Hybrid Simulations: Numerical Details and Current Applications

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Content

1. Heliospheric/Space Plasmas, Solar Wind-Magnetosphere Interaction: *Kinetic Physics*
2. Computational Models and Numerics
3. Example Simulations
4. Outlook

- Plasma is (mostly) magnetized
- Plasma is (mostly) collisionless
Why a kinetic approach?
Physics of the Interaction: Regions – Overview

- not to scale -
Why a kinetic approach?

Thermal and other Plasma Properties

* Temperature anisotropy (sheath, tail)
* Turbulence (upstream, sheath, solar wind, corona)
* Specific heat ratios (sheath, MP, solar wind, corona)
* Energetic particles (shocks, magnetosphere, tail)
* Unmagnetized ions (current sheets)
* Heat flux (solar wind, current sheets, boundary layers)
Why a kinetic approach?

**Signatures and Coupling**

* Field-aligned currents (*magnetosphere, tail*)
* Energetic ions (*shocks, cusp, tail, ring current*)
* Heat flux (*sheath, MP, solar wind*)
* Poynting flux (*magnetosphere, tail*)
* Small spatial/temporal scales (*micro physics/all*)
Physics of the Interaction:
Time Scales (1AU/MS)

gyro frequency: \( \Omega_{ci} = \frac{eB}{mc} \)

\( \tau_{ci} \sim 0.5 \) to 10 s

plasma frequency: \( \omega_{pi} = (4\pi ne^2/m)^{1/2} \)

\( \omega_{pi} / \Omega_{ci} \sim 100 \) to 10,000

→ Electrons on much faster time scales
Physics of the Interaction:
Spatial Scales (1AU/MS)

gyro radius: \[ \rho = \frac{v_{th}}{\Omega_{ci}} \]
\[ \rho_i \sim 20 \text{ to } 200 \text{ km} \]

for \( T_e \sim T_i \Rightarrow \rho_e \sim \rho_i / 40 \)

inertial length: \[ \lambda = \frac{c}{\omega_{pi}} \] (skin depth)
\[ (\rho_i / \lambda_i)^2 = \beta_i \sim 0.1 \text{ to } 5 \]

→ Electrons on much smaller spatial scales
2. Computational Models
Computational Models:  
Fluid Versus Kinetic Approach

• Idealized particle motion, moments, Maxwell’s equations, closure relations → fluid models:  
  
  **MHD**

• General particle motion, Maxwell’s equations, self-consistent wave-particle interaction, few idealizations → kinetic models:  
  
  **Vlasov and Particle Codes**
Computational Models: Strengths and Weaknesses

• MHD codes:
  - very successful
  - early-on large-scale
  - many time steps
  - good conserv. properties
  - “easily” paralleled
  - “easy” non-uniform grids

• Particle Codes:
  - historically, successful on smaller scales
  - can address kinetic instabilities & waves, anisotropies, energization, thermalization, boundary layers, mass- and energy transport
Computational Models:
Types of Kinetic Codes

- explicit or implicit codes
- relativistic, e-m full-particle codes
- electrostatic codes
- Vlasov codes vs. PIC codes
- Darwin codes
- hybrid codes (kinetic ions, electron fluid)
Hybrid Codes: Algorithms

• Early codes:
  Auer et al., 1962, 1971; Forslund & Friedberg 1971;
  Chodura, 1975; Sgro & Nielson 1976

• Leroy et al. (1981): 1-D implicit
  Swift & Lee, 1983; Hewett, 1980

• Harned (1982): predictor-corrector
  Winske & Quest, 1986; Brecht & Thomas, 1988

• Fujimoto (1989): velocity extrapolation

• Horowitz (1989): iteration
Hybrid Codes: Algorithms

Some recent codes:

One-Pass (Omidi and Winske, 1995; Fujimoto, Thomas)

Moment Method (Quest, 1983; Matthews, 1994)

Improved Predictor-Corrector (efficient + substepping, Krauss-Varban, 2005)
Hybrid Codes: PIC Method

- “Finite size” particles, follow motion
- Collect & interpolate moments onto grid
- Solve e.m. fields on grid
Hybrid Codes: Equations

- **Electrons:**
  
  - massless, quasi-neutral fluid

  \[ en_e = q_i n_i \]

  - momentum equation

  \[
  \frac{d}{dt} n_e m_e v_e = 0 = -e n_e \left( \mathbf{E} + v_e \times \mathbf{B}/c \right) - \nabla \cdot \mathbf{P}_e
  \]

