Theory & Modeling of Solar Eruptions

coronagraph (CME)  Hα limb (prominence)

Hα disk (flare ribbons)  X-ray (flare loops)
Large Solar Eruptions

prominence

flame ribbons

X-ray loops

cavity

plasmapi-up

shock

QuickTime™ and a Video decompressor are needed to see this picture.
\[ \text{Log} \left( \frac{I}{I_0} \right) \]

The graph shows the logarithmic ratio of the intensity \( I \) to the original intensity \( I_0 \) for \( \text{H} \alpha \) line. The dashed line indicates the original plage intensity.
Inertial Line-Tying

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} = 0 .$$

Photospheric boundary condition:

Photospheric convection is negligible

$\mathbf{B}$ normal to surface is fixed.

Plasma below the photosphere is both massive and a good conductor.

Evolution of the photosphere is slow compared to time scale of eruptions.
CME/Flare Energetics

kinetic energy of mass motions: $\approx 10^{32}$ ergs

heating / radiation: $\approx 10^{32}$ ergs

work done against gravity $\approx 10^{31}$ ergs

volume involved: $\geq (10^5 \text{ km})^3$

energy density: $\leq 100 \text{ ergs/cm}^3$
<table>
<thead>
<tr>
<th>Nature of Energy Source</th>
<th>Required: $\approx 100 \text{ ergs/cm}^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
<td><strong>Observed Values</strong></td>
</tr>
<tr>
<td>---</td>
<td>$n = 10^9 \text{ cm}^{-3}$</td>
</tr>
<tr>
<td>kinetic $(m_p n V^2)/2$</td>
<td>$V = 1 \text{ km/s}$</td>
</tr>
<tr>
<td><em><strong>thermal</strong></em></td>
<td>$T = 10^6 \text{ K}$</td>
</tr>
<tr>
<td>$nkT$</td>
<td>$h = 10^5 \text{ km}$</td>
</tr>
<tr>
<td>gravitational $m_p n g h$</td>
<td>$B = 100 \text{ G}$</td>
</tr>
<tr>
<td>magnetic $B^2/8\pi$</td>
<td></td>
</tr>
</tbody>
</table>
How is Energy Stored?

\[ \beta = 10^{-3} \]

\[ \nabla p \approx 0 \]

\[ \mathbf{j} \times \mathbf{B} \approx 0 \]

Force-free fields: \( \mathbf{j} \parallel \mathbf{B} \)

Current sheets:

sheared magnetic fields

emerging flux model

reconnection when \( j > j_{\text{critical}} \)
How Much Energy is Stored?

\[ B = B_{\text{photospheric currents}} + B_{\text{coronal currents}} \]

\[ B_{\text{from corona}} \approx B_{\text{from photosphere}} \]

Free magnetic energy \( \approx 50\% \) of total magnetic energy

from Gaizauskas & Mackay (1997)
Apparent Motion of Loops & Ribbons

inertial line-tying at surface

early on

loop

ribbon

later on

3 2 1 1 2 3

QuickTime™ and a YUV420 codec decompressor are needed to see this picture.
Flux Injection Models

(e.g. Chen 1989)

area of circuit increases

after onset

before onset

photosphere

surface flows

generator
During injection energy flows through photosphere.

Poynting flux: $\mathbf{S} = \frac{\mathbf{c} \times \mathbf{E}}{4\pi} \quad \text{ergs/cm/sec}$

$\mathbf{E} = \frac{-(\mathbf{V} \times \mathbf{B})}{c}$

Injection models predict large surface flows which are never observed.

$> 10 \text{ km/sec for } > 10 \text{ minutes}$
Loss of Equilibrium Model

(a) $R_o = 0.1$

(b) $\lambda = 4.00$

(c) $\lambda = 0.97$

(d) $\lambda = 0.97$

$W_B$ vs. $y$

pressure
tension
Energy Release in 2D Model

![Graph showing free magnetic energy versus source separation \( \lambda \). The graph indicates a rapid reconnection at \( \lambda = 1 \) and an ideal energy release of approximately 5%.](image)
Aly - Sturrock Paradox

impossible transition

(a)

(b)

(c)

magnetic energy

ideal

forbidden

(resistive)

(a)

(b)

(c)

time
Trajectories

- loss of equilibrium
- x-line appears

Diagram showing trajectories with labels:
- $h + r$
- $h$
- $h - r$
- $q$
- $p$

Legend:
- Flux rope
- Current sheet

Time axis: 0 to 5 hours
Numerical Simulation of Critical Point Configuration

- **initial condition:** $V = 0$
- **energy equation:** Ohmic heating, no cooling
- **resistivity:** uniform, $S = 500$
Power Output

- Power Output ($10^{29}$ erg/s)
- x-line appears
- loss of equilibrium

Graph showing power output over time, with critical points marked.
Chromospheric Evaporation
QuickTime™ and a Animation decompressor are needed to see this picture.
Evaporation Doppler Shift Puzzle

24 April 1984  BCS/SMM

Ca XIX  Antonucci et al. 1990

normalized counts

wavelength (Å)

blue wing
QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.
2D Asymmetric Quadrupole Model

- x-line
- flux rope
- NEF

test of “tether-cutting” concept
Equilibrium Manifold in 5D Parameter Space of Model

1. normalized radius of flux rope
2. normalized main arcade field
3. new emerging flux strength (NEF)
4. normalized depth of NEF
5. normalized distance of NEF

2nd order umbelic catastrophe
Basic Principles I

Driving Force:

inner edge is pinched by curvature of rope

repulsive force:

$$ F \propto \frac{I^2}{R} \ln \left( \frac{R}{r_o} \right) $$
Basic Principles II

Flux Conservation:

\[ I \propto \frac{1}{R \ln \left( \frac{R}{r_0} \right)} \]
How to Achieve Equilibrium

However, such an equilibrium is unstable!
How to Achieve a Stable Equilibrium

Key factor: Line-tying

Line-tying creates a second, stable equilibrium
3D Loss-of-Equilibrium Model

Titov & Démoulin (1999)

3 field sources

1. flux rope
2. magnetic charges
3. line-current
3D Line-Tied Solution by Method of Images

Solution for \( B \) in terms of incomplete elliptical integrals

- flux rope current
  - surface
  - line current & \( q \) sources

- perturbed position

- stationary background source

- flux rope arc
  - image
Line-Tied Evolution

initial configuration

vertical field at surface

erupted configuration

potential field
Forces Acting on Flux Rope

force vectors

restoring forces near feet

out of plane twisting
Effect of Line Current on Twist

- Flux rope
- Line current field
- Line current

Rotation out of plane upon eruption

\[ \mathbf{B} \]
current density

Kliem & Török (2004)
Simulation of Kliem & Török

1. line current replaced by quadrupole
2. subcritical twist for helical kink
3. torus center near surface
What is the Trigger Mechanism in the Breakout Model?

1. Initial state ($j = 0$)
2. Energy build-up
3. Post eruption

![Diagram showing magnetic energy and reconnected flux at upper x-line]

- $t_{\text{Alfvén}}$
- Magnetic energy
- Reconnected flux at upper x-line
- Lower x-line appears
Role of Reconnection in the Breakout Model

QuickTime™ and a decompressor are needed to see this picture.
Flux Rope Emergence & Eruption

3D simulations of Fan & Gibson (2006)
Some Unanswered Questions

1. How are stressed magnetic fields formed?
   — magnetic energy storage —

2. What determines the rate of reconnection?
   — kinetic processes —
   — turbulence —

3. To what extent are flares & CMEs predictable?
   — loss of equilibria —
   — loss of stability —