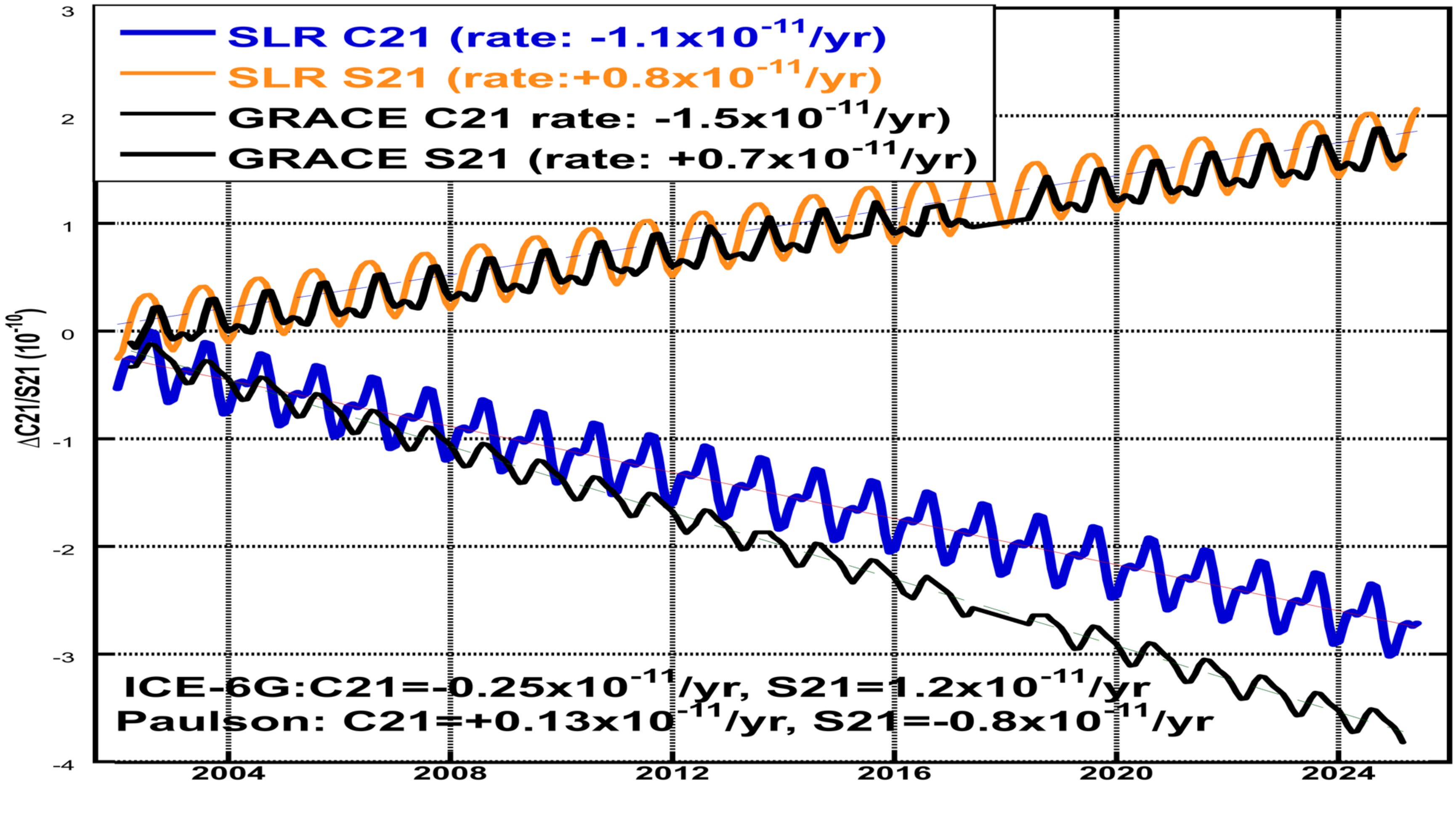
- **Atmosphere and oceanic Torques on Earth's figure axis based on angular momentum due to motion relative to axis, x, y (h_x,h_y)**

$$\begin{pmatrix}\Delta C_{21}^{t}\\ \Delta S_{21}^{t}\end{pmatrix} = \sqrt{\frac{3}{5}}\frac{1}{\Omega M a^2}\begin{pmatrix}h_x\\ h_y\end{pmatrix}$$

