Problem Sheet: Planetary outer atmospheres
Marina Galand
July 2013

1. 1. The more energetic an auroral electron, the deeper in the atmosphere it is likely to be thermalized.

2. The more energetic a solar photon, the deeper in the atmosphere it is likely to be absorbed.

3. The use of recombination coefficients is enough to derive the electron density from the electron production rate in a region where transport is dominant.

4. Let's consider two wavelengths, \( \lambda_1 \) and \( \lambda_2 \), with \( \lambda_1 > \lambda_2 \) and a photo-absorption cross section \( \sigma(\lambda) \) associated with the dominant neutral species present in the atmosphere. If \( \sigma(\lambda_1) < \sigma(\lambda_2) \), then solar photons of wavelength \( \lambda_1 \) are going to deposit their energy deeper in the atmosphere than the more energetic solar photons of wavelength \( \lambda_2 \).

5. At Jupiter, the main aurora is primarily induced by the interaction of the planet with the space environment.

6. Aurora is observed throughout the Solar System and can be used as a fingerprint of atmospheric species and a tracer of plasma processes and magnetic field line configuration.

7. The solar flux at Neptune is 9 times less than at Saturn.

8. Solar photons of 180 nm are effective ionizers.

9. For a thermal electron population, it is possible to define a temperature.

10. Photochemical equilibrium applied to ionospheric plasma means thermal electron production rate equals thermal electron loss rate.

11. The profile in altitude of the electron density always peaks at the same altitude as the profile in altitude of the electron production rate.

12. In the ionospheric region, the ion densities are several orders of magnitude lower than the neutral densities.

13. Both ionospheric electrons and photoelectrons are thermal.

2. Short Problems.

   (i) At which distance from the Sun should Uranus be located to experience a solar power input equal to the auroral power input, which it undergoes at its current location? Express the solution in AU.

   (ii) The spectroscopic analysis of H\(_2\) Lyman and Werner emissions can be used to derive the energy of incident auroral electrons over the 10-200 keV energy range. Why is softer electron precipitation not detected by this technique?
3. Let’s focus on the ionosphere of Saturn. Assume in this problem that $\text{H}_2$ and $\text{H}_3^+$ are the dominant neutral and ion species, respectively, and that all $\text{H}_2^+$ ions are converted to $\text{H}_3^+$ ions. The electron temperature is assumed to be 600 K.

(i) The nightside ionosphere at high latitudes is under auroral electron precipitation with the electron number density having reached $2 \times 10^4$ cm$^{-3}$ at an altitude $z$ of 1300 km above the 1 bar level. There is a sudden increase in the electron precipitation level yielding an additional 100 cm$^{-3}$s$^{-1}$ in electron production rate.

(a) Calculate the electron number density at 1300 km after the increase in electron precipitation. By which factor has the electron number density increased? How would a significant increase in electron temperature, as a result of the precipitation intensification, affect the electron density?

(b) If the electron bombardment stops totally, how long will it take to have the electron density reduced by a factor of 2? of 10?

(ii) At low latitudes, under sunlit conditions the peak $\text{H}_3^+$ number density has reached a value of $5 \times 10^3$ cm$^{-3}$.

What is the effect of an influx of water from the rings? Quantify your response. The water number density at this ionospheric region is about $10^5$ cm$^{-3}$. 
**Ionization sources**

- **Ionisation potential:**
  - $\text{H}_2$: $15.43 \text{ eV} \leftrightarrow 80 \text{ nm}$
  - $\text{H}$: $13.60 \text{ eV} \leftrightarrow 91 \text{ nm}$
  - $\text{CH}_4$: $12.55 \text{ eV} \leftrightarrow 99 \text{ nm}$

- **Solar EUV radiation:**
  - Solar flux / (Sun-planet distance)$^2$

- **Energetic particles** from the space environment
  - A few keV to a few 100s keV

**Energy sources**

<table>
<thead>
<tr>
<th></th>
<th>Solar EUV input*</th>
<th>Auroral input*</th>
<th>Auroral particle input**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth</td>
<td>500 GW (1x10$^3$ W/m²)</td>
<td>80 GW</td>
<td>1-10 keV</td>
</tr>
<tr>
<td>Jupiter</td>
<td>800 GW (1.3x10$^{-5}$ W/m²)</td>
<td>10$^5$ GW</td>
<td>30-200 keV 2-30 mW m$^{-2}$</td>
</tr>
<tr>
<td>Saturn</td>
<td>200 GW (4.4x10$^6$ W/m²)</td>
<td>(5-10)x10$^3$ GW</td>
<td>10-20 keV 10 C200 keV</td>
</tr>
<tr>
<td>Uranus</td>
<td>8 GW</td>
<td>100 GW</td>
<td>-</td>
</tr>
<tr>
<td>Neptune</td>
<td>3 GW</td>
<td>1 GW</td>
<td>-</td>
</tr>
</tbody>
</table>

