Wrocław University of Environmental and Life Sciences

The Missing Relativistic Oblateness Orbit Correction in the IERS Conventions and its Impact on GRACE-FO Products and MAGIC Orbits

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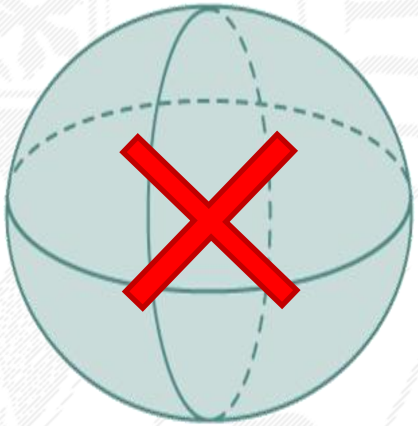
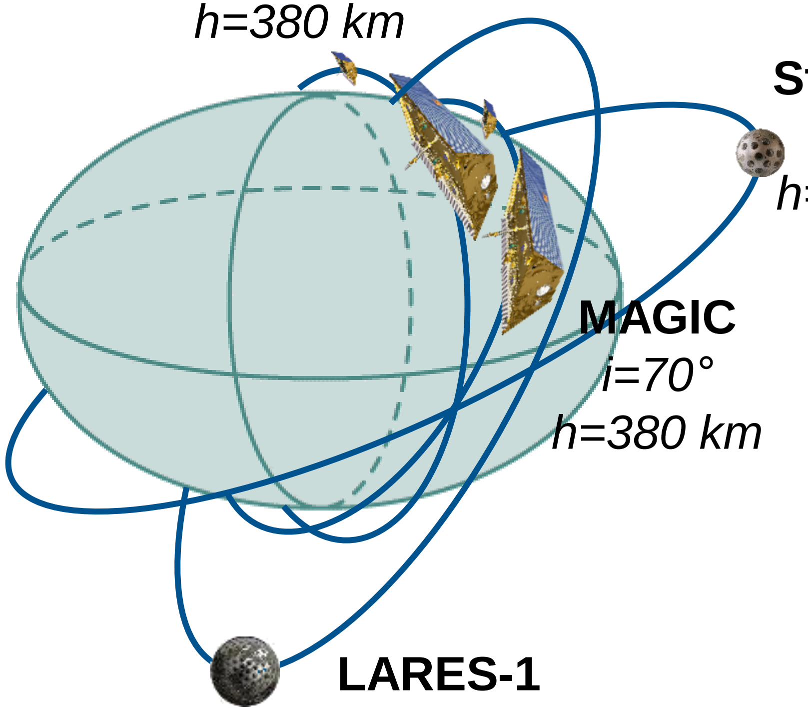
Impact of the Earth's oblateness on satellite orbits

The current General Relativistic (GR) corrections in the IERS Conventions (Schwarzschild, Lense-Thirring, Geodetic Precession) assume that the Earth is a sphere.





What about treating the Earth's shape as an ellipsoid (oblate)?

