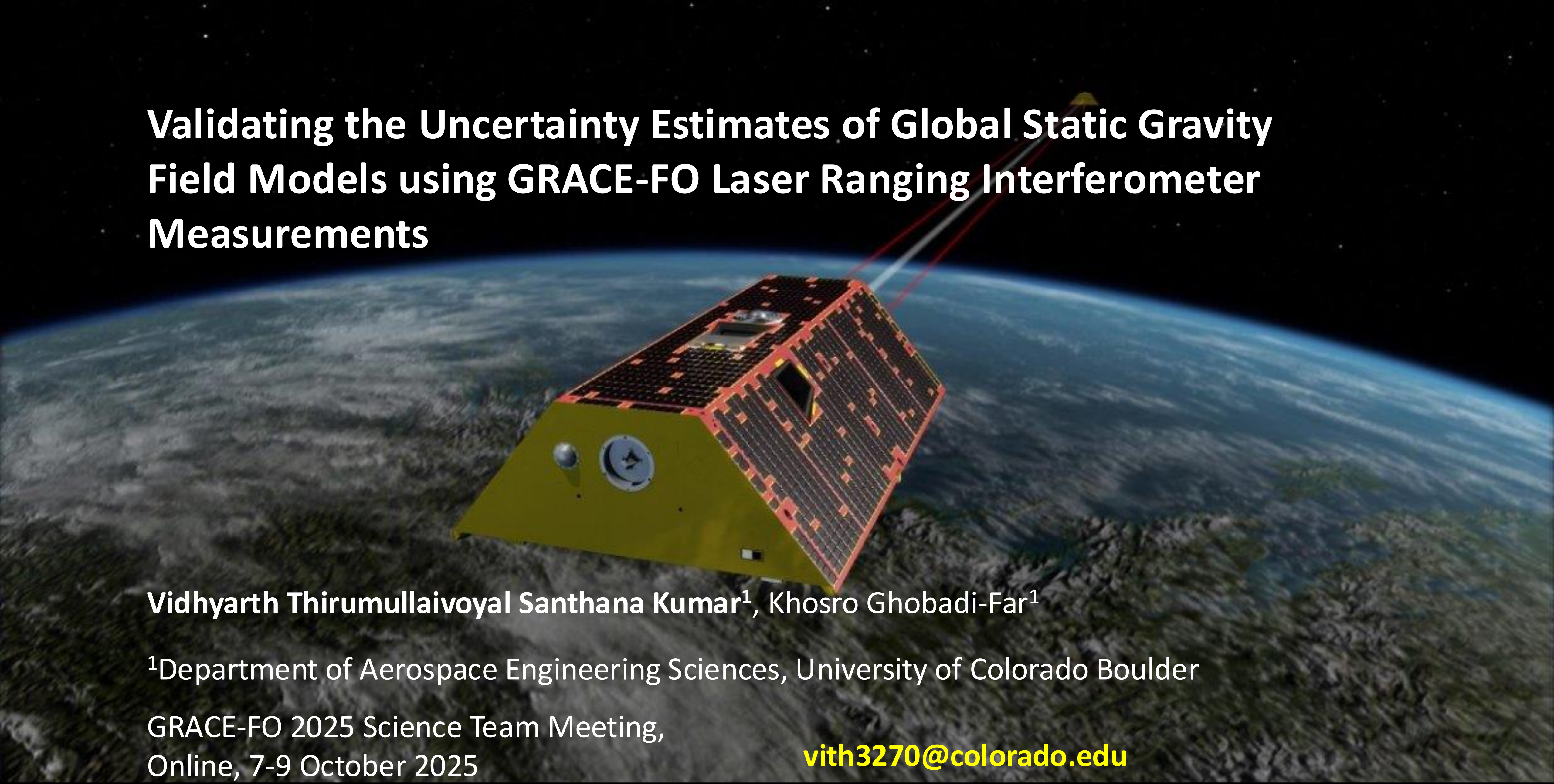

Validating the Uncertainty Estimates of Global Static Gravity Field Models using GRACE-FO Laser Ranging Interferometer Measurements

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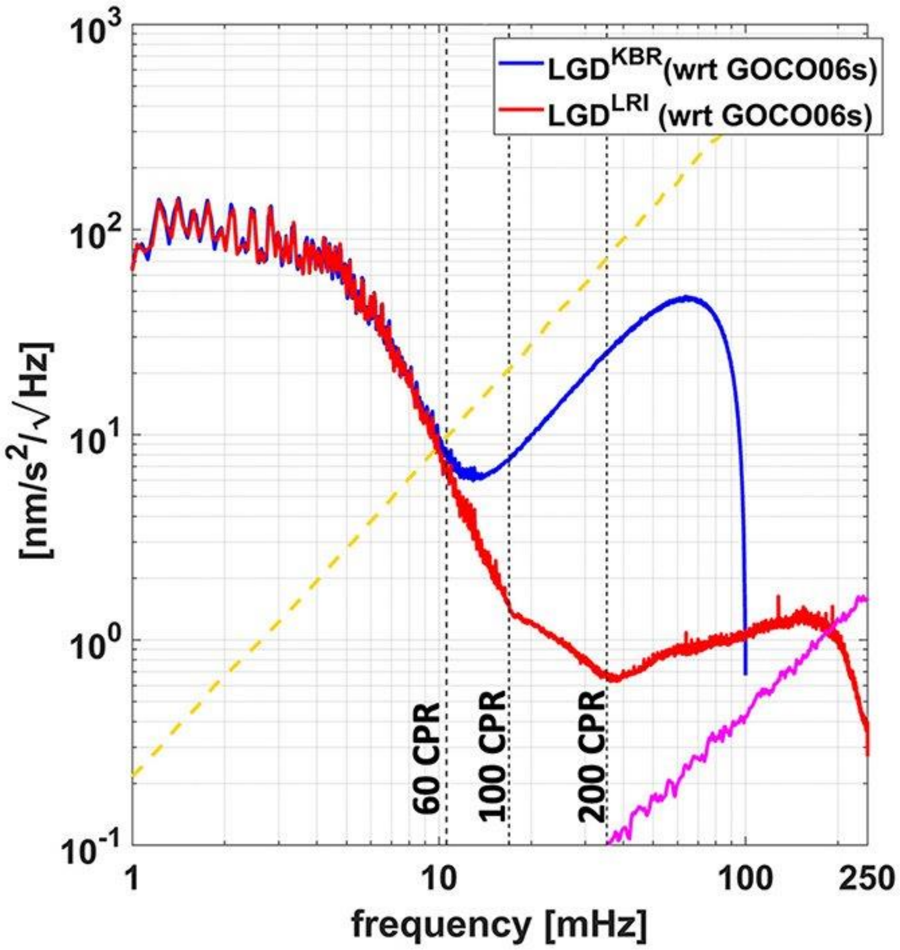
Motivation

- Global static gravity field models comes with associated uncertainty estimates.
- The errors provided are vital for various geodetic applications such as error quantification of orthometric heights.
- An essential step in gravity field modeling is the appropriate weighting of various satellite and terrestrial data.
- For this, we need a reliable error of static gravity model to use them as an a priori data.

