Solar Internal Flows and Dynamo Action

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NASA Heliophysics Summer School
Year 3
The Earth’s Climate System and Long-Term Solar Activity

22-29 July, 2009
Boulder, CO
Outline

- Solar Convection
  - Granulation
  - Supergranulation, Mesogranulation
  - Giant Cells

- Rotational Shear and Meridional Flow
  - Helioseismology
  - The Solar Internal Rotation
  - Maintenance of mean flows

- Convection, Shear and Magnetism
  - Local Dynamos
  - Global Dynamos
Granulation in the Quiet Sun

L $\sim$ 1-2 Mm
$U \sim$ 1 km s$^{-1}$
$\tau \sim$ 10-15 min

Lites et al (2008)
Granulation in the Quiet Sun

\[ L \sim 1-2 \text{ Mm} \]

\[ U \sim 1 \text{ km s}^{-1} \]

\[ \tau \sim 10-15 \text{ min} \]
Radiative MHD Simulations of Solar Granulation

**Upflows**
- warm, bright

**Downflows**
- cool, dark

Vertical magnetic fields swept to downflow lanes by converging horizontal flows

**Bright spots in downflow lanes attributed to magnetism**

Vogler et al. (2005)
Cool doesn’t necessarily mean dark

*Channelling of radiation in magnetic flux concentrations* ($B_z > 1$ kG)

Viewed at an angle they look brighter still

*Faculae*

Keller et al. (2004)

Vogler et al. (2005)
The Surface of the Sun is Corregated!

Stein & Nordlund (1998)

Photosphere depressed in downflow lanes even without magnetism
Photospheric temperature variations relatively small

$H^-$ opacity $\sim T^{1.0}$
Granulation is driven by strong radiative cooling in the photosphere.

Downflows dominate buoyancy work.

Upflows are largely a passive response induced by horizontal pressure gradients; peak velocities occur adjacent to downflows.

When granules get too wide, radiative cooling overcomes the convective flux coming up from below, reversing the buoyancy driving in the center of the granule.

Upflow becomes downflow and the granule bisects (exploding granules).

\[ \rho v_z y N_A \chi H \gtrsim \sigma T^4 \]

\[ L \sim D \frac{v_h}{v_z} \quad v_h \ll c_s \]

\[ D \sim H \rho \]
The Magnetic Network

CaIIK narrow-band core filter
PSPT/MLSO

Supergranulation
$L \sim 30-35 \text{ Mm}$
$U \sim 500 \text{ m s}^{-1}$
$\tau \sim 20 \text{ hr}$
Most prominent in horizontal velocities near the limb.
Mesogranulation

Most readily seen in horizontal velocity divergence maps obtained from local correlation tracking (LCT)

Vertical velocity and temperature signatures of mesogranulation and supergranulation are still elusive hard to verify that they are convection per se

$L \sim 5 \text{ Mm}$

$\tau \sim 3-4 \text{ hr}$

Shine, Simon & Hurlburt (2000)
Convective plumes cluster on larger scales due to kinematic advection from the converging horizontal flows that feed them
A toy model of interacting plumes

Granulation modeled as distributed points of horizontal convergence (representing downflow plumes) on a 2D surface

Kinematic advection and merging produces a larger-scale lattice of stronger convergence points

Rast (2003)
A hierarchy of convective scales

In the Sun, density and dynamical time scales increase with depth

Most of the mass flowing upward does not make it to the photosphere

Downward plumes merge into superplumes that penetrate deeper

Deep-seated pressure variations drive surface flows

Supergranulation and mesogranulation are part of a continuous (self-similar?) spectrum of convective motions

Spruit, Nordlund & Title (1990)

Nordlund, Stein & Asplund (2009)
Bigger Boxes

Latest local simulations are now achieving supergranular scales

Size, time scales of convection cells increases with depth

Stein et al (2006)
Beyond Solar Dermatology

But what lies deeper still?
Giant Cells

radial velocity, $r = 0.98R$

Miesch, Brun, DeRosa & Toomre (2008)

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Granulation-like network of downflow lanes and plumes

Cool, Helical Downflows
Solar Cyclones
\( \omega_r - \Delta \) anticorrelation
Solar Cyclones are strong, helical, rapidly evolving and highly intermittent.