Impact of the Earth's oblateness on satellite orbits

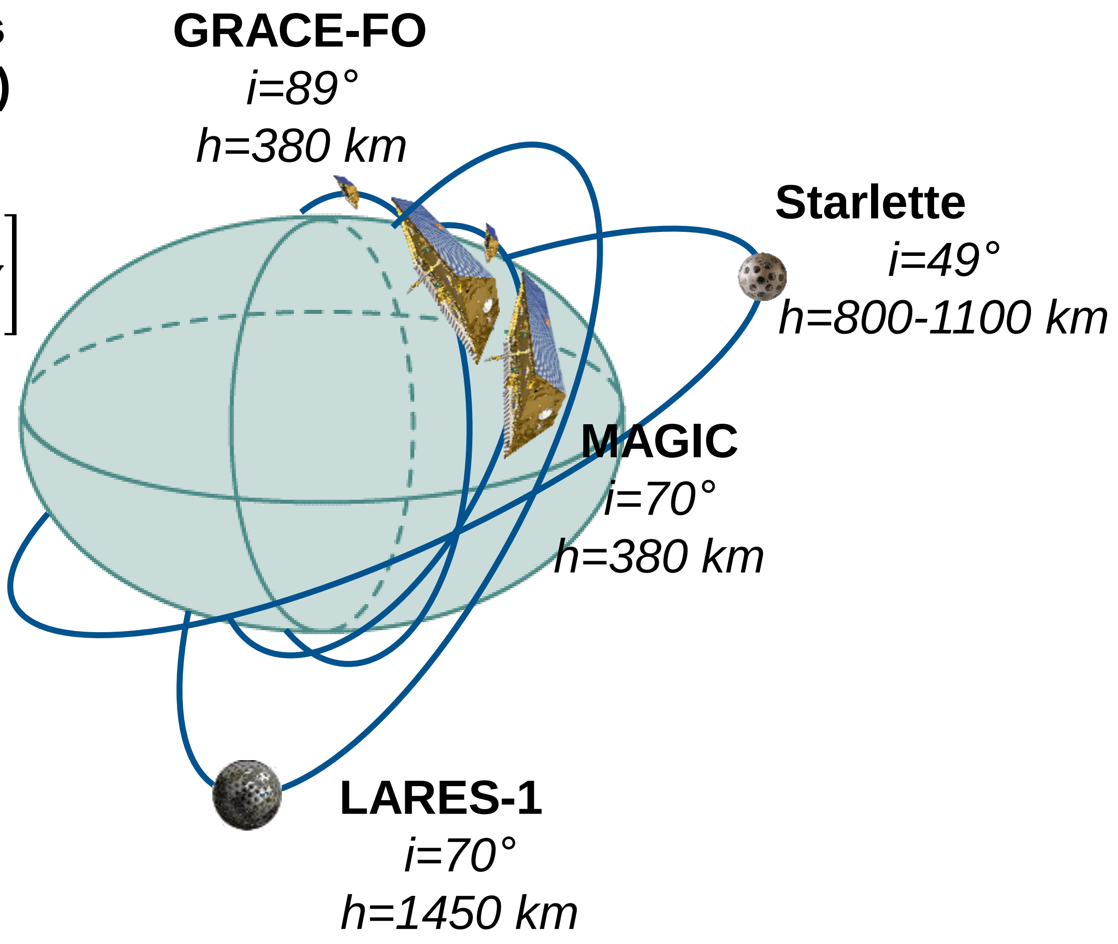
The relativistic correction caused by Earth's oblateness to the satellite acceleration in the Post-Newtonian (PPN) approximation reads as:

$$\boldsymbol{a}_{J2} = \frac{1}{c^2}\left[-2(\beta+\gamma)\,\boldsymbol{\nabla}\left(\frac{GM}{r}R\right) + \gamma v^2 \boldsymbol{\nabla}R - 2(1+\gamma)(\mathbf{v}\cdot\boldsymbol{\nabla}R)\cdot\mathbf{v}\right]$$

where R is the potential-related perturbing function characterizing a satellite motion around an oblate planet:

$$R = \left[-J_2\frac{GM}{r^3}a_E^2\left(\frac{3}{2}\sin^2\varphi - \frac{1}{2}\right)\right] = \left[-J_2\frac{GM}{r^3}a_E^2\left(\frac{3}{2}\frac{z^2}{r^2} - \frac{1}{2}\right)\right].$$

with the oblateness equal to $\boldsymbol{J_2 = 1.0826359\cdot 10^{-3}}$, GM is the product of the gravitational constant and Earth's mass, φ – satellite latitude in the Earth-fixed equatorial system, c – speed of light, β=1 and γ=1 for PPN GR.