Objectives

- To present a novel framework to validate the errors provided by the static gravity model.
- This analysis uses the Line-Of-Sight Gravity Difference (LGD) (Ghobadi-Far et al, 2018) calculated using the GRACE-FO Laser Ranging Interferometer (LRI) observations.
- LRI provides more accurate measurements over a wider range of frequencies than KBR.
- However, the inverted gravity field solutions from KBR and LRI show similar quality and spatial resolution.
- The along orbit analysis using the LGD approach has proven to be a vital way to bypass the temporal aliasing effects.

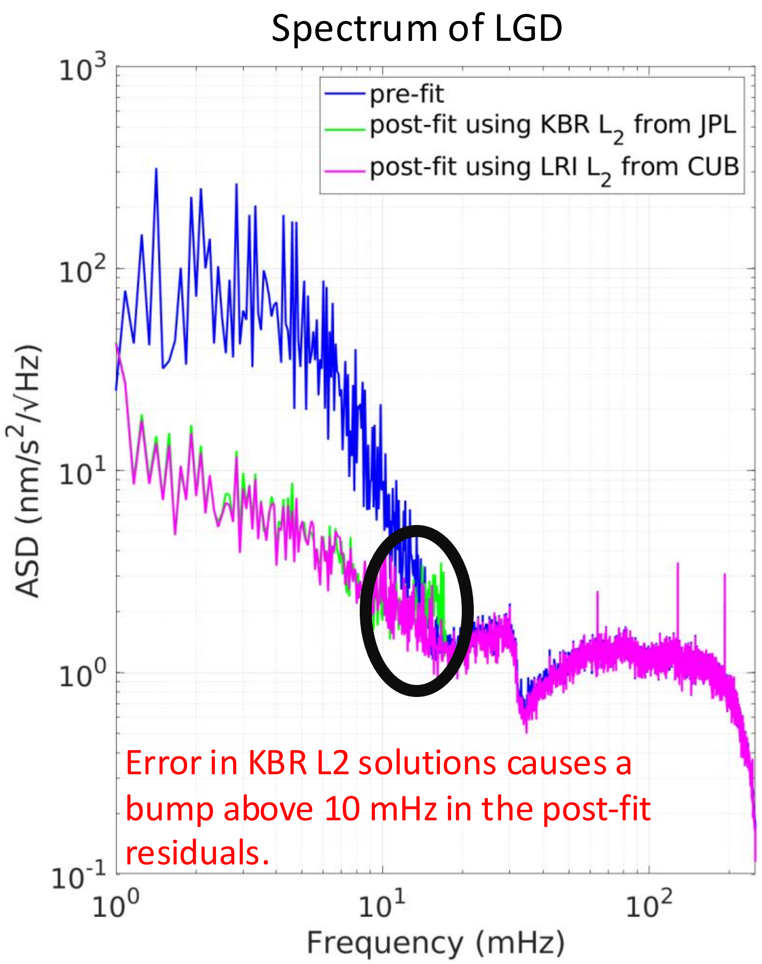


Ghobadi-Far et al, 2020

Post-fit residuals from LRI w.r.t. a static gravity model

- We get the pre-fit LGD LRI residuals ($LGD_{pre\text{-}fit}$) by subtracting the static gravity signal (LGD_{signal}) from LRI observations.
- This leaves us with un-/mis-modeled static gravity signal not properly represented by the static gravity model, as well as time-variable gravity and sensor noise.
- In this analysis, we have used the monthly L2 solutions developed at the University of Colorado Boulder (CUB) from LRI, for removing the time-variable gravity (TVG) signals from the pre-fit residuals.
- This results in the post-fit LGD LRI residuals ($LGD_{post\text{-}fit}$) to be used for validating the errors in different static gravity models.

$$\mathrm{LGD}_{\text{post-fit}} = \mathrm{LGD}_{\text{pre-fit}} - \mathrm{LGD}_{\text{TVG}}$$



Propagation of static gravity field errors to LGD

- Each static gravity model is provided with associated errors, which are used to assemble a noise covariance matrix ($\mathrm{C_{xx}}$).

$$\mathrm{C}_{xx} = \begin{bmatrix} \sigma^2_{C_{nm}} & 0 \\ 0 & \sigma^2_{S_{nm}} \end{bmatrix}$$

- Propagating the noise covariance gives us the covariance matrix (C) of LGD.