Cells bisect and fragment due to efficient cooling in the thermal boundary layer.

Cyclones localized near the surface.
Disconnected lanes and plumes deeper down

Inward KE flux
Turbulent entrainment
Turbulent entrainment
Compression
Vortex stretching

Vorticity in Downflows!
North-South (NS) Downflow Lanes

Prograde propagation: Traveling convection modes!
Coherence through most of the convection zone
Turbulent Transport: especially angular momentum!
Propagation and Lifetime

Correlation time ~ 2.5 - 9 days but NS lanes can live for months

Optimal tracking rate faster than local rotation rate
Granulation
- Driven by radiative cooling in the photospheric boundary layer
- Strong downflow plumes, lanes
- Weaker upflows are a passive response

Supergranulation and Mesogranulation
- Self-organization of granular plumes
- Density stratification, plume interactions
- Part of a continuous hierarchy

Giant Cells
- Strong downflow lanes & plumes, weaker upflows
- Propagating NS downflow lanes at low latitudes
- Solar cyclones at high latitudes
- Kinetic helicity

\[ L \sim \begin{cases} 1-2 \text{ Mm} & U \sim 1 \text{ km s}^{-1} \\ L \sim 30-35 \text{ Mm} & U \sim 400 \text{ m s}^{-1} \\ L \sim 100 \text{ Mm} & U \sim 100 \text{ m s}^{-1} \end{cases} \]
\[ \tau \sim \begin{cases} 10-15 \text{ min} \\ 20 \text{ hours} \\ \text{days - months} \end{cases} \]
Most reliable observable is doppler velocity of the photosphere, although intensity may also be used.

$p$-modes excited by granulation, $g$-modes (theoretically) excited by giant cells.
Global Oscillation Modes

$\nu$ (\textmu Hz)

$P \sim 5$ min

$P \sim 2$ hrs

Christensen-Dalsgaard (2002)

Gough & Toomre (1991)
Global Rotational Inversions

\[ \omega_{nlm} = \omega_{nl0} + m \int_0^R \int_0^\pi K_{nlm}(r, \theta) \Omega(r, \theta) r dr d\theta \]

\[ \Delta_{nlm} \equiv \frac{\omega_{nlm} - \omega_{nl0}}{m} \]

\[ \sum_{nlm} c_{nlm}(r_0, \theta_0) \Delta_{nlm} = \int_0^R \int_0^\pi \mathcal{K}(r_0, \theta_0; r, \theta) \Omega(r, \theta) r dr d\theta \]

\[ = \overline{\Omega}(r_0, \theta_0) \]

\[ \mathcal{K}(r_0, \theta_0; r, \theta) = \sum_{nlm} c_{nlm}(r_0, \theta_0) K_{nlm}(r, \theta) \]

You pick!
The Internal Rotation of the Sun

**Differential Rotation (DR)**
Monotonic decrease in \( \Omega \) of \(~ 30\%\) from equator to high latitudes in CZ

**Nearly uniform rotation in radiative interior**

**Convection implicated as source of DR**

**Interior rate intermediate relative to CZ**

**Conical isosurfaces at mid-latitudes**

**Near-surface shear layer** \((0.95R < r < R)\)

**Tachocline** \((0.69R < r < 0.72R; \ CZ \ base = 0.713R +/- 0.003)\)
- Toroidal field generation by rotational shear (critical for global dynamo)
- Penetrative convection, internal gravity waves
- Instabilities (*magnetic buoyancy, magneto-shear*)
- Confinement

Local Helioseismology

Inferring subsurface flows from local high-wavenumber, non-resonant acoustic wave fields (see Gizon & Birch http://solarphysics.livingreviews.org)
Meridional Flow

Photospheric Doppler measurements

Poleward near surface at latitudes < 60° (unknown elsewhere)

Amplitude ~ 10-20 m s\(^{-1}\) but highly variable

Possible evidence for multiple cells at high latitudes, deeper levels

Solar cycle variations; convergence into activity bands (near surface)
Maintenance of Mean Flows: Dynamical balances!

