Homework for chapter 2, Solar Explosive Activity Throughout the Evolution of the Solar System

1. The central thesis of this chapter is that studies of flares on stars other than the Sun can be used to give insight into flares on the Sun, and thus extend the time baseline for examining the influence of the Sun on its environs. Think about this idea, and come up with two or three biases which might affect such an approach. Consider all possible avenues, from objects to physical processes to observational issues. You are not meant to create an exhaustive list, but rather to "think like an astronomer" about what these issues might be. How would you attempt to compensate for some of these biases?

Some biases that arise because of the objects being used include the stars not being solarlike; the stars are typically young, binaries, fast rotators, and/or M dwarfs. Some ways to mitigate these factors: find solar-like stars to study for flares, or use scaling laws to correct for the dependence of some of these effects.

Some of the biases that creep in due to observational issues might be the flare rate – needing to observe stars with a high flare rate due to generally small time baseline for observations; observing stellar flares in different wavelength ranges compared to solar flares; instrumental sensitivity to flares; propagation effects affecting flare detection. Some ways to mitigate these effects: perform large surveys of ensembles of stars, observe the Sun as a star, perform deep observations of one star.

Some biases that might appear due to physical processes are big stellar flares not being the same physical processes as solar flares; stellar environment impacting flares (e.g. planet-forming disks); outbursts not being due to magnetic reconnection; hotter stellar flares; different energy partition between plasma heating and particle acceleration.

2. Flare frequency distributions are a common way to gauge the influence of stellar flares. For a power-law distribution, where

$$\frac{dN}{dE} = kE^{-\alpha} \quad , \tag{1}$$

dN is the number of flares occuring per unit time in a dE energy interval. Discuss the relative importance of different flare sizes in the regimes $\alpha < 2$ and $\alpha > 2$. Observationally, what are some of the things that might affect a determination of this index?



The figure above shows two schematic power-law distributions, one with an α less than 2 and another greater than 2. The steeper distribution has fewer energetic flares and more flares at the low energy end of the distribution, and vice versa for the less steep distribution. Some things that might affect a determination of this index are: the flare contrast at lower energies (ability to identify these flares); small dynamic range in flares measured; completeness effects at lower energies hampering determination of the index; mixing flares from different stars that have intrinsically different flare frequency distributions; mixing flares at different energies measured using different techniques/wavelength ranges without an attempt to correct for these effects; a departure from a strict power-law function, either due to a break (with different power-laws above and below), or an exponential rollover in the distribution. 3. Where does the flare energy come from, and what sets the maximum energy flare a star can produce? The energy reservoir for flares ultimately comes from the active region in which it forms. Assuming that the radiated energy in the flare is a fraction f of the energy stored in the active region magnetic field, write an expression that relates the flare energy to the magnetic field and the size of the active region.

We can write this expression for the flare energy as:

$$E_{\text{flare}} = f \frac{B^2}{4\pi} L^3 \tag{2}$$

where E_{flare} is the radiated energy in the flare, f is the efficiency of converting magnetic energy into this radiated energy, B is the magnetic field strength in the active region which is being converted to flare energy, and L is the size of the active region. Evaluating this for some values of f, B, and L, and relating the size L to a fraction of the surface area of the star, $L = \sqrt{x\pi R_{\star}^2}$ involved in the active region, gives

$$E_{\rm flare} \sim 10^{32} \left(\frac{f}{0.1}\right) \left(\frac{B}{150G}\right)^2 \left(\frac{x}{0.006}\right)^{3/2} \left(\frac{R_{\star}}{R_{\odot}}\right)^3 \,\,\mathrm{erg}$$
 (3)

So it seems there are only a few parameters that might control the largest energy flare a star can produce: the conversion of magnetic energy to radiated flare energy f, the magnetic field strength B, and the size of the active region. This is important for heliophysics studies due to the implications of such a large energy release. Whether the Sun can produce flares of energy significantly in excess of the largest ones observed to date ($\sim 10^{33}$ erg) depends on what the largest values might be. Historically, a value of 0.006 seems to be the largest historically recorded sunspot area, and an upper limit to energy conversion seems to be about 0.1 for the Sun, and the largest field strengths in an active region are around 3 kG, which suggests a maximum solar flare energy of a few times 10^{34} erg.