$$C = \mathrm{A}\mathrm{C}_{xx}\mathrm{A}^{\mathrm{T}}$$

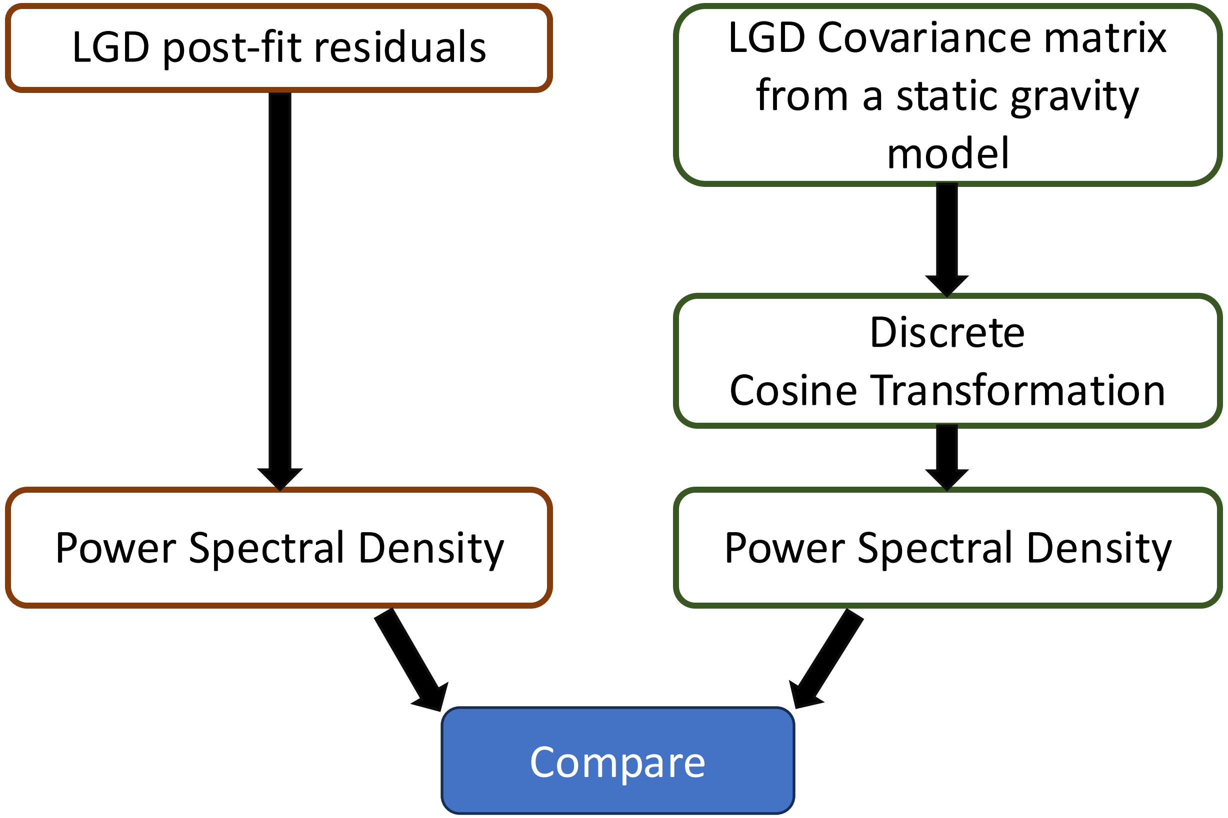
- The following static gravity models are used in this analysis:
  - GGM05C (Ries, J. et al, 2016)
  - GGM05S (Tapley, B.D. et al, 2013)
  - GOCO06s (Kvas, A. et al, 2021)
  - GOCE-TIM-R05 (Brockmann, J.M. et al, 2014)
  - GOCE-TIM-R06 (Brockmann, J.M. et al, 2021)
  - ITG-Grace2010s (Mayer-Gürr, T. et al, 2010)

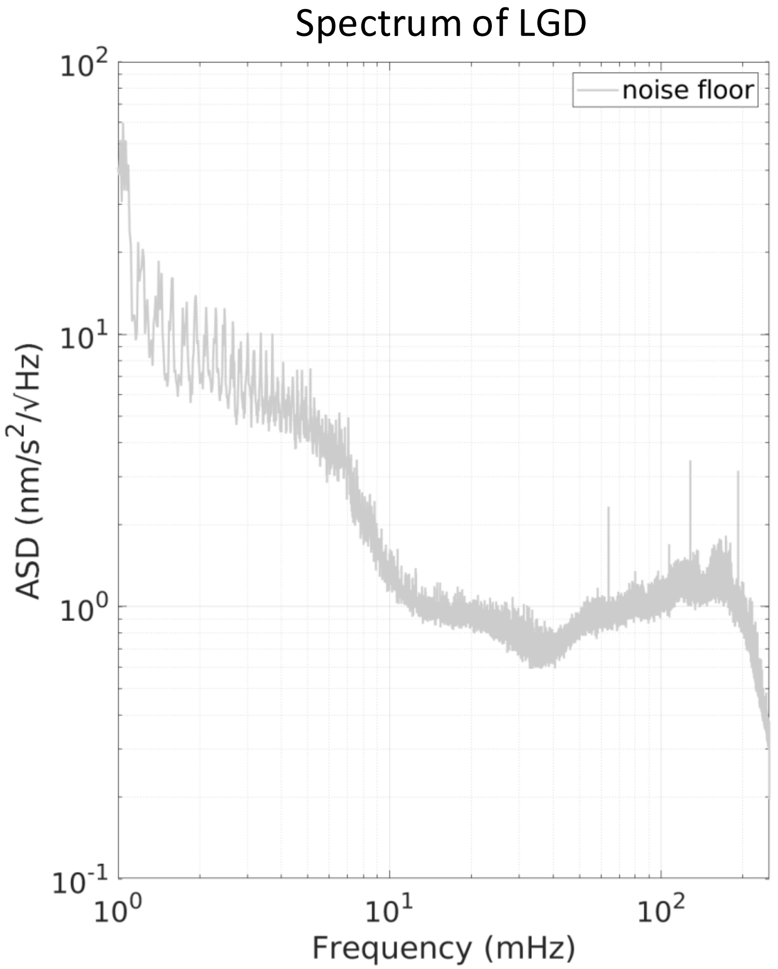
NOTE: This analysis have been carried out only using the variances provided along with the static gravity models and not the full covariance matrix (except for GOCO06s).