Mean Pole model

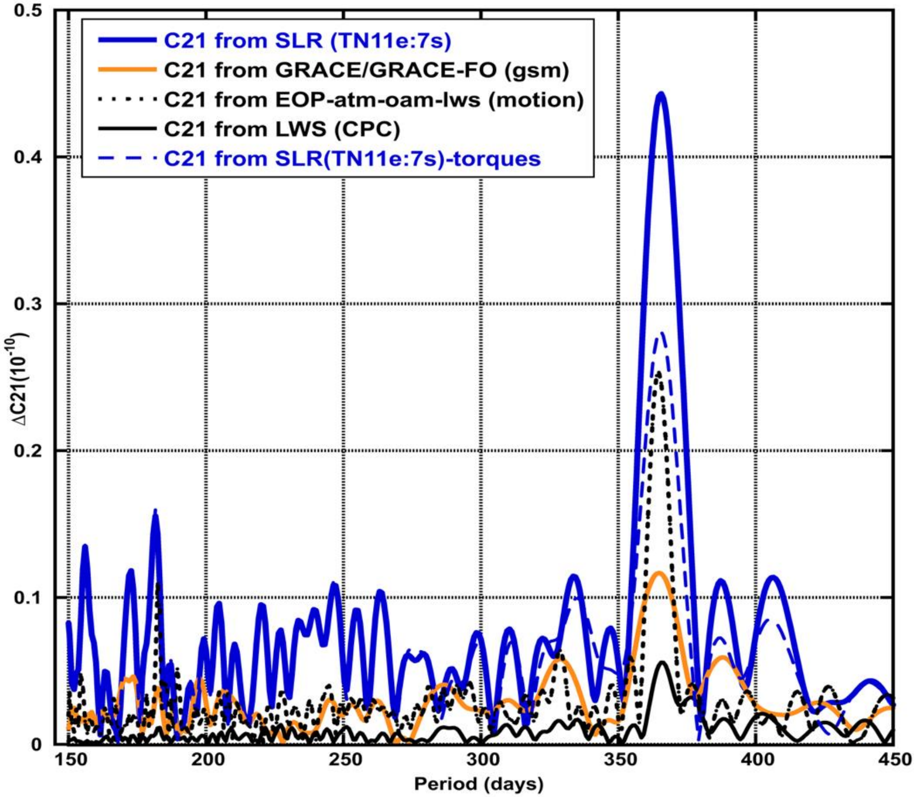


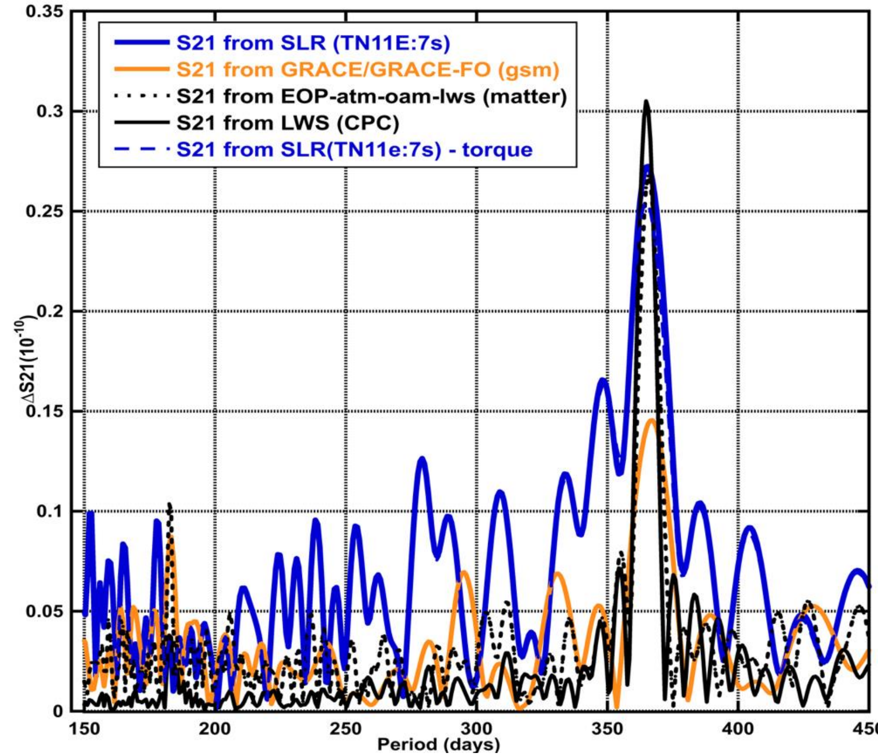
Linear trend in C21/S21 from SLR and GRACE



Remark:SLR estimate of C2/S21 is with respect to the mean pole
The contribution from the melting of polar ice sheets is estimated to be -0.05×10^{-11}/yr for C21 and $+0.21 \times 10^{-11}$/yr for S21

