7. It is possible to determine the properties of a shock using measurements of the magnetic field $\mathbf{B}$, the proton density n, and the velocity $\mathbf{v}$ relative to the spacecraft. This must be done without assuming knowledge of the shock's normal $\hat{\mathbf{n}}$ or speed v_s. Of course the problem is complicated considerably by the presence of numerous fluctuations on top of what would otherwise be a simple shock. Sophisticated methods have been developed to pull the shock properties out of the fluctuating data. Here we demonstrate, using ideal data, the principle with calculations simple enough to perform on a hand calculator. Two sets of ideal measurements have been made by a spacecraft moving radially outward from the Sun (the R direction) at $v_0 = 20$ km/sec. Velocities are expressed in the (R, T, N) coordinate system, relative to the spacecraft.

	17:25UT (first)	17:49UT (second)
n [cm^{-3}]	11.62	4.25
v_R [km/s]	349.5	295.9
v_T [km/s]	64.80	79.14
v_N [km/s]	37.98	59.75
B_R [nT]	0.027	1.000
B_T [nT]	-1.759	-0.908
B_N [nT]	-2.931	-1.474

a. Which of the measurements, first or second, is the pre-shock (upstream) region? Is the radial component of the shock normal, $\hat{\mathbf{n}} \cdot \hat{\mathbf{r}}$, positive or negative?

b. Using the jump condition $[\![B_n]\!] = 0$ show that the shock normal $\hat{\mathbf{n}}$ must be orthogonal to $[\![\mathbf{B}]\!]$.

c. Use the jump condition

$$\rho v_n [\![\mathbf{v}_\perp]\!] \;=\; \frac{B_n}{4\pi}[\![\mathbf{B}_\perp]\!] \quad ,$$

to show that the velocity jump $[\![\mathbf{v}]\!]$ must lie in the plane spanned by $\hat{\mathbf{n}}$ and $[\![\mathbf{B}]\!]$. This condition is known as *co-planarity*. Does it matter which reference frame $\mathbf{v}$ is measured in to apply this principle?

d. Use the facts established in parts b. and c. to compute the shock normal $\hat{\mathbf{n}}$, from the data. This will naturally be expressed as components in (R, T, N) coordinates. How do you choose between 2 options?

e. Use $\hat{\mathbf{n}}$ and the magnetic field vectors to compute θ_1 and θ_2.

f. Use the two relations

$$\frac{n_2}{n_1} \;=\; \frac{M_{A1}^2}{M_{A2}^2} \quad , \quad [\![(M_A^2 - 1)\tan\theta]\!] \;=\; 0 \quad ,$$

to find the Alfvén Mach numbers M_{A1} and M_{A2} for this shock.

g. What kind of shock is this? Fast, slow or intermediate?

h. Use the definition of the Alfvén Mach number

$$M_A \;=\; \frac{v_n}{B_n/\sqrt{4\pi\rho}} \quad ,$$

to compute the velocity of the shock *relative to the spacecraft.*