

These questions introduce the fundamental nomenclature and relationships of a planetary ionosphere.

- Hydrostatic balance
- Neutral composition
- E-region peak
- F-region peak
- Solar zenith angle
- Plasma density and critical frequency
- Wind and electric field induced plasma drifts
- Total electron content of the ionosphere
- Slab thickness

1. The composition of the thermosphere is well mixed in the lower altitude region. However above this altitude, from 110 km, the neutral species separately obey their hydrostatic balance equations. Given that the neutral composition $N_2:O_2:O$ is 60:10:30 at 150 km and that the neutral temperature is 1000 K, calculate the neutral composition at 250 km assuming that the temperature and gravity is constant over this altitude range.

Scale Heights:

$$H(N_2) = 30.083 \text{ km}$$

$$H(O_2) = 26.322 \text{ km}$$

$$H(O) = 52.644 \text{ km}$$

Density equation: $n(z) = n(z_0) \exp(-(z-z_0)/H)$

$$N_2: n(z_0)=60 \quad n(z) = 2.1602$$

$$O_2: n(z_0)=10 \quad n(z) = 0.2239$$

$$O : n(z_0)=30 \quad n(z) = 4.4891$$

Sum of the “relative” densities 6.8732

Hence computing the composition ratios at $z = 250 \text{ km}$

$$\text{Percentage } N_2 = 31.42\%$$

$$\text{Percentage } O_2 = 3.26\%$$

$$\text{Percentage } O = 65.31\%$$

2. If in Question 1 the N₂ density at 150 km is $2 \times 10^{11} \text{ cm}^{-3}$ what is the thermospheric density at 250 km?

Densities at 150 km

$$\text{N}_2: n(z_0) = 2 * 10^{11}$$

$$\text{O}_2: n(z_0) = 0.3333 * 10^{11}$$

$$\text{O} : n(z_0) = 1.0 * 10^{11}$$

At 250 km using question 1 percentages obtain

$$\text{N}_2: n(z) = 0.7201 * 10^{10}$$

$$\text{O}_2: n(z) = 0.07463 * 10^{10}$$

$$\text{O}: n(z) = 1.4964 * 10^{10}$$

Total density at 250 km is $2.291 * 10^{10} \text{ (cm}^{-3}\text{)}$

3. If during a geomagnetic storm the thermospheric temperature is increased to 1500 K, while at 150 km the density is unchanged, how much is the density increased at 250 km?

Increasing temperature changes the scale height.

$$H(\text{N}_2) = 45.1238 \text{ km}$$

$$H(\text{O}_2) = 39.483 \text{ km}$$

$$H(\text{O}) = 78.966 \text{ km}$$

Computing the relative densities at 250 km

$$\text{N}_2: n(z_0) = 60 \quad n(z) = 6.5418$$

$$\text{O}_2: n(z_0) = 10 \quad n(z) = 0.7944$$

$$\text{O}: n(z_0) = 30 \quad n(z) = 8.4556$$

$$\text{Sum of the fractions} = 15.7918$$

Hence at 150 km relative density =100

And at 250 km relative density = 15.7918

Therefore decrease as a percentage $(100 - 15.7918)\% = 84.21\%$

4. Calculate the solar zenith angle at local noon, on 23 December, for the geographic location of 29 degrees?

December 23, winter north hemisphere implies the sun is 23.5 degrees below the equatorial plane. At noon all the geometry between sun – center of Earth – and location is in a plane.

Hence solar zenith angle is $23.5 + 29.0 = 52.5$ degrees.

5. The ionospheric E-region at mid and low latitudes is mainly a balance between production by solar photons and chemistry mechanisms. Empirically the equation:

$$f_oE = 3.3 \left[(1 + 0.008\mathfrak{R}) \cos\psi \right]^{0.25} \text{ MHz}$$

where f_oE is the peak E-region critical frequency and \mathfrak{R} and ψ are the solar Sun spot number and solar zenith angle respectively. Calculate the maximum E region density N_mE for solar minimum and for a strong solar maximum.

The maximum density is given when solar zenith angle is zero, hence $\cos(\text{SZ}) = 1$.

Solar Minimum:

Sun Spot Number = 0

$f_oE = 3.3 \text{ MHz}$, and $n_mE = 1.24 * 10^{**4} * f_oE^{**2} = 1.35 * 10^{**5} \text{ (cm}^{**-3}\text{)}$

Solar Maximum:

Sun Spot Number = 200

$f_oE = 4.19 \text{ MHz}$, and $n_mE = 2.177 * 10^{**5} \text{ (cm}^{**-3}\text{)}$

6. The ionospheric F-region does not have as simple a chemistry/physics scheme as that for the E-layer. However, the annual averaged noon $\overline{f_oF_2}$ rule of thumb, empirical relationship is:

$$\overline{f_oF_2} = C(1 + 0.02\mathfrak{R})^{1/2}$$

where $\overline{f_oF_2}$ is the critical plasma frequency at the F-layer peak, C is a constant that is location dependent, and \mathfrak{R} is the Sun spot number. Calculate the solar maximum plasma density N_mF_2 given that at solar minimum $\overline{f_oF_2}$ is 4.4 MHz.

Solar Minimum:

Sun Spot Number = 0

Hence given average $f_oF_2 = 4.4 \text{ MHz}$ giving the constant a value $C = 4.4 \text{ MHz}$

Solar Maximum:

Sun Spot Number = 200

Hence average foF2 = 4.4 SQRT(1. +0.02*200) = 9.839 MHz

$$nmF2 = 1.24 * 10^{**4} * 9.839^{**2} = 1.20 * 10^{**6} \text{ (cm}^{**3}\text{)}$$

7. A horizontal neutral wind in the upper thermosphere can drive plasma either upwards or downwards because the electrons and ions are bound to magnetic field lines even although the flowing neutral gas is colliding with the plasma. How large is the vertical induced plasma drift if an equatorward, meridional, neutral wind of 50 m/s is present at a mid-latitude location where the local magnetic dip angle is 45°.

$$\text{Induced vertical drift} = 50 * \cos(45) * \sin(45) = 25.00 \text{ m/s}$$

8. At the same mid-latitude location an eastward electric field leads to a vertical drift. Calculate this induced drift if the eastward electric field has a magnitude of 15 mV/m.

$$\text{Magnitude of E} = 15 * 10^{**3} \text{ V/m}$$

$$\text{Magnitude of B} = 0.45 * 10^{**4} \text{ T (this is an estimate!)}$$

$$V = E/B = 15 * 10^{**3} / 0.45 * 10^{**4} = 333.3 \text{ m/s}$$

$$\text{Vertical component is } \text{vertV} = V * \sin(45) = 235.7 \text{ m/s}$$

9. The diagram shows three ionospheric electron density profiles. Determine the total electron content (TEC) for each profile. Calculate the slab thickness for each profile.

Profile c estimated TEC = 12. Tecu, and nmF2 = 8 * 10^{**5} (cm^{**3}), slab thickness = 140 km.

