

Planetary Dynamos & Magnetospheres - Problems

1 – *What if...?*

- A. How would the magnetosphere change if you moved the Earth to 10 AU?
- B. What would happen if you doubled Earth's dipole magnetic field strength? Keeping Earth at 1 AU
- C. What would happen if Earth spun x3 faster? i.e. 1 day = 8 hours?
- D. What would happen to the magnetosphere if you moved Jupiter to 0.1 AU?

2 - *Order of Magnitude Estimates*

- A. Information is transferred along magnetic field lines by Alfvén waves. Estimate the time an Alfvén wave takes to “tell” the ionosphere that reconnection has happened on the dayside magnetopause. Do this for both Earth and either Mercury or Jupiter.
- B. A fluxtube reconnects on the dayside magnetopause. Estimate the time it takes the end of the fluxtube that is in the magnetosheath to travel from the dayside magnetopause to the distance of the X-line in the magnetotail – say, $5 R_M$ for Mercury, $100 R_E$ for Earth, $200 R_J$ for Jupiter.
- C. The other end of the above recently-reconnected fluxtube is in the ionosphere. If the ionospheric end of the fluxtube traverses the polar cap in the same time as the “free” end traverses down the tail to the X-line, what sort of speed is implied for the anti-sunward flow in the ionosphere? Again, both Earth and either Mercury or Jupiter, please.

3 - *Length of Magnetotail*

The polar cap is the region of open field lines – lines that are attached to the Earth at one end and are swept along by the solar wind at the other. Observations at high latitudes suggest that the Earth’s polar cap boundary is at the magnetic latitude of about 78°

- A. What is the total magnetic flux through the Earth’s polar cap?
- B. What is the total magnetic flux through the region of open field lines on the magnetopause?
- C. Making some simple assumptions, calculate the area of the region of open field lines on the magnetopause.
- D. Observations of the ionosphere indicated that there is a total potential drop of about 65kV. Estimate the width of the region of open field lines on the magnetopause.
- E. Now estimate the length of the magnetotail.
- F. The distance to the Moon is $60 R_E$. How does this compare with the length of the magnetotail? How often does the Moon spend in the magnetotail?
- G. Estimate the fraction of the whole magnetopause that is threaded by open field lines.

4 - *The Curie Temperature*

The Curie Temperature is the temperature at which a rock loses magnetization - when a magnetic mineral such as magnetite cools below this temperature it maintains a permanent magnetic field - the background field in which the rock was embedded becomes "frozen" into the rock. So, surface rocks on Earth and Mars can have remnant magnetization, indicative of the magnetic field of the planet at the time that the rock solidified below the Curie Temperature. Similarly, the increasing temperature with depth inside planets tells us that the internal magnetic fields of planets cannot be generated by remnant magnetization – there must be magnetic dynamos in convecting regions of electrically conducting materials. How deep do you need to go to get to the Curie Temperature of magnetite (580K)?

A. It is reasonable to assume that we know the temperature on the surface of the planet. So, the question is how steep is the temperature gradient with depth? Write down an equation that says: HEAT FLUX (energy per second per unit area) can be written as the conductivity times the temperature gradient. (Check units work.)

B. Assuming that for short distances the temperature gradient can be written as $\Delta T / \Delta z$, (where $\Delta T = T_2 - T_1$, $\Delta z = z_2 - z_1$) re-write the equation in terms of the unknown quantity Δz .

C. Taking a value of heat conductivity of rocks being $1.5 \text{ W m}^{-1} \text{ K}^{-1}$ and that the heat flux from the Earth is about $0.055 \text{ Watts m}^{-2}$, estimate the depth at which the temperature reaches the Curie Temperature.

D. Do you need to go deeper/shallower to reach this temperature in Mars vs. Earth? Explain your reasoning.

5 – Getting a feel for the Earth’s magnetic field

A. How fast are the flows in the Earth’s geodynamo? Could you walk/run/drive that speed?

B. How fast is the Earth’s north pole moving? Or south – are they the same? Could you walk/run/drive that speed?

C. How much stronger/weaker is the field at the top of the Earth’s core than the surface?

D. By comparing the power in $n=2$ and $n=3$ harmonics, what can you say about the symmetries of Earth’s magnetic field?

6 – Outflow of Io-genic plasma in Jupiter’s magnetosphere

A – Io produces about 1 ton per second of plasma – i.e. 3×10^{28} ions per second (a mixture of sulfur and oxygen). Galileo measured the radial profile of plasma density to decrease with distance as $n(R) = 2000 \text{ (cm}^{-3}\text{)} (R/6)^{-7}$ where R is in R_j (where $1 R_j = 71400 \text{ km}$). The plasma is confined to a plasma sheet of about $2 R_j$ thickness. Assuming that there are no additional sources or losses, and that the plasma sheet can be approximated as a uniform cylinder, use conservation of mass flux, the above radial profile and production rate to derive the profile of radial outflow $V(R)$.

B – Where does the radial outflow speed equal the corotation speed? $V_{co} = 12.6 R \text{ km/s}$ where R is in R_j

C – Plasma is thought to be lost down the magnetotail in plasmoids. Make some reasonable estimates of plasmoid size (e.g. thickness of plasma sheet, fraction of tail width, etc) and estimate the rate of plasmoid ejection down the tail to match the source at Io. Is your estimate consistent with the Vogt et al. (2010) observations of 249 events between 1996 and 2003? Slide 69.