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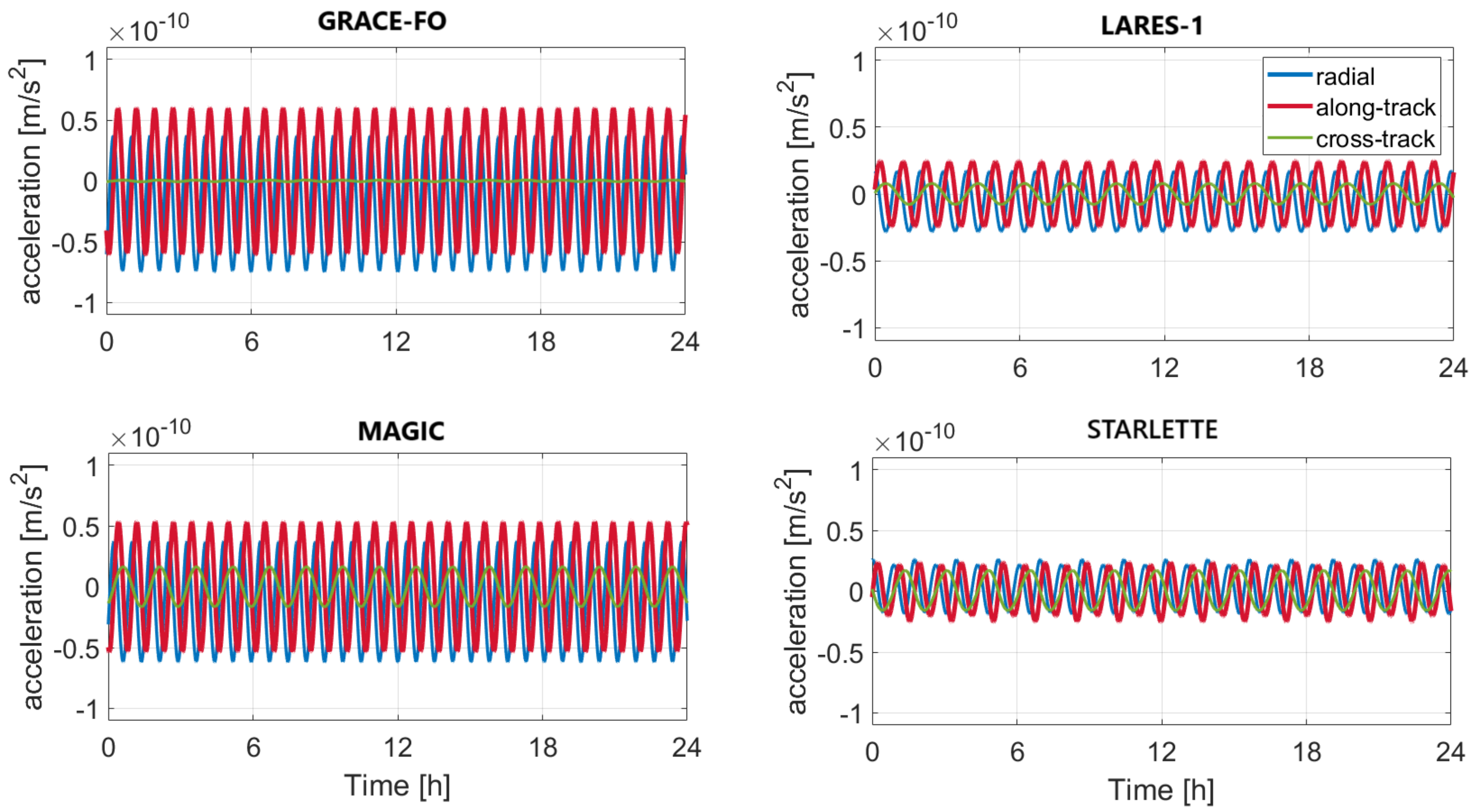
The nabla operator $\boldsymbol{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ allows for deriving a gradient of the potential perturbing function that represents the acceleration related to the oblateness potential function $\boldsymbol{R}$ and the $\left(\frac{GM}{r}\boldsymbol{R}\right)$ component:

$$\boldsymbol{\nabla}R = -\frac{3}{2}J_2\,\mathrm{GM}\,a_E^2\,\frac{1}{r^5}\begin{bmatrix} x\left(1-5\frac{z^2}{r^2}\right) \\ y\left(1-5\frac{z^2}{r^2}\right) \\ z\left(3-5\frac{z^2}{r^2}\right)\end{bmatrix},$$

$$\boldsymbol{\nabla}\left(\frac{GM}{r}R\right) = -J_2\,\mathrm{GM}^2\,a_E^2\,\frac{1}{r^6}\begin{bmatrix} x\left(2-9\frac{z^2}{r^2}\right) \\ y\left(2-9\frac{z^2}{r^2}\right) \\ z\left(5-9\frac{z^2}{r^2}\right)\end{bmatrix}.$$

GR acceleration acting on MAGIC orbits



The relativistic C_{20} acceleration for different satellite missions.