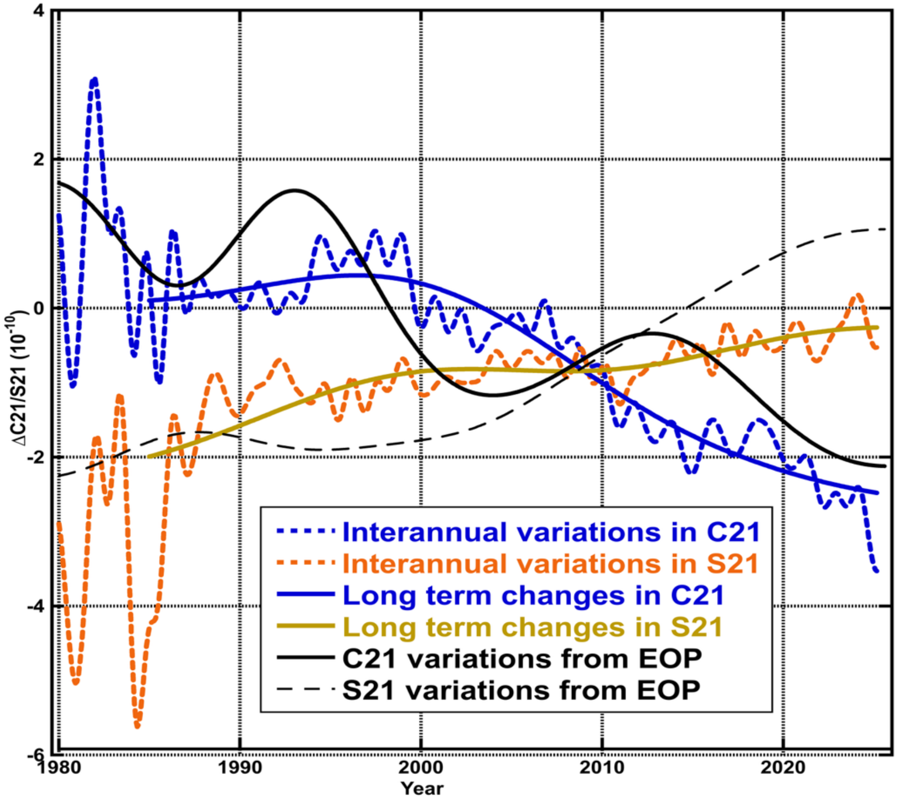
Annual Variations in C21/S21from SLR and GRACE

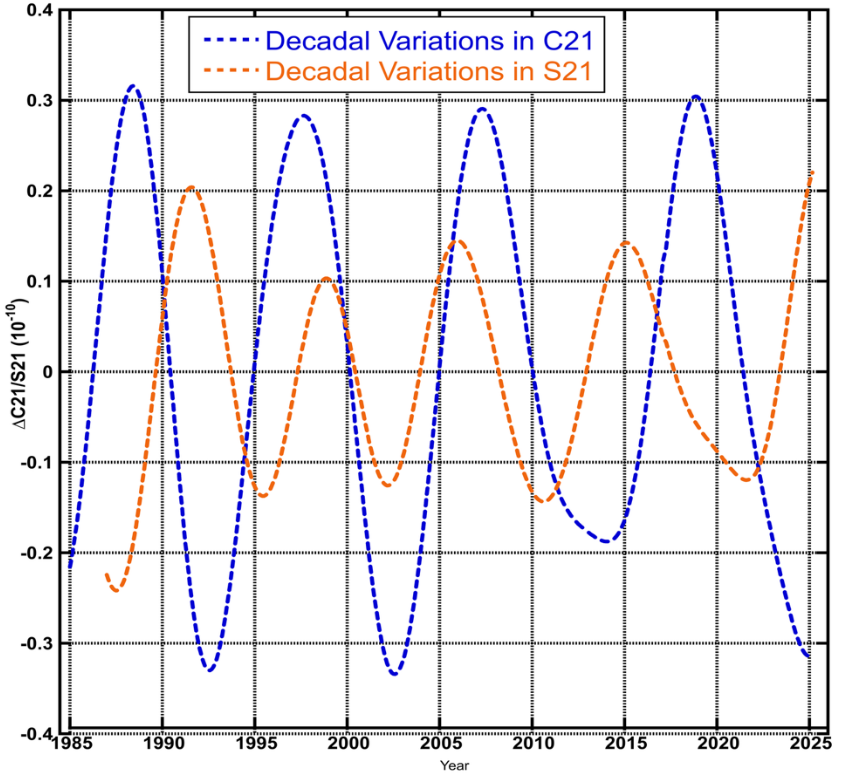




There is a significant annual signal in C21 produced by the atmospheric-oceanic wind torques [Wahr, 1987]. Corrections lead the SLR estimates to be in good agreement with those from EOP (-motion term) for C21 and the EOP (matter) for S21 and LWS. EOP excitation from the Motion term for S21 is too small. The GRACE estimate appears to be underestimated from C21/S21.