  - closure relation model:

    scalar pressure with const. \( T_e \), or adiabatic, or pressure tensor
Hybrid Codes: Equations

- Ions:
  
  - particle advance

  \[ m_i \left( \frac{\partial}{\partial t} \right) v_i = q_i \left( E + v_i B/c \right) \]

  \[ \left( \frac{\partial}{\partial t} \right) x_i = v_i \]

  \[ \rightarrow \text{leapfrog} \]

  - collect moments (charge density, current) on grid
Hybrid Codes: Equations

- Electromagnetic fields
  - Faraday’s law
    \[
    \frac{\partial}{\partial t} B = -c \nabla \times E
    \]
  - Ampere’s law
    \[
    \nabla \times B = 4 \pi J /c = 4\pi q_i n_i (v_i - v_e) /c
    \]
  - Electric field from electron momentum equation
    \[
    E = -v_i \times B /c - \nabla p_e / (q_i n_i) - B \times (\nabla \times B) / (4\pi q_i n_i)
    \]

→ State equation for E, time-advance for B, plus leapfrog means: information is not necessarily available at points in time when needed
Normalization

• spatial scale: \( c/\omega_{pi} \)
• velocity: \( c \)
• temporal scale: \( \omega_{pi}^{-1} \) (in code), \( \Omega_{ci}^{-1} \) (input/output)
• B: “Bo” and \( \omega_{pi} / \Omega_{ci} \)
• \( \rightarrow E: \) \( v_A \) \( B_0 \) and \( (\omega_{pi} / \Omega_{ci})^2 \)
• temperature: “\( \beta \)” – for fictitious species of unit density in unit field
• density: “\( n_0 \)"

\( \rightarrow \) With this normalization, simulation becomes independent of actual value of \( \omega_{pi} / \Omega_{ci} \)
Popular Hybrid Code Variations

- one-pass
- CAM-CL (moment method)
- predictor-corrector
- other variations (electron energy equation, finite electron mass, electron pressure tensor)

→ Codes are distinct in the way they deal with the fact that E, B, v, and n are not available at the same time(s)
Flow Charts:

Simple Explicit Method vs. Predictor-Corrector

- substepping
- $v$-moment evaluation
Moment Methods (CAM-CL)

- Use moment method to advance unknown velocity or current density $\frac{1}{2}$ step ahead
- Faster than additional particle push required in P-C
- Collect appropriate moments and apply a separate equation of motion
- CAM-CL:
  - current density $\rightarrow$ easier to include multiple species
  - advective term absent (included via time centering)
  - no ion pressure tensor required

Matthews, 1994
Numerical Properties:
Drifting Plasma Regions with Anti-Parallel Fields

![Graph showing numerical properties with lines for different methods: Predictor-Corrector, IOP, and CAM-CL. The y-axis represents $E_\perp$ ranging from 10 to 0.1, and the x-axis represents $\Delta t/\Omega_p^{-1}$ ranging from 0.02 to 0.00125. The graph illustrates the decline of $E_\perp$ with increasing $\Delta t/\Omega_p^{-1}$ for each method.]
Numerical Properties:
Drifting Plasma Regions with Anti-Parallel Fields

Graph showing the behavior of $E_\perp$ over time $t$. The graph compares different methods (Predictor-Corrector, IOP, CAM-CL) with their respective lines and markers.
Numerical Properties:
Dispersion Relation of Parallel Whistlers

\[(x^2/2 + \sqrt{(x^4/4 + x^2)})\]

Small k approximation \(= x + (x^2)/2 + (x^3)/8\)

Large k approximation \(= x^2 + 1 - 1/(x^2)\)
Numerical Properties:
Dispersion Relation of Parallel Whistlers

\[ \Delta t_{\text{max}} \]

\[ t_{\text{min}} = \frac{dx}{v_{ph}} \]
(Small \( dx \)) \[ t_{\text{min}} = dx^{**2/\pi} \]

Graph showing the relationship between \( dx \) and \( \Delta t_{\text{max}} \).
CFL-Condition Example:
Solar Wind Reconnection
Example: Solar Wind Reconnection

\[ v_{ph} = \omega/k ; \quad k = \frac{\pi}{\Delta x} = 15.7 \quad (\Delta x = \Delta y = 0.2) \]

\[ \Rightarrow \Delta t_{\text{max}} = \frac{\Delta x}{v_{ph}} = \frac{\Delta x^2}{\pi} \sim 0.013 \]

**Low density regions:**

(a) unlimited, \( n \sim 0.05 \, n_o \) \( \Rightarrow \) marginally unstable at \( \Delta t = 0.01 \) and 20 substeps

(b) artificially limited to \( n > 0.1 \, n_o \) \( \Rightarrow \) stable at \( 0.01/8 = 0.00125 \)

... substepping of more than 8-16 rarely useful
Some Examples in Detail:

All examples run on (fast, 64-bit AMD) single CPU!