					$\sigma_{C_{nm}}$	$\sigma_{S_{nm}}$
key	L	M	C	S	sigma C	sigma S
end_of_head	======	======	======	======	======	======
gfc	0	0	1.000000000000D+00	0.000000000000D+00	0.00000D+00	0.00000D+00
gfc	1	0	0.000000000000D+00	0.000000000000D+00	0.00000D+00	0.00000D+00
gfc	1	1	0.000000000000D+00	0.000000000000D+00	0.00000D+00	0.00000D+00
gfc	2	0	-4.841694573200D-04	0.000000000000D+00	1.17430D-10	0.00000D+00
gfc	2	1	-3.103431067239D-10	1.410757509442D-09	4.29920D-11	4.29620D-11
gfc	2	2	2.439373415940D-06	-1.400294011836D-06	3.68360D-11	3.63870D-11
gfc	3	0	9.571647583412D-07	0.000000000000D+00	1.30040D-11	0.00000D+00
gfc	3	1	2.030446637169D-06	2.482406346848D-07	7.71580D-12	7.69980D-12
gfc	3	2	9.047646744100D-07	-6.190066246333D-07	1.17100D-11	1.17290D-11
gfc	3	3	7.212852551704D-07	1.414400065165D-06	2.34700D-11	2.34810D-11
gfc	4	0	5.399815392137D-07	0.000000000000D+00	6.73720D-12	0.00000D+00
gfc	4	1	-5.361808133703D-07	-4.735769769691D-07	5.29480D-12	5.28940D-12
gfc	4	2	3.504921442703D-07	6.625051657439D-07	7.61690D-12	7.61490D-12
gfc	4	3	9.908610311151D-07	-2.009508998058D-07	1.28570D-11	1.28620D-11
gfc	4	4	-1.884924225276D-07	3.088185785570D-07	1.34180D-11	1.33940D-11
gfc	5	0	6.865032345839D-08	0.000000000000D+00	3.20970D-12	0.00000D+00
gfc	5	1	-6.291457940968D-08	-9.434259860005D-08	2.56020D-12	2.55850D-12
gfc	5	2	6.520586031691D-07	-3.233430798143D-07	3.36790D-12	3.36650D-12
gfc	5	3	-4.518313784464D-07	-2.149423673602D-07	6.27900D-12	6.27680D-12
gfc	5	4	-2.953234091704D-07	4.981057884405D-08	7.88550D-12	7.89070D-12
gfc	5	5	1.748143504694D-07	-6.693546770160D-07	1.22840D-11	1.22710D-11
gfc	6	0	-1.499760561088D-07	0.000000000000D+00	3.22190D-12	0.00000D+00
gfc	6	1	-7.594326587940D-08	2.652568324970D-08	2.52870D-12	2.52720D-12
gfc	6	2	4.863560317995D-08	-3.737694732656D-07	3.21280D-12	3.21060D-12
gfc	6	3	5.725132649565D-08	8.973165407480D-09	4.30050D-12	4.30410D-12
gfc	6	4	-8.599693746901D-08	-4.714265243868D-07	4.43810D-12	4.43720D-12
gfc	6	5	-2.671631936205D-07	-5.364960380290D-07	6.76260D-12	6.75870D-12

Observation vs model errors

- Comparison is carried out in the spectral domain.



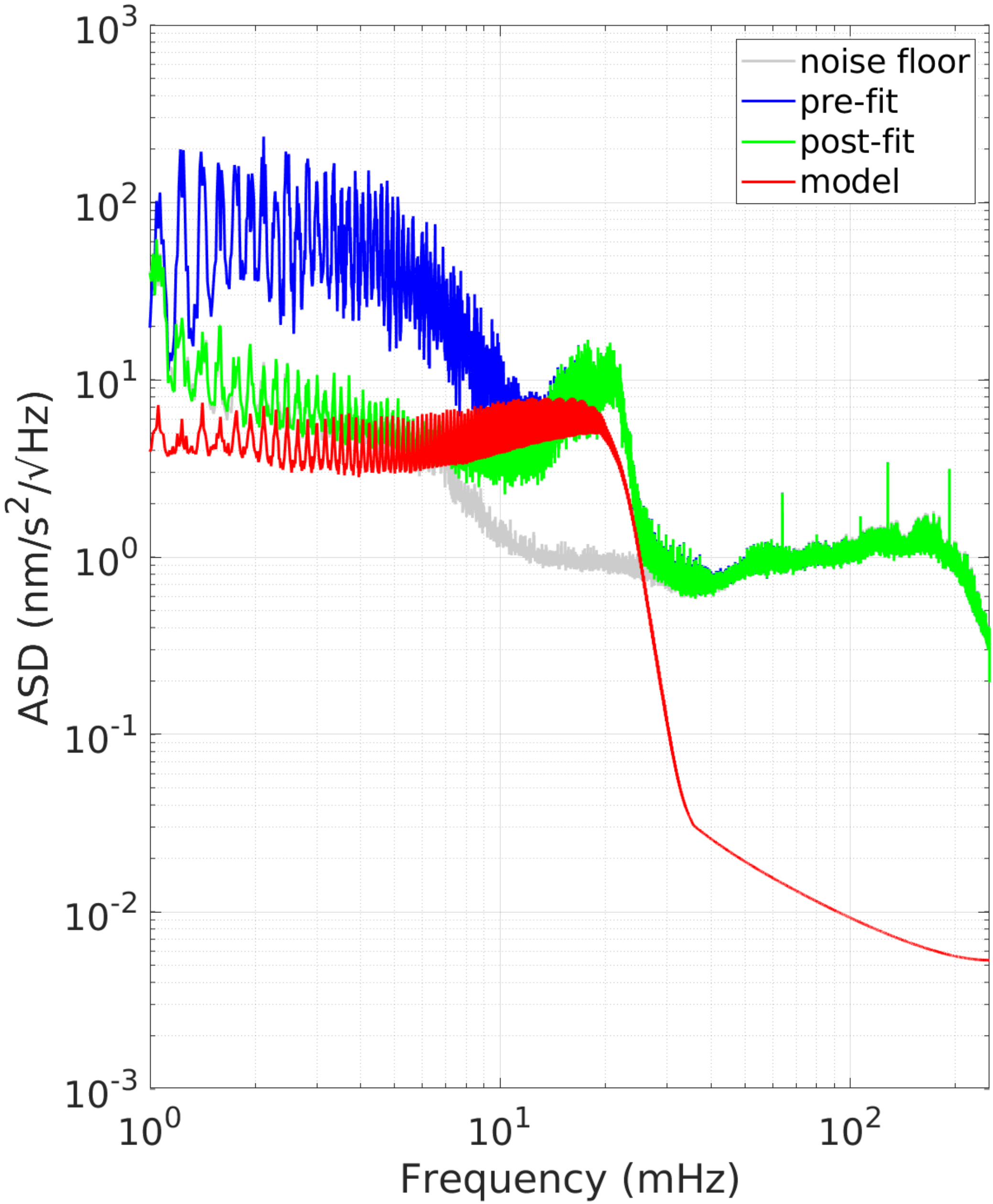


- Since the residual time-variable gravity signals dominate the post-fit residuals below 8 mHz, this analysis is valid only beyond that specific frequency.