* Auroral input refers to “particle + Joule heating” (Strobel 2002)

** Values valid for the main auroral oval, inferred from the analysis of auroral emissions (e.g., Fox et al. 2008, Gustin et al. 2004, 2009)
Absorption of solar radiation in an atmosphere

✓ In the EUV, primarily extinction in the beam
  → apply Beer-Lambert Law:
\[
\frac{dI_\lambda(s)}{I_\lambda} = -\sum_i \sigma_{i,\text{abs}}(\lambda) n_i(s)
\]

✓ Attenuated solar flux at wavelength \( \lambda \) and at altitude \( z \):
\[
I_\lambda(z) = I_\lambda^\infty \exp \left( -\sum_i \sigma_{i,\text{abs}}(\lambda) \int_z^{\infty} n_i(z') \sec(\chi) \, dz' \right)
\]

✓ Photoelectron production rate at \( \lambda \):
\[
P_{e,\lambda}(z) = \sum_i \sigma_{i,\text{ion}}(\lambda) n_i(z) I_\lambda(z) \propto I_\lambda^{\text{TOA}}
\]

Photo-chemistry in an \( \text{H}_2 \) atmosphere

- Charge exchange reaction \( \text{H}^+ + \text{H}_2(v\geq4) \rightarrow \text{H}_2^+ + \text{H} \) (1)
  controls the abundance of \( \text{H}_3^+ \) as it is quickly followed by:
\[
\text{H}_2^+ + \text{H}_2 \rightarrow \text{H}_3^+ + \text{H}
\]
- Reaction rate \( k_1^* = k_1 \frac{[\text{H}_2(v\geq4)]}{[\text{H}_2]} \)
  – Low \( k_1^* \) means less charge exchange reaction and increase in ionospheric densities
  ➢ \( k_1 = 10^{-9} \text{ cm}^3 \text{ s}^{-1} \) [Huestis, 2008]
Photochemistry in Gas Giant atmospheres

\[ \text{H}^+ + \text{H}_2\text{O} \rightarrow \text{H}_2\text{O}^+ + \text{H} \]
\[ \quad k_2 = 8.2 \times 10^{-9} \text{ cm}^3 \text{ s}^{-1} \]

\[ \text{H}_2\text{O}^+ + \text{H}_2 \rightarrow \text{H}_3\text{O}^+ + \text{H} \]
\[ \quad k_3 = 7.6 \times 10^{-10} \text{ cm}^3 \text{ s}^{-1} \]

\[ \text{H}_3^+ + \text{H}_2\text{O} \rightarrow \text{H}_3\text{O}^+ + \text{H}_2 \]
\[ \quad k_4 = 5.3 \times 10^{-9} \text{ cm}^3 \text{ s}^{-1} \]

\[ \text{H}_3\text{O}^+ + \text{e}^- \rightarrow \text{neutral products} \]
\[ \alpha_5 = 1.74 \times 10^{-5} \times \text{Te}^{-0.5} \text{ cm}^3 \text{ s}^{-1} \]

with Te in K.

AURORAL SPECTROSCOPIC ANALYSIS

- Identification of energetic particle type
- Assessment of \( (E_m, Q_{\text{prec}}) \) of energetic particles
- Supported by comprehensive modeling

COLOR RATIO

<table>
<thead>
<tr>
<th>Two spectral bands chosen in:</th>
<th>Earth</th>
<th>Jupiter, Saturn</th>
</tr>
</thead>
<tbody>
<tr>
<td>One band strongly absorbed by:</td>
<td>\text{N}_2 \text{ LBH}</td>
<td>\text{H}_2 \text{ Lyman and Werner}</td>
</tr>
<tr>
<td>Electron energy range covered</td>
<td>\text{O}_2 \ (&lt; 160 \text{ nm})</td>
<td>\text{CH}_4 \ (&lt; 140 \text{ nm})</td>
</tr>
<tr>
<td>Type of aurora identified:</td>
<td>Electron aurora (discrete only)</td>
<td>Electron aurora (diffuse + discrete)</td>
</tr>
</tbody>
</table>

Similar techniques can be applied at various planets
BUT different limitations on the product

[Fox et al. 2008]