• Thin current sheets and reconnection in the magnetotail, the solar wind, and the low corona

Common theme: high resolution (separation of scales) and/or low ion beta require very small cell size

• Interplanetary shocks and SEPs

• Global simulations of the magnetosphere
Generic High-Resolution Reconnection

- density -

- current -
Interplanetary Shocks
and
Solar Energetic Particles

- numerical considerations -
Interplanetary Shocks and SEPs

• Black-box Models and Source Description

• Role of Simulations in SEP models

Reames, 1999

Figure 3.2. Intensity-time profiles at different energies for the large 1989 October 19 event show time profiles with intensity peaks near the time of shock passage even at very high energies at 1 AU.
SEP Shock Sources:

1 \frac{c}{\omega_p} \quad \text{or} \quad v_o/\Omega_p

\sim 100 \frac{c}{\omega_p}

>10^4 \frac{c}{\omega_p}

1 \text{ AU}

downstream

shock source description

upstream
Scales and Extrapolation

Conservative estimate:

Assume target energy of 1MeV.
Convected gyro radius in 6nT B-field $10^5$ km $\sim 10^3 \frac{c}{\omega_p}$
Need several resonant $\lambda$ in system in 1 direction
→ e.g., $10,000 \times 500 \frac{c}{\omega_p}$ (assuming 2-D).
Typical time step $0.01 \Omega_p^{-1}$, $2.5 \cdot 10^6$ pp/s / CPU
1 hour of real time ($\sim$transit time at $M_A = 5$)
→ 5 days on 40 CPUs

1. power-law → extrapolation
2. quasi-linear estimate too restrictive
3. energetic tail (seed particles) can be described by separate population
Shock Set-up and Overview

\[ T_{||} \quad \text{and} \quad B_z ; \quad M_A = 6.0, \quad \theta_{Bn} = 30^\circ \]
Ion Distributions: quasi-parallel case

Upstream Distribution: Evolution over Time

$\Omega_c t = 0$ [40] 320

$M_A = 6$

$\theta_{Bn} = 30^\circ$

Upstream Distribution: Approximate Power Law of Tail

$M_A = 6$

$\theta_{Bn} = 30^\circ$
Ion Distributions: quasi-parallel case

Convergence with simulation domain size

$M_A = 6$
$\theta_{Bn} = 30^\circ$

$x_{\text{max}} = 800 \, \frac{c}{\omega_{pi}}$
$x_{\text{max}} = 1600 \, \frac{c}{\omega_{pi}}$
Modeling Tail/Seed Population with $\kappa$-Distribution

- reaches higher energies, provides better statistics in wing
- combined Maxwellian / $\kappa$-distribution can model actual solar wind superthermal ions
Bow Shock Simulations

*here*: effect of resistivity model(s)

why add resistivity?
constant resistivity
parameter-dependent resistivity
Cuts from Upstream to Magnetopause

\[ \eta = 0.0 \]

\[ \eta = \eta(\eta, B) \]

\[ \eta = \eta_0 j^4 \]

\[ \eta = \text{const.} \]
3-D global still requires significant computational resources on large clusters…

Karimabadi et al., 2006
large-scale, but localized 3-D simulations: magnetotail and ionosphere
Other Numerical Details…

- loss of cache memory correlation/ particle sorting
- energetic particles: Courant condition
- formulation of equilibria/ initialization
- boundary conditions
- low noise / linear methods
- inclusion of dipole field etc.
- parallel codes / domain decomposition
- diagnostics
Summary

- Hybrid simulations are being used successfully for a large range of topics from local to global 3-D.
- While much current research is done on parallel supercomputers, many significant problems, also in 3-D, can be addressed on single CPUs.
- Various modern versions of the Hybrid code converge well with time step, and give comparable results in most circumstances.
- Some versions are more diffusive.
- The predictor-corrector code is often the best method for challenging situations.