NOTE: In our analysis, we have chosen the post-fit LRI residuals computed with respect to GOCO2025s static gravity model as the noise floor (i.e., if the magnitude of the error provided by the model is below this noise floor, we cannot validate the specific static gravity model).

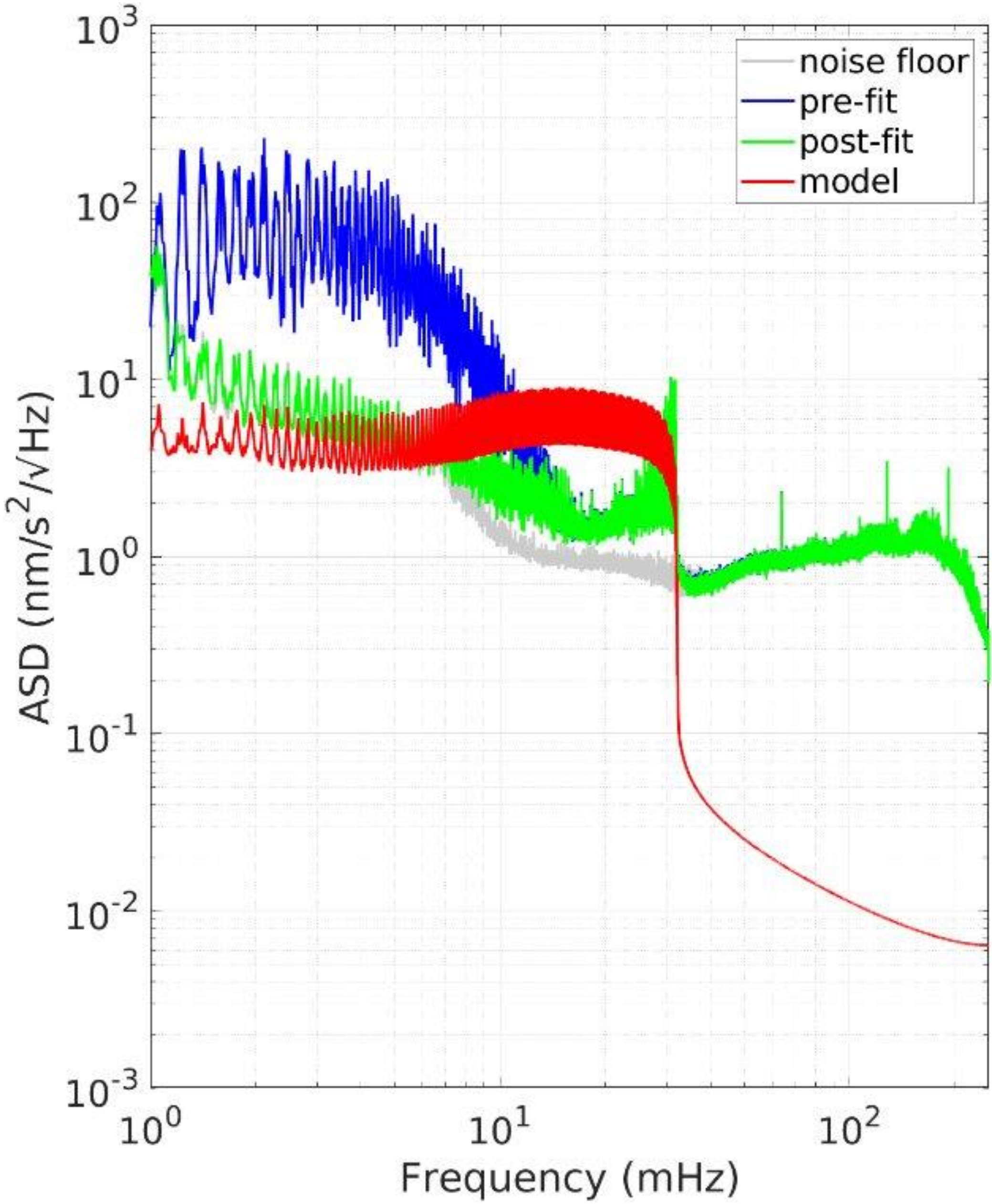
Validation of Error in Static Gravity Model: GGM05C

- Both residual and model ASDs have a falling pattern beyond ~17 mHz (~96 CPR).
- The model does a good job in providing the errors for the frequency band beyond ~20 mHz (~111 CPR).



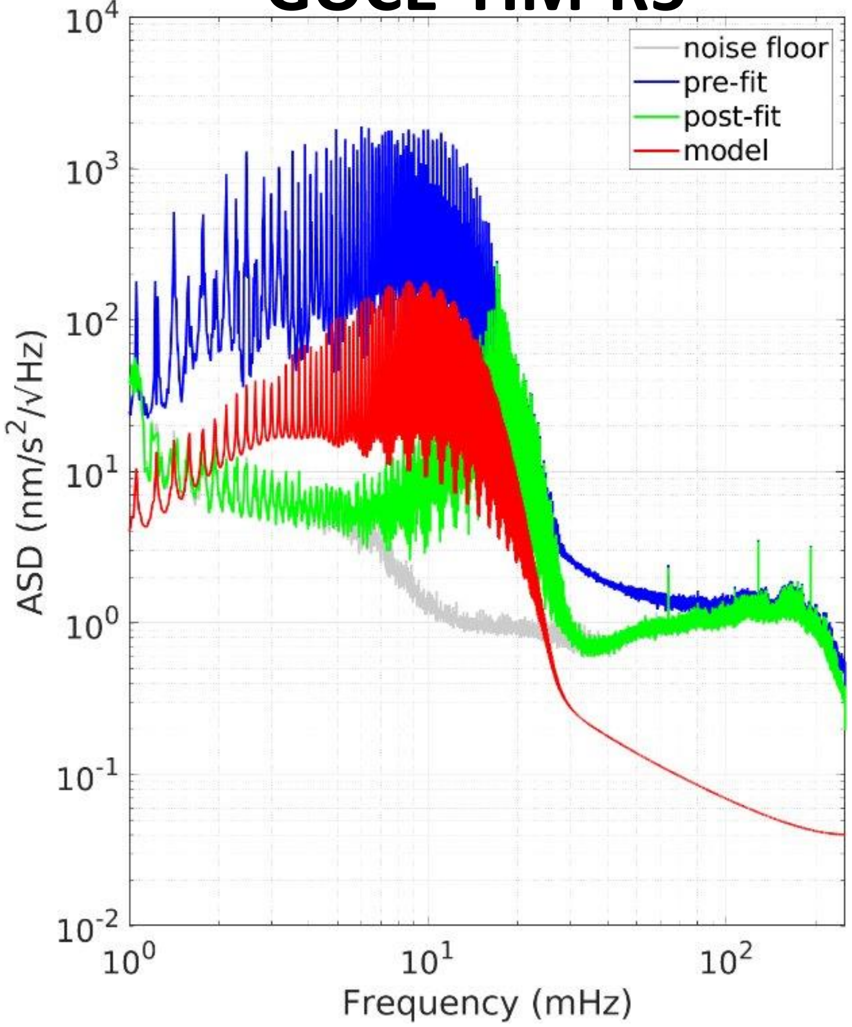
Validation of Error in Static Gravity Model: GGM05S