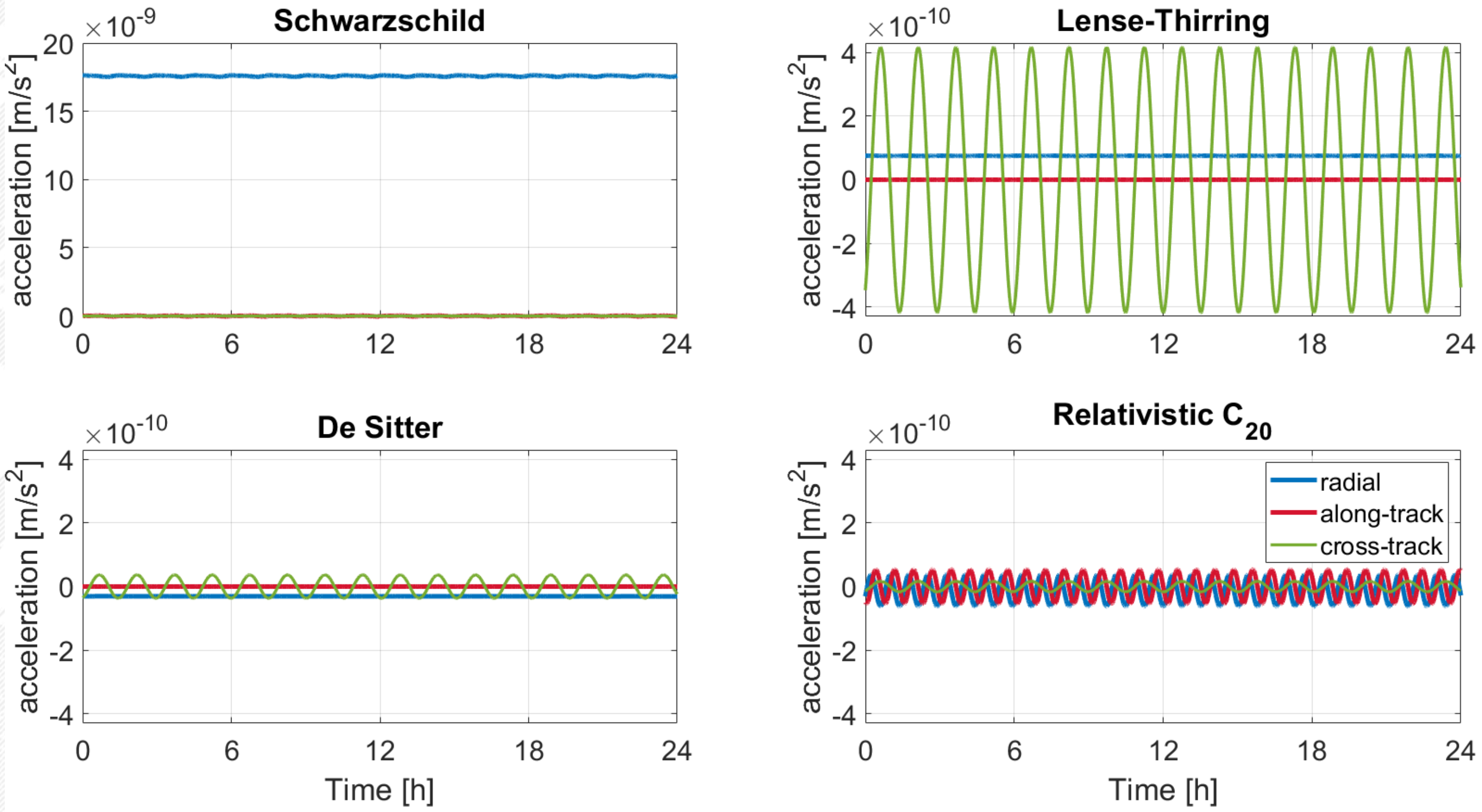
For GRACE-FO, the radial and along-track components dominate (near-polar orbits).

For MAGIC (the same height as GRACE-FO, but lower inclination), the cross-track component becomes relevant.

LARES-1 is re-scaled version of MAGIC (the same inclination, larger height).

Starlette has comparable magnitudes for all components.

GR acceleration acting on MAGIC orbits



The same scale for Lense-Thirring, de Sitter, and relativistic C_{20}. A different scale for the Schwarzschild effect.

Schwarzschild acts only in the orbital plane (no cross-track accelerations) and introduces near-constant radial accelerations.

The relativistic C_{20} effect is larger than de Sitter effect but smaller than the Lense-Thirring effect for MAGIC orbits.

