Problem set: solar irradiance and solar wind

Karel Schrijver

July 13, 2013

1 Stratification of a static atmosphere within a force-free magnetic field

Problem: Write down the general MHD force-balance equation to derive the effect of the curvature and pressure-gradient forces on the stratification of a plasma with a potential magnetic field or with a field with only field-aligned currents. What is the effect of magnetic pressure in a potential or force-free field compared to that in a flux tube surrounded by a field-free atmosphere?

2 Bright and dark magnetic features and solar irradiance

2.1 When are magnetic concentrations in the solar photosphere seen as bright or dark?

Problem: Strong magnetic fields suppress convective motion. Under which conditions does this occur in the solar photosphere? Estimate the field strength at which convective suppression begins to be effective (note: sound speed $v_s \approx 7\,\mathrm{km/s}$, convective velocity far from upflow $v_c \approx 2\,\mathrm{kms}$). Once convection stops, the gas inside a flux bundle slumps back ("convective collapse") leaving a largely evacuated tube with field of roughly 1 kG. Explain the resulting Wilson depression as a result of the photon mean free path ($\lambda \sim H_p/2$, for pressure scale height H_p), the formation of sunspot umbrae and photospheric faculae. At what flux, roughly, does the transition from bright facula to dark pore occur?

2.2 Why does solar irradiance peak when the sunspot number reaches its maximum?

Problem: Discuss the relative roles of spots, pores, and faculae in regulating the solar irriadiance, and explain why a sunspot surrounded by magnetic faculae, together forming active regions, result in a dip as the sunspot passes central meridian, even as the overall irradiance near sunspot maximum is higher than at sunspot minimum.

3 How much of the Sun's luminosity is available for conversion into phenomena related to its magnetic activity?

Problem: Estimate the maximum mechanical energy flux density available to provide energy into the Sun's atmospheric magnetic field by random horizontal displacements of that field (by the "braiding mechanism" originally proposed by Parker). Use the approximation that the convective enthalpy flux near the photosphere is dominated by the latent heat of hydrogen ionization, that the vertical/depth scale of the convection D equals the sub-photospheric pressure scale height $H_p \approx 400\,\mathrm{km}$ and that the horizontal scale is approximately $L = 700\,\mathrm{km}$. What fraction of the solar luminosity does that amount to? Does solar outer atmospheric activity approach that limit? Why do you think that is?

4 Which ions carry the dominant radiative losses from the solar corona?

Problem: Why are solar coronal observations commonly made in spectral lines of iron rather than in those of the dominant species, hydrogen and helium? At what temperature do "heavy elements" (heavier than helium) dominate the spectrum of a plasma?

5 Solar wind and magnetic braking

5.1 Why is there a solar wind?

Problem a: Demonstrate that a hot solar atmosphere must flow outward. Show that a static atmosphere on a star with mass M_* and radius r_* in which

$$\frac{dp}{dr} = -nm_H GM_* \frac{1}{r^2} \tag{1}$$

for fully ionized hydrogen ($n = p/2kT$) and with

$$T(r) = T(r_*) \left(\frac{r_*}{r}\right)^\gamma \tag{2}$$

slowly decreasing with distance ($\gamma < 1$) owing to efficient electron heat conduction, the pressure at infinity exceeds the pressure of the interstellar medium.

Problem b: Combine conservation of mass with the momentum equation for a steady outflow,

$$\rho v \frac{dv}{dr} + \frac{dp}{dr} = -nm_H GM_* \frac{1}{r^2}, \tag{3}$$

to estimate at what distance from the Sun an isothermal transsonic wind becomes supersonic.

Problem c: Discuss the conditions under which $r_c < r_*$. What kind of stellar and coronal conditions does this require, and what would happen to the stellar wind if they were met? Consider other forces that may be important in driving the wind.

5.2 What is the time scale of magnetic braking for the present-day Sun of average activity?

Problem a: In a "Weber-Davis" approximation of the solar wind, the angular momentum per unit mass transported by the solar wind (including the specific angular momentum of the plasma and the torque density associated with the magnetic field in the Parker spiral) equals what is carried in a thin, rigidly rotation shell, $L = \frac{2}{3}\Omega r_A^2$, where r_A is the distance at which the wind's radial velocity equals the Alfvén velocity for the radial component of the solar wind, $v_{A,r} = B_r/(4\pi\rho)^{1/2}$. Combine that with conservation of mass and flux of the mostly monopolar-like field to derive the "Skumanich relation": $\Omega \propto t^{-1/2}$, using the approximation that the field strength at the base of the heliosphere scales roughly with angular velocity Ω as $B_{r,0} \propto \Omega$.

Problem b: For a solar moment of inertia of $I \approx 7\,10^{53}\,\mathrm{g\,cm^2}$, and a heliospheric flux of $\Phi =\approx 5\,10^{22}\,\mathrm{Mx}$, and an Alvén velocity close to the sound velocity at 1 MK, what is the time scale of the Sun's magnetic braking?