- This is a GRACE-only model.
- The errors provided by the model matches the post-fit residuals for the frequency band of 25 to 30 mHz (~140 to ~168 CPR).
- We can see that the error provided by this model doesn't agree with the post-fit residuals from 8 to 25 mHz (~44 CPR to ~140 CPR).

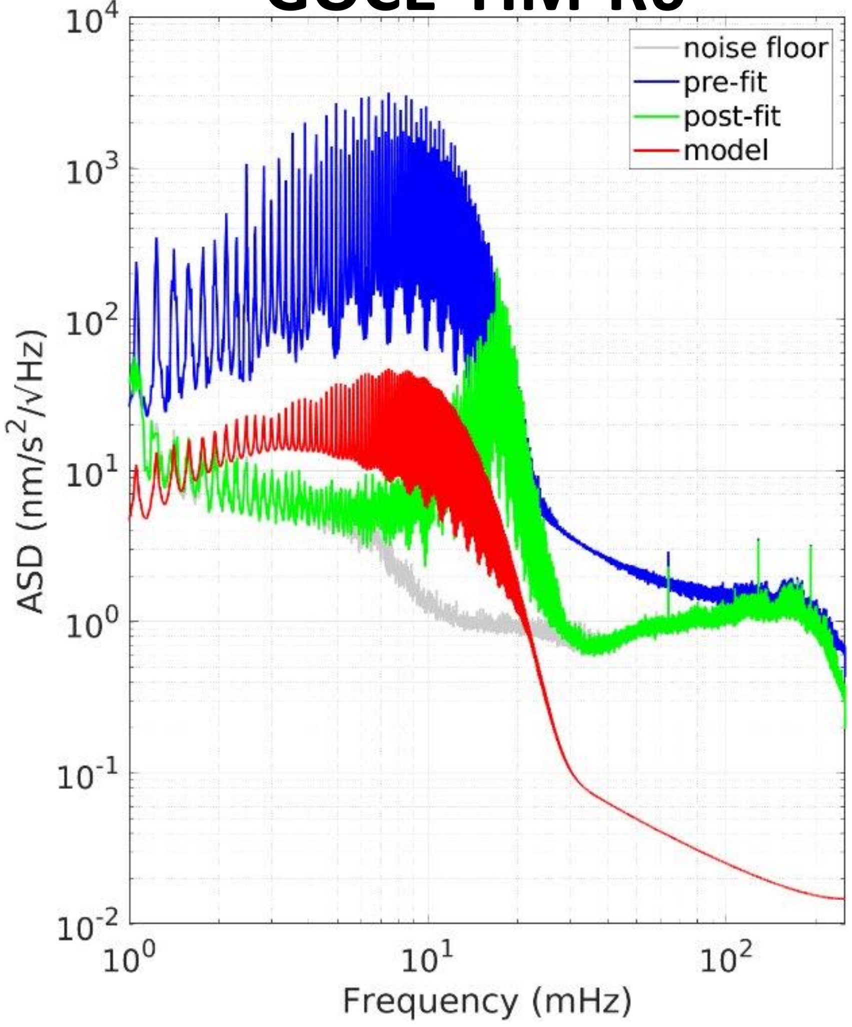


Validation of Error in Static Gravity Model:

GOCE-TIM-R5



GOCE-TIM-R6

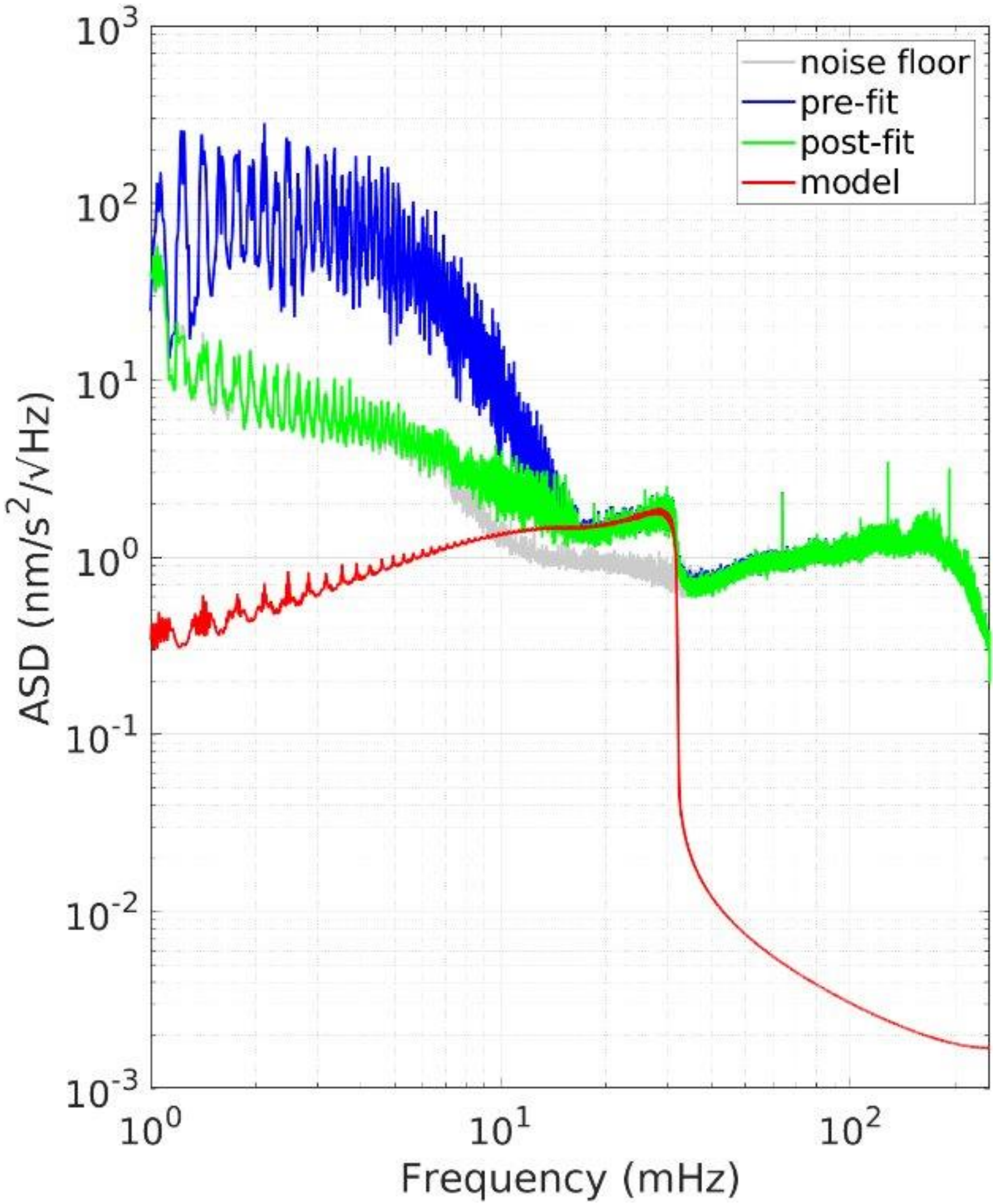


- These are pure GOCE-only models.
- Both errors provided by the RL05 and RL06 fit the post-fit residuals above 17 mHz (~96 CPR).
- But RL05 is slightly better than RL06 at these frequencies.
- This might be due to the change in the stochastic modeling approach used by the RL06 compared to RL05.

Validation of Error in Static Gravity Model: ITG-Grace2010s

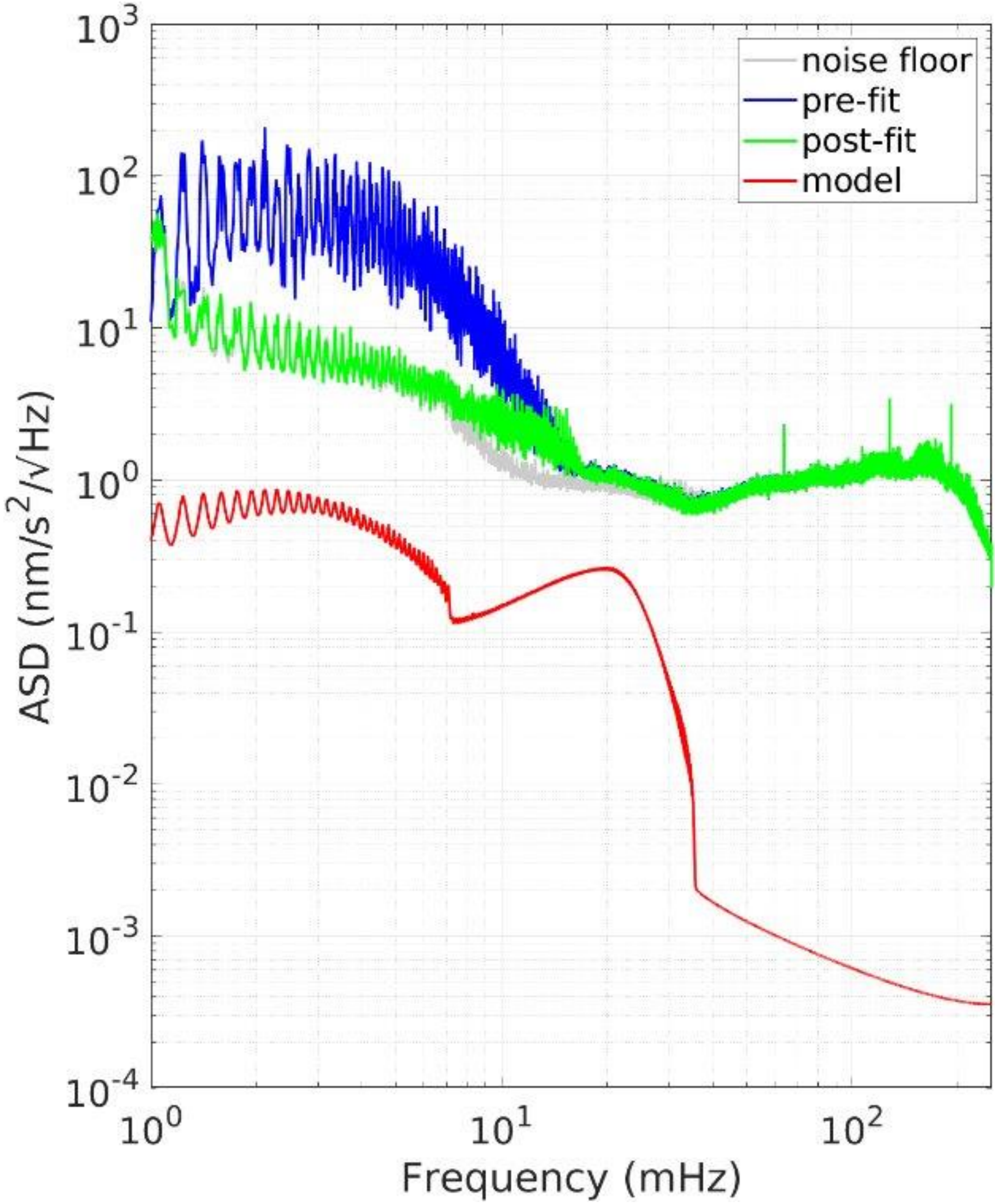
- This is a pure GRACE-only model.
- Beyond 10 mHz (~55 CPR), the error provided by this model perfectly agrees with the post-fit residuals.
- This tells us that beyond 10 mHz, the model errors are very reliable.

