

1. The magnetic moment of Earth is approximately  $31,000 \text{ nT} \cdot R_E^3$ . The solar wind has a bulk velocity of  $400 \text{ km/sec}$  and a density of  $5 \text{ cm}^{-3}$  at 1 AU. Assume a very simple model of Earth's magnetopause in which it is a plane whose normal is parallel to the x axis.

- (a) Compute the location of the subsolar magnetopause.
- (b) Using the result of (a), calculate the magnetic field at the subsolar point.
- (c) Find the locations of the magnetic nulls on the magnetopause surface.
- (d) Assuming that the solar wind velocity is constant from the orbit of Mercury to the orbit of Earth, repeat (a) and (b) for Mercury, using the fact that Mercury's dipole moment is  $3 \times 10^{12} \text{ T} \cdot \text{m}^3$ .
- (e) Assuming that the solar wind velocity is constant from the orbit of Earth to the orbit of Jupiter, repeat (a) and (b) for Jupiter, using the fact that Jupiter's dipole moment is  $1.58 \times 10^{20} \text{ T} \cdot \text{m}^3$ .

2. Assume that the boundary of Earth's polar cap is a circle centered on Earth's dipole axis and that the angular distance from the magnetic north pole to the polar cap boundary is 15 degrees. Assume further that the solar wind at 1 AU has the following properties: the number density is  $5 \text{ cm}^{-3}$ , the speed  $400 \text{ km/sec}$ , the temperature  $10 \text{ eV}$ , and the magnetic field magnitude is  $5 \text{ nT}$ . Define the magnetotail lobe as a magnetic flux tube whose intersection with Earth's surface is the polar cap and whose geometry in the distant magnetotail can be approximated as a circular cylinder with axis parallel to the solar wind velocity vector.

- (a) Assume that the magnetic flux is uniformly distributed within the tail lobes. Use force balance normal to the lobe surface to compute the radius of the magnetotail lobe in the distant magnetotail.
- (b) Again assume that the magnetic flux is uniformly distributed within the tail lobes, and suppose that the plasma number density in the lobes is  $0.01 \text{ cm}^{-3}$ . Estimate the potential difference across the polar cap required to transport lobe magnetic flux toward the center of the plasma sheet at a tenth of the Alfvén speed.

3. Assume the 1 AU solar wind conditions of problem 2. Assume further that the solar wind flow along the Sun-Earth line is axisymmetric and incompressible, with a component along the Sun-Earth line that decreases linearly from its value just downstream of the bow shock to zero at the magnetopause. If the IMF is  $(0, 0, -5) \text{ nT}$ ,

- (a) Compute the distance along the Sun-Earth line of the bow shock from the magnetopause.
- (b) Compute the density, bulk velocity and magnetic field just downstream of the bow shock on the Sun-Earth line.
- (c) Assuming that ideal MHD is valid everywhere in the magnetosheath, compute the magnetic field along the Sun-Earth line.
- (b) Compute the plasma inflow (in units of the local Alfvén speed) a distance of  $200 \text{ km}$  along the Sun-Earth line upstream of the magnetopause.