For Lense-Thirring and de Sitter, the periodic cross-track component dominates. For the C_{20} effect, the radialand along-track components dominate, whereas cross-track becomes important for low inclinations.

Impact of the Earth's oblateness on satellite orbits

$$\boldsymbol{a}_{J2} = \frac{1}{c^2}\left[-4\,\nabla\left(\frac{GM}{r}R\right) + v^2\nabla R - 4(\mathbf{v}\cdot\nabla R)\cdot\mathbf{v}\right]$$

Potential component

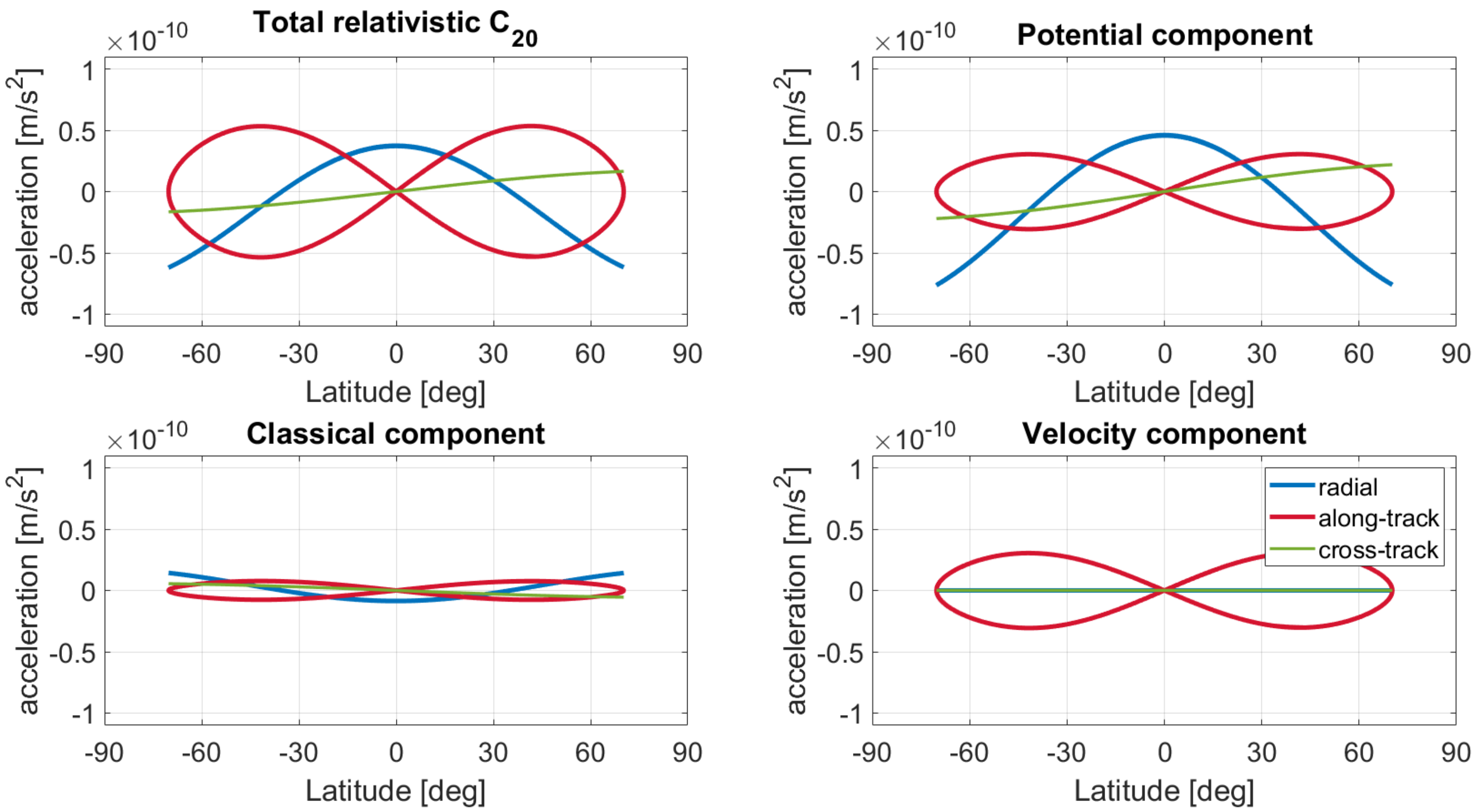
Classical Component

Velocity Component

Dependence on the latitude:

$$R = \left[-J_2\frac{GM}{r^3}a_E^2\left(\frac{3}{2}\sin^2\varphi - \frac{1}{2}\right)\right]$$

GR acceleration acting on MAGIC orbits



Decomposition of the relativistic C_{20} effect into components.

