Homework linked to Kivelson lectures

1. Small amplitude wave perturbations in a uniformly magnetized plasma at rest satisfy equations below

$$-\omega d\rho + \rho\, \mathbf{k}\cdot\mathbf{v} = 0$$

$$-\omega\rho\mathbf{v} = -\mathbf{k}dp + \mathbf{b}(\mathbf{k}\cdot\mathbf{B})/\mu_o \; - \mathbf{k}(\,\mathbf{b}\cdot\mathbf{B})/\mu_o$$

$$dp/p = \;\gamma d\rho/\rho$$

$$\omega\mathbf{b} = \mathbf{v}(\,\mathbf{k}\cdot\mathbf{B}) - \mathbf{B}(\mathbf{k}\cdot\mathbf{v})$$

   This form assumes that spatial and temporal variations have the form $e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$. Use a coordinate system with z along $\mathbf{B}$ and $\mathbf{k}$ in the xz plane. Eliminate dp, $d\rho$, and $\mathbf{b}$ to obtain three equations governing $\mathbf{v}$. You should find that the equation for v_y does not involve the other two components of $\mathbf{v}$ and that v_y is multiplied by the Alfvén wave dispersion relation. The remaining equations do not contain v_y.

   a. Explain why this set of equations tells you that v_x, v_z, b_x, and b_z must vanish for the wave mode that satisfies the Alfvén wave dispersion relation.
   b. Eliminate either v_x or v_z from the remaining pair of equations to obtain the dispersion relation for the compressional fast and slow waves.
   c. From the three equations involving components of $\mathbf{v}$, explain why v_y must vanish for fast and slow mode waves.

   Hint: Any solution (Alfvén wave, fast wave, slow wave) to the set of three equations for the velocity perturbations must satisfy all three of the equations.

2. Examine slide 16 of Part 1 of the handout and use the figure
   a. to identify regions in which it appears that the fast mode, the slow mode, and the Alfvén wave are imposing the observed perturbations.
   b. to estimate the Alfvén speed of the upstream flow assuming that the incident flow is 140 km/s.
   c. Can you explain why there is no shock upstream of Ganymede's magnetosphere?

3. The Alfvén wing structure that we have described for Io is formed by sheets of current that flow towards Io from both north and south on the Jupiter-facing hemisphere and away from Io on the opposite hemisphere. The total current in these systems is of order 10^6 A. Because current density in a plasma is divergenceless ($\nabla\cdot\mathbf{j} = 0$), the field-aligned currents must link to currents flowing perpendicular to the field. As physics is required to be a quantitative science, it is useful to consider whether ion pickup can account for the magnitude of the closure current.

   Approximately a ton (2000 kg) per second of neutral gas is introduced into Jupiter's magnetosphere from Io. Much is lost or is ionized far from Io. Assume that 5% of the molecules are dissociated (100 kg/s) and ionized in a slab surrounding Io with dimensions l,w,h and assume the length in the direction of transverse current flow is 1 R_{Io}. The magnetic field in the neighborhood of Io is 2000 nT, and the ions are picked up at a flow speed of 60 km/s relative to Io. Estimate the net current that will flow across the slab from ion pickup. (Ions are picked up with a thermal speed equal to the flow speed.) Remember that I, the net current, is an integral of j, the current density. If you write your equations in algebraic form before plugging in numbers, you will find that the list below gives you all the numerical quantities you need to estimate the total current that is carried across the field. (Remember basic units are kg,m,s,Tesla.)

Radius of Io = 1836 km=slab length in direction of current flow.
Magnetic field near Io ~ 2000 nT. Mass addition rate 100 kg/s
Flow speed relative to Io = 60 km/s

Comment on the implications for the Alfvén wing currents.