(1) Meridional Circulation = Reynolds stress

\[ \nabla \cdot (\bar{\rho} \langle v_m \rangle \mathcal{L}) = -\nabla \cdot \left( \bar{\rho} r \sin \theta \langle v' \cdot v'_m \rangle \right) \]

(2) Thermal Wind Balance (Taylor-Proudman theorem)

\[ \Omega \cdot \nabla \langle v_\phi \rangle = \frac{g}{2r C_P} \frac{\partial \langle S \rangle}{\partial \theta} \]

Steady State
- Neglect LF, VD
- Rapid Rotation RS \ll CF
- ideal gas
- hydrostatic, adiabatic background

\[ \mathcal{L} = r \sin \theta (\Omega r \sin \theta + \langle v_\phi \rangle) \]

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Example 1: Thermal coupling to the tachocline

\[ \Omega \cdot \nabla \langle v_\phi \rangle = \frac{g}{2 r C_P} \frac{\partial \langle S \rangle}{\partial \theta} \]

\( a \) Prograde equator maintained by Reynolds stresses

\( b \) Conical profile maintained by baroclinicity

- thermal wind balance in lower CZ
- latitude-dependent convective heat flux
- enhanced by thermal gradients in the tachocline
- mediated by induced circulations

Miesch, Brun & Toomre (2006)

Rast et al. (2007)

Warm Poles!
Example 2: Isorotation contours as characteristics of the Thermal Wind equation

Assume, for the sake of argument that \( S' = S - \langle S \rangle = S' (\Omega^2) \)

Then TW eqn is hyperbolic and may be solved by means of characteristics

Characteristics trace out \( \Omega, S' \) isosurfaces

Possible mechanism: coherent structures (downflow plumes)

- Those that cross \( \Omega \) contours are sheared out
- Conduits for heat transport (mixing S)

\[ \Omega \cdot \nabla \langle v_{\phi} \rangle = \frac{g}{2rC_P} \frac{\partial \langle S \rangle}{\partial \theta} \]
Example 3: Delicate Maintenance of Meridional Circulation

\[ \nabla \cdot (\bar{\rho} \left\langle v_m \right\rangle \mathbf{L}) = -\nabla \cdot \left( \bar{\rho} r \sin \theta \left\langle v'_\phi v'_m \right\rangle \right) \]

\[ \bar{\rho} \left\langle v'_m \right\rangle \cdot \nabla \mathbf{L} = F_\phi \]

gyroscopic pumping
Helioseismology
- p-modes, f-modes, g-modes
- Global oscillations: $\Omega$, $c_s$, $\rho$, $\Gamma$
- Local patches: horizontal flow fields (SSW) ($r > 0.97R$)

Differential Rotation
- Monotonic decrease from equator to pole
- Conical mid-latitude contours
- Tachocline, near-surface shear layer
- Maintained by convective Reynolds stress, baroclinicity

Meridional Circulation
- Poleward near the surface ($r > 0.97R$, latitude < $60^\circ$)
- Relatively weak and highly variable
- Maintained by gyroscopic pumping and baroclinicity
Convection Breeds Magnetism

Charbonneau (2009)


Linsky (1985)
Lagrangian Chaos

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B - \eta \nabla \times B)
\]

\[
\frac{DB}{Dt} = \frac{\partial B}{\partial t} + (v \cdot \nabla) B = (B \cdot \nabla) v - B (\nabla \cdot v) - \nabla \times (\eta \nabla \times B)
\]