All components of the relativistic C_{20} effect strongly depend on satellite latitude.

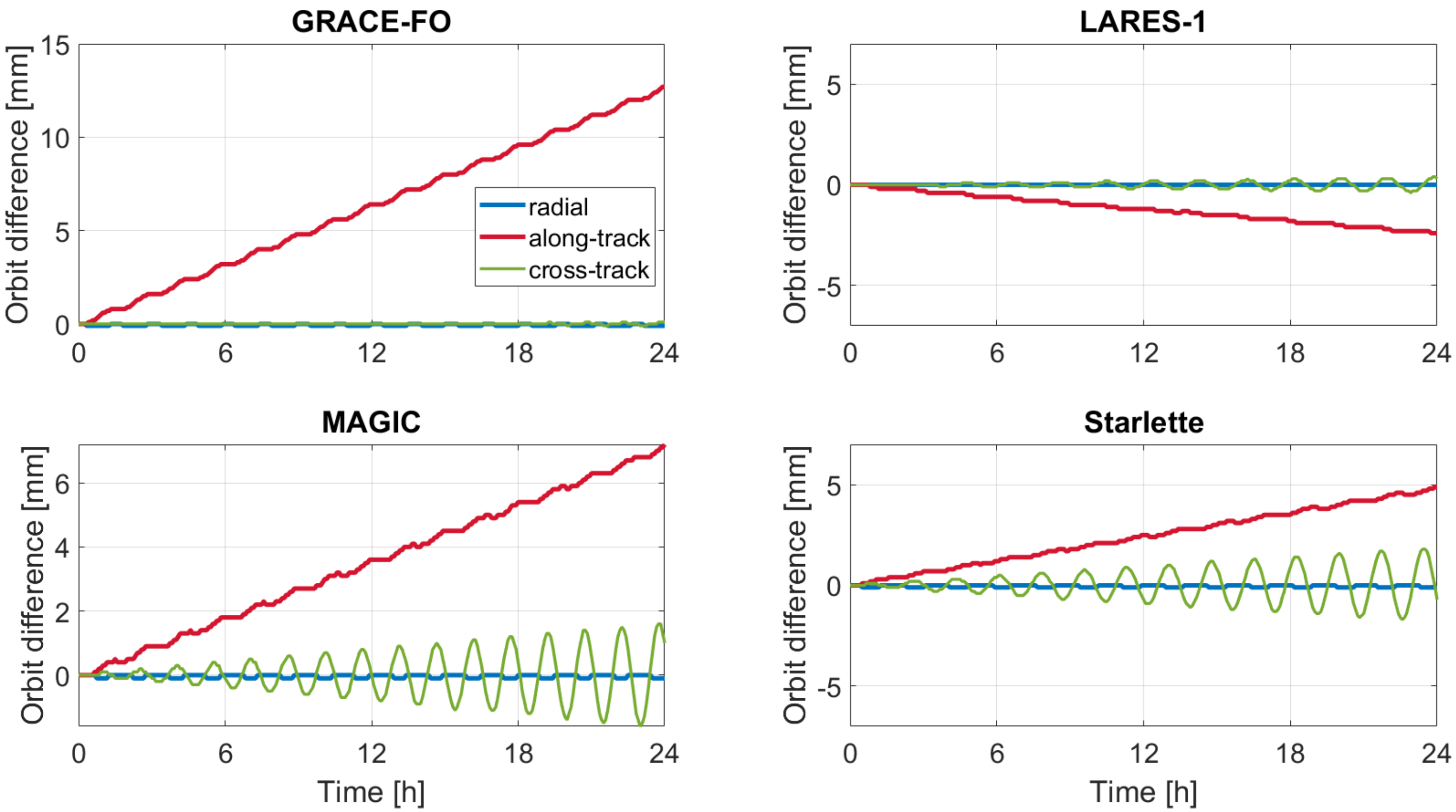
Cross-track accelerations are of once-per-revolution type.

Along-track and radial accelerations are of the twice-per-revolution type.

The cross-track component is dominated by the potential term.