Observed Long wavelength variations in C21/S21





The dominant signal in C21/S21 appears to be non-hydrological long-term changes, which characterize the Earth's figure axes variations related to the mean rotation axes (xp/yp) represented by a 125-year mean. The amplitude of decadal variations is estimated to be 2.6×10^{-11} for C21 and 1.6×10^{-11} for S21. The rate is estimated to be -1.2×10^{-11}/yr for C21, 0.3×10^{-11}/yr for S21 for 1993-2025. Those variations could be a valuable observation of the core axis motion. Further efforts are required to understand the observed variations in the Earth's figure axis.

Dynamics of Core-mantle interactions In C21/S21

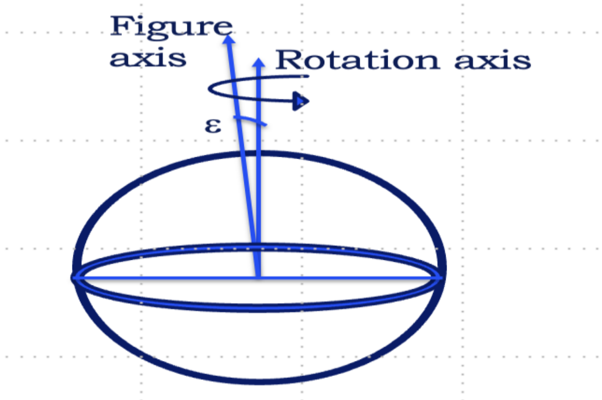
Solution	C21 (10^{-10})	S21 (10^{-9})
GGM05C	-3.1034311	1.4107575
SLR	-4.0378394	1.3782463
Wind torques	-0.0548323	-0.0159808

The non-zero value of C21/S21 is due to the offset of the mantle rotation axis from the core figure axis[Wahr 1987, Cheng, Ries & Tapley, 2011] by

$$\begin{pmatrix} I_{yz}^{c} \\ I_{xz}^{c} \end{pmatrix} = -\frac{1}{\Omega^2}\begin{pmatrix} N_x^c \\ N_y^c \end{pmatrix} - \sqrt{\frac{5}{3}} M a^2 \begin{pmatrix} C_{21} \\ S_{21} \end{pmatrix} = \begin{pmatrix} -0.43 \\ -0.13 \end{pmatrix} 10^{30} \mathrm{kg.}\, m^2 \quad \beta = arctan\,(I_{xy}^{c} / I_{yz}^{c}) = 254^{\circ}$$

β represents the direction of the mean figure axis of the entire core, opposite to the direction of the mean figure axis of the entire Earth.

The equatorial polar offset of the mean figure axis is ε. The tilt of the inner core relative to the Equatorial polar offset



$$\widetilde{\Delta n} = 10^4 \varepsilon = 0.9^{\circ}$$

$$\varepsilon = 693\, mas$$

Summary

- The dominant non-hydrological signal in C21/S21 appears to be long-term changes, which characterize the Earth's figure axis variations related to the mean rotation axis, represented by a 125-year mean of IERS EOP. This extended model is recommended for evaluating the pole tide.

- Significant annual signal in C21/S21 is estimated from SLR and polar motions data, while GRACE underestimates the annual signal for hydrological part.

- Significant atmosphere and ocean motion-induced torque on C21 is observed from the 40-year SLR data. The motion-induced excitation in EOP-derived S21 is smaller; the matter-induced excitation is dominant.

- The direction of the mean figure axis of the entire core is estimated to be $\sim 254^0$ eastward, opposite to the direction of the mean figure axis of the entire Earth with a magnitude of 4 arcsecs.

Abstract

The figure axis of the Earth is the axis of maximum inertia for the deformed (oblate) Earth, characterized by the degree-two and order-one geopotential coefficients C21 and S21. Long-period fluid motion within the Earth's system (including the atmosphere, ocean, and surface water storage, or the Earth’s liquid core) will result in the difference between the figure axis and the mean rotation pole. 45-year Satellite laser ranging (SLR) data and 24-year GARCE/FRACE-FO data were analyzed to determine the variations in the Earth's figure axis represented by changes in C21 and S21 coefficients. The primary signature appears to be a non-hydrological long-term change in the SLR and GRACE time series of C21 and S21, which characterizes the Earth's figure axes variations related to the mean rotation axes represented by a 125-year mean. An important atmosphere-ocean induced wind torque on C21 is observed from SLR. A significant annual signal from SLR is in good agreement with the polar motion or EOP-inferred and the continental surface water redistribution-induced variation in C21/S21. However, the annual signal appears to be an underestimate by GRACE. A detailed analysis will be presented.