If \( \nabla \cdot v = \eta = 0 \) then

\[
\frac{DB}{Dt} = (B \cdot \nabla) v
\]

\[
\frac{d\delta}{dt} = (\delta \cdot \nabla) v
\]

\[
\frac{d\delta_i(x_0, t)}{dt} = \mathcal{J}_{ij}(x_0, t) \delta_j(x_0, t)
\]

\[
\lambda_1 + \lambda_2 + \lambda_3 = 0
\]

Chaotic fluid trajectories amplify magnetic fields

(provided that chaotic stretching wins the battle against ohmic diffusion)

\( \lambda = \text{Local Lyapunov exponents} \)

Ott (1998)

\( L_{ij} = \exp [\lambda_i(x_{0j})t] \)
Spatially smooth, temporally chaotic flows work best

\[ R_m = \frac{UL}{\eta} \quad P_m = \frac{\nu}{\eta} \]

If \( P_m > 1 \) then turbulent dynamos build fields on sub-viscous scales

Magnetic energy peaks near resistive scale

Turbulent flows beget turbulent fields!


Like pulling taffy!
Folded Field Structure

\[ k_{\parallel} = \left( \frac{\langle |(B \cdot \nabla)B|^2 \rangle}{\langle B^4 \rangle} \right)^{1/2} \]

\[ k_{B \times J} = \left( \frac{\langle |B \times J|^2 \rangle}{\langle B^4 \rangle} \right)^{1/2} \]

\[ k_{B \cdot J} = \left( \frac{\langle |B \cdot J|^2 \rangle}{\langle B^4 \rangle} \right)^{1/2} \]

\[ k_{\text{rms}} = \left( \frac{\langle |(\nabla B)|^2 \rangle}{\langle B^2 \rangle} \right)^{1/2} \]

\[ k_{\lambda} = \left( \frac{\langle |(\nabla v)|^2 \rangle}{\langle v^2 \rangle} \right)^{1/2} \]


\[ k_{\parallel} \sim k_{\lambda} \]

\[ k_{B \times J} \sim R_m^{1/2} \]
But Stars have $P_m < 1$!

**Now chaotic stretching must overcome ohmic diffusion and turbulent diffusion**

Still, the dynamo prevails if $R_m$ is large enough

$$E_k \sim k^{-p}$$

$$\gamma \sim k v_k \sim k^{(3-p)/2}$$

**Rough velocity fields ($p < 3$)**

Smallest eddies are best at amplifying field because they have the fastest turnover time

Magnetic energy still peaks near the resistive scale, at least in the kinematic regime

**Small-scale Fields!**

Iskakov et al (2007)
Local Dynamo Action in the Sun and Stars

Granulation: $\tau \sim 10\text{-}15\text{ min}$

Giant Cells: $\tau \sim \text{days - months}$

Granulation may generate field locally by chaotic stretching with little regard for the deeper convection zone

Flux expulsion and reconnection produce strong horizontal fields near photosphere

Magnetic pumping of flux through lower boundary can inhibit the surface dynamo in simulations

In the Sun the local dynamo is likely intimately coupled to the global dynamo

Schussler & Vogler (2008)
The Global Solar Dynamo

Ask not: How to generate Magnetic Energy? but rather: How to generate Magnetic Flux?

D. Hathaway

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Recipe for a Global Dynamo

❖ Lagrangian Chaos
› Builds magnetic energy

❖ Rotational Shear
› Builds non-helical large-scale toroidal flux (Ω-effect)
› Enhances dissipation of small-scale fields
› Promotes magnetic helicity flux

❖ Helicity
› Rotation and stratification generate kinetic helicity
› Kinetic helicity generates magnetic helicity
› Upscale spectral transfer of magnetic helicity generates large-scale fields
   ✦ Local transfer: inverse cascade of magnetic helicity
   ✦ Nonlocal transfer: $\alpha$-effect

$H_k = \langle \omega \cdot v \rangle$

$H_m = \langle A \cdot B \rangle$

$H_c = \langle J \cdot B \rangle$

$\omega = \nabla \times v$

$B = \nabla \times A$

$J = \frac{c}{4\pi \nabla \times B}$

Small-Scale Dynamo: $L_B < L_v$
Large-Scale Dynamo: $L_B >> L_v$
Inverse Cascade of Magnetic Helicity