Radial and cross-track accelerations in the velocity component are equal to zero.

Orbit propagation from the same state vector – accumulation of orbit errors

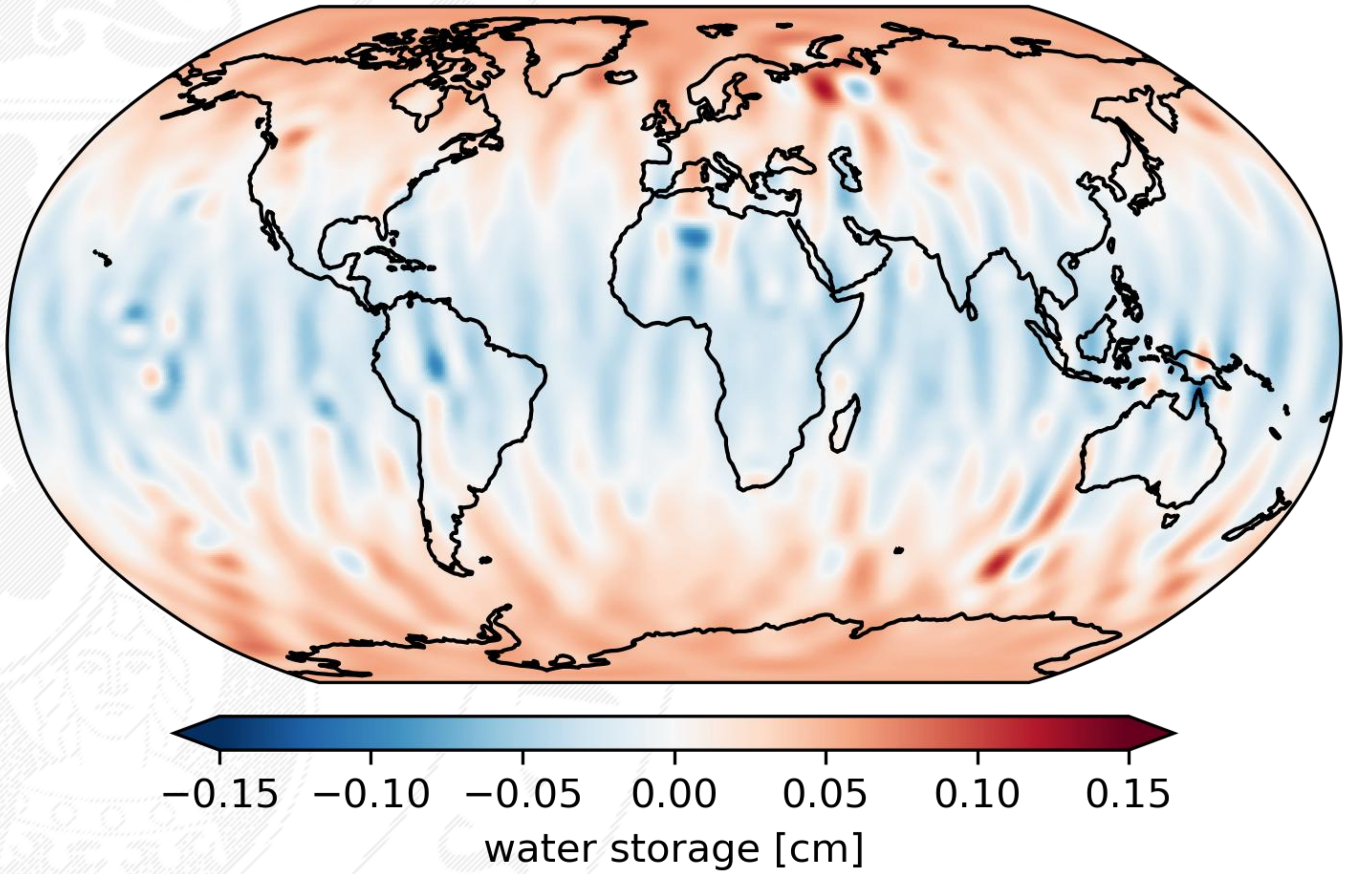


For LARES and Starlette 2-5 mm after 1 day.

For GRACE-FO, the along-track component accumulates the radial acceleration – 13 mm after 1 day.