NOTE: Since the residual time-variable gravity signals dominate the post-fit residuals below 8 mHz, this analysis is valid only beyond that specific frequency.



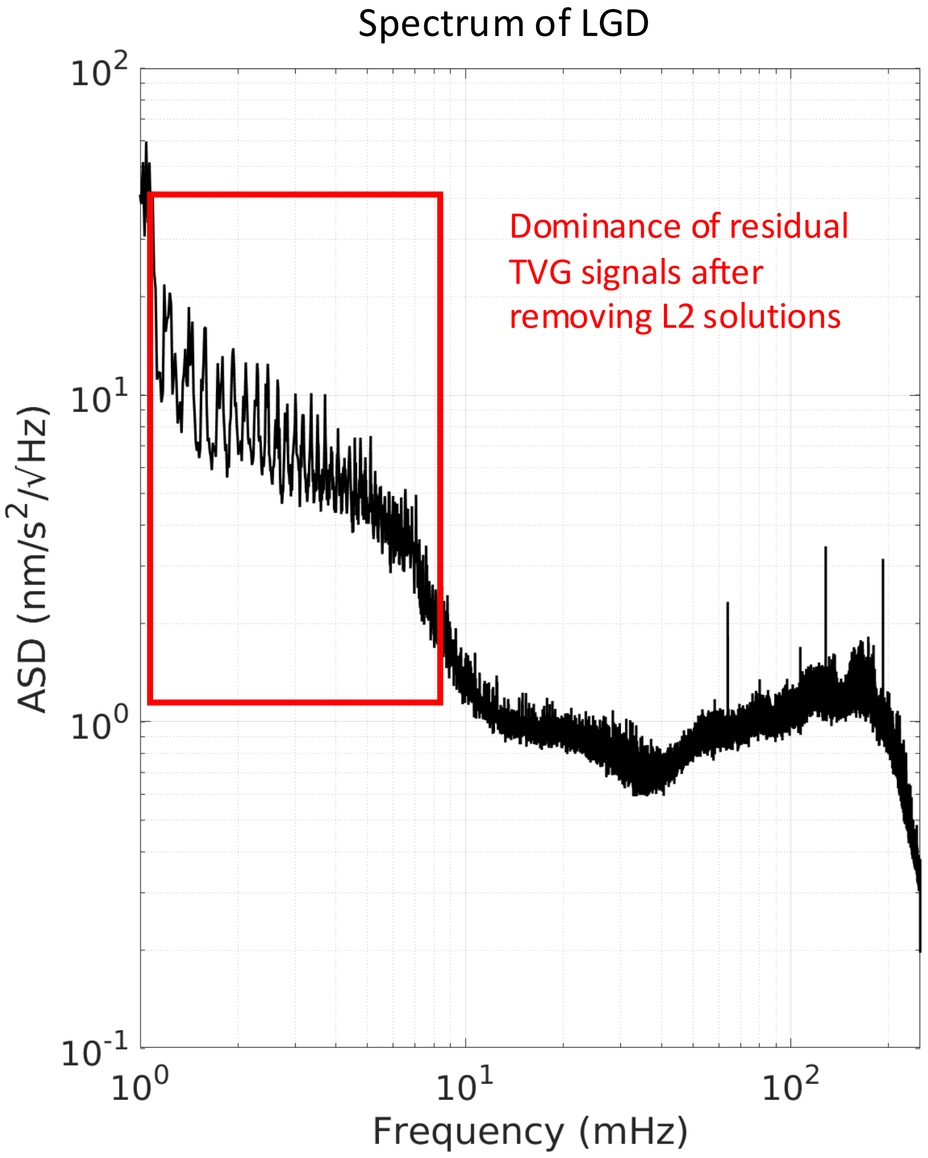
Validation of Error in Static Gravity Model: GOCO06s

- For this model, the error provided is lesser than the noise floor.
- The post-fit LRI residuals is not accurate enough to allow for evaluating the error of GOCO06s.



Limitations

- Residual time-variable gravity signals dominate the post-fit residuals at lower frequencies (below 8 mHz).
- Hence, we cannot use this analysis for validating the errors of the static gravity model below 8 mHz.
- If the model errors are below the noise floor, then this analysis cannot be used.



Post-fit residuals computed with respect to GOCO2025s model and CUB L2 LRI solution.

Conclusions

- We present a novel approach for validating the error of static gravity models.
- Our novel approach based on along orbit analysis of LRI bypasses the temporal aliasing error and allows for fully exploiting the utra-precise LRI measurements.
- We evaluated the reliability of the error information provided by various static gravity models.

Way forward

- Conduct the same analysis using the full covariance matrices of the different static gravity models.
- Perform a spatial assessment to examine how the errors are distributed across the spatial domain.

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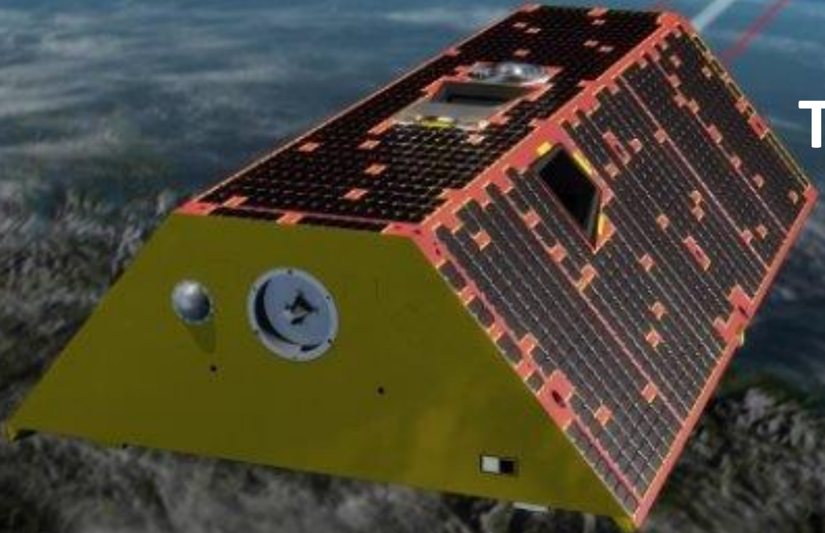

Thank you for your attention!

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