Injection of $H_m, E_k, E_m$

EDQNM Closure Model

Pouquet, Frisch & Leorat (1976)

Injection of $E_k, H_k$

MHD Simulations

Alexakis, Mininni & Pouquet (2006)

Magnetic Helicity is conserved in the limit $\eta \rightarrow 0$

If you twist the field on small scales, large scales will respond

Provides an essential link between large and small scales
Dynamical (aka Catastrophic) $\alpha$ Quenching

$\mathcal{E} = \langle \mathbf{v}' \times \mathbf{B}' \rangle = \alpha B$

$\alpha = \alpha_k + \alpha_m = -\frac{\tau}{3} \langle \mathbf{v}' \cdot (\nabla \times \mathbf{v}') \rangle + \frac{\tau}{12\pi\rho} \langle \mathbf{B}' \cdot (\nabla \times \mathbf{B}') \rangle$

$\frac{d}{dt} \langle \mathbf{A}' \cdot \mathbf{B}' \rangle = -2 \langle \mathcal{E} \cdot \mathbf{B} \rangle - 2\eta \langle \mathbf{B}' \cdot (\nabla \times \mathbf{B}') \rangle$

$\alpha = \frac{\alpha_k}{1 + R_m \langle B^2 \rangle / B_{eq}^2}$

In stars $R_m \sim 10^5 - 10^9$!!

Turbulent $\alpha$-effect may be extremely inefficient!

$\frac{B_{eq}^2}{8\pi} = \frac{1}{2} \rho U^2$

In order to sustain the inverse cascade of $H_m$ toward large scales, helicity of the opposite sign is necessarily generated on small scales

If small-scale magnetic helicity is not dissipated or otherwise removed from the system, the resulting Lorentz force will inhibit chaotic stretching and kill the large-scale dynamo

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Avoiding Catastrophe

- **Dissipating small-scale helicity**
  - Forward cascade on sub-forcing scales may help
  - Turbulent diffusion (but this may be quenched as well)

- **Open Boundaries**
  - Helicity loss must occur preferentially on small scales
  - Anisotropy needed to promote helicity flux
    - Rotational shear
  - Coronal Mass Ejections

*Magnetic helicity flux through the photosphere may play a crucial role in the operation of the global solar dynamo*

Kapyla, Korpi & Brandenburg (2008)

Alexakis, Mininni & Pouquet (2006)
Spherical geometry is essential to understand global dynamos but not all global dynamos build strong mean fields.

Brun, Miesch & Toomre (2004)
A Turbulent, Convective Dynamo with a Tachocline

Pumping, amplification, organization of toroidal flux


$B_\phi$

$B_\rho$ at $R = 0.96$

solar_tachocline_pen6
A Dynamo with a Different Spin


Persistent toroidal wreathes of magnetism in midst of the convection zone

\( \Omega = 3 \Omega_\odot \quad P = 9.3 \text{ days} \)
$\Omega = 5\Omega_\odot$

$P = 5.6$ days


$\pm 5kG$
Faster still - Cycles!

$$\Omega = 5\Omega_\odot$$

$$P = 5.6\text{ days}$$

**The (Global) Solar Dynamo: A Boundary Layer Dynamo**

**Miesch & Toomre (2009)**

**Breakup and dispersal of photospheric active regions may contribute to poloidal flux generation (Babcock-Leighton mechanism)**

**Meridional Circulation may contribute to cyclic activity (Flux-Transport Models)**

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a. Dispersion relation involving $\alpha$, $\Delta \Omega$, and $\eta_r$.
Summary: Convective Dynamos

Local Dynamos
- Lagrangian Chaos
- Small-scale fields
- Magnetic carpet
- Strong horizontal fields near photosphere

Global Dynamos
- Rotational Shear
- Helicity
- Spherical Geometry
- Meridional Circulation
- Boundary Layers

Solar Activity Cycle still the most pressing and formidable challenge