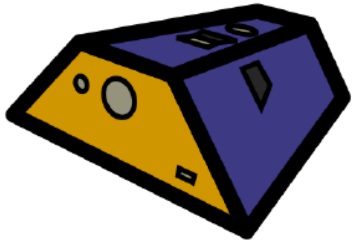
Orbit differences from the orbit propagations using the same initial stave vector with and without the relativistic C_{20} effect.

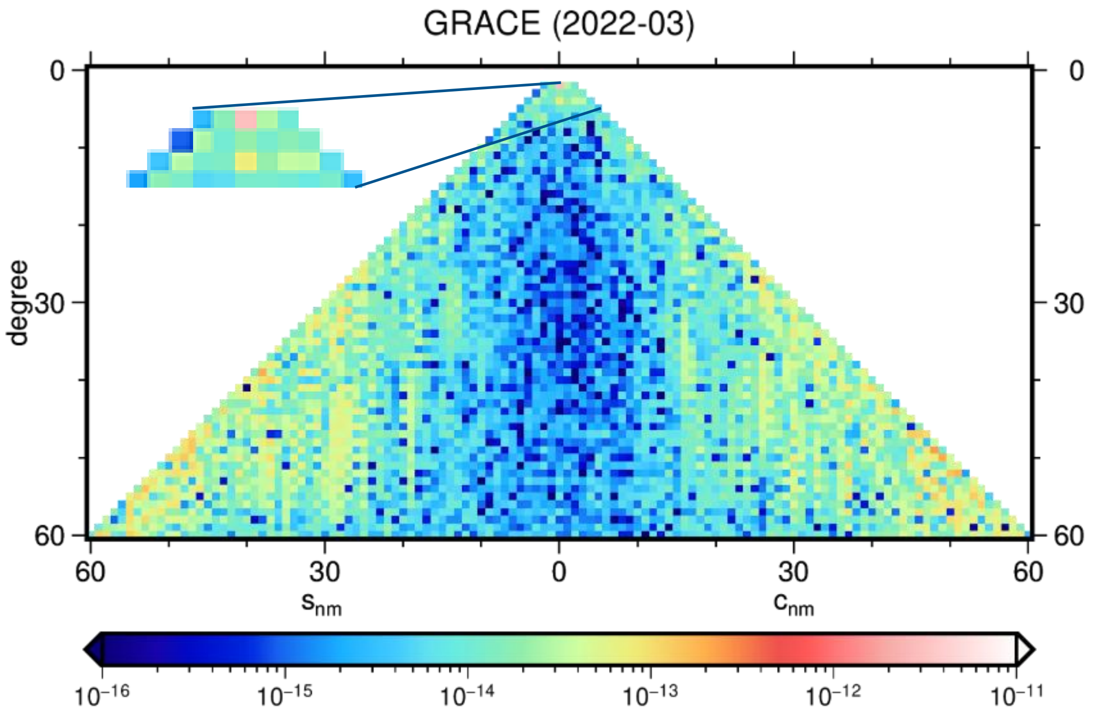
Impact on GRACE-FO gravity field solutions



Difference in the GRACE gravity field recovery with and without applying the C_{20} relativistic correction in equivalent of water height (with DDK3 filter by Kusche et al. 2009). Clear systematic „oblateness” pattern is visible.

GROOPS GRACE-FO solutions for 1 month (March, 2022)





Differences in the spherical harmonic domain – the largest for the C_{20} term of 3.0×10^{-12}.

Summary

The C_{20} relativistic effect is larger than the Geodetic precession (de Sitter) for low-orbiting satellites (de Sitter IS in the IERS Conventions, C_{20} IS NOT).

The acceleration caused by the C_{20} relativistic effect dominates in the radial and along-track components (near-polar orbits, e.g., GRACE-FO). Cross-track is important for non-polar orbits.

The accelerations caused by the C_{20} relativistic effect 1×10^{-10} m/s^2 are of the order of planetary accelerations (Venus, Jupiter) and above the sensitivity of satellite accelerometers (not to mention the quantum accelerometers).
The relativitic effect introduces a bias into C_{20} estimates of 3.0×10^{-12}.

Sośnica, K., Gałdyn, F. Orbital relativistic correction resulting from the Earth's oblateness term. *J Geod* **99**, 52 (2025). https://doi.org/10.1007/s00190-025-01973-3

Note: some figures in the paper were based on a double minus sign (incorrect). The figures in this presentation have been corrected.

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Thank you for your attention!

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