Basic Plasma Concepts and Models

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Goal of this lecture

• Review a few basic plasma concepts and models that underlie the lectures later in the week.
• There are several excellent text books in plasma physics: Chen, Nicholson (out of print), Goldston and Rutherford, Boyd and Sanderson, Bellan.
• The book I am most familiar with is by Gurnett and Bhattacharjee, from which most of the material is taken.
What is a Plasma?

Plasma is an ensemble of charged particles, capable of exhibiting collective interactions.

Levels of Description:

• Single-particle dynamics in prescribed electric and magnetic fields
• Plasmas as fluids in 3D configuration space moving under the influence of self-consistent electric and magnetic fields
• Plasmas as kinetic fluids in 6D $\mu$-space (that is, configuration and velocity space), coupled to self-consistent Maxwell’s equations.
Single-Particle Orbit Theory

Newton’s law of motion for charged particles

\[ m \frac{dv}{dt} = q(E + v \times B) \]

Guiding-Center: A very useful concept
Single-Particle Orbit Theory

ExB Drift

\[ m \frac{dv}{dt} = q\left( E + v \times B \right) \]

Consider \( E = \text{const.}, \ B = \text{const.} \)

The charged particles experience a drift velocity, perpendicular to both \( E \) and \( B \), and independent of their charge and mass.

\[ V_E = \frac{E \times B}{B^2} \]
\[ \vec{v}_E = \frac{\vec{E} \times \vec{B}}{B^2} \]
Gradient B drift

\[
V_G = \frac{w_\perp}{qB} \left( \frac{B \times \nabla B}{B^2} \right),
\]

\[
w_\perp = \frac{1}{2} \omega_c^2 \rho_c^2
\]
Curvature drift

\[ V_C = \frac{2w_\parallel}{qB^2} \left( \frac{\mathbf{R}_C \times \mathbf{B}}{R_C^2} \right) \]

\[ w_\parallel = \frac{1}{2} m v_\parallel^2 \]
The Ring Current in Earth’s Magnetosphere: An Example

Oblique View

Polar View
Einstein suggested that while both the energy $E$ and the frequency $\nu$ change, the ratio $E/\nu$ remains approximately invariant.
Adiabatic Invariants

Harmonic oscillator

\[ \frac{d^2 x}{dt^2} + \omega^2 (\varepsilon t) x = 0, \quad \varepsilon << 1 \]

The adiabatic invariant is

\[ J = \oint pdq \]

\[ \Delta J / J \sim \exp(-c / \varepsilon) \]
Adiabatic Invariants

Three types of bounce motion
Adiabatic Invariants

Three types of bounce motion

First adiabatic invariant \( \mu = \frac{w_{\perp}}{B} \)

Second adiabatic invariant \( J = m\oint v_{\parallel} ds \)

Third adiabatic invariant \( \Phi = \pi R^2 B \)
Lectures for which this material is directly pertinent

- *Vasyliunas*: Planetary Magnetospheres
- *Lee*: Particle acceleration in shocks
- *Liemohn*: Energization of trapped particles
Kinetic Description of Plasmas

Distribution function \( f(\mathbf{r}, \mathbf{v}, t) \)

Normalization \[ N = \int \int d\mathbf{x} d\mathbf{v} f(\mathbf{r}, \mathbf{v}, t) \] \quad \text{phase space}

Example: Maxwell distribution function

\[ f = n_0 \exp \left( -\frac{m\mathbf{v}^2}{2kT} \right) \]
\( n_0 = N / V \)
Boltzmann-Vlasov Equation

Motion of an incompressible phase fluid in $\mu$–space

\[ \frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{v}} + \frac{q}{m} \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \frac{\partial f_s}{\partial \mathbf{v}} = 0, \ s = e,i \]

In the presence of collisions

\[ \frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{v}} + \frac{q}{m} \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \frac{\partial f_s}{\partial \mathbf{v}} = \left( \frac{\partial f_s}{\partial t} \right)_c \]
The diagram illustrates a scattering process over time. It shows a 3D representation with the axes labeled as $v_r$, $t$, and $x_r$. The diagram includes annotations for "scattering out," "collision," and "scattering in."
Vlasov-Poisson equations: requirements of self-consistency in an electrostatic plasma

\[ \frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{r}} - \frac{q_s}{m_s} \nabla \Phi \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0 \]

\[ \mathbf{E} = -\nabla \Phi \]

\[ \nabla \cdot \mathbf{E} = -\nabla^2 \Phi = 4 \pi \rho = 4 \pi \sum q_s \int d\mathbf{v} f_s \]
Quasilinear theory: application to scattering due to wave-particle interactions

• Consider electrostatic Vlasov equation

\[
\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{v}} - \frac{q}{m} \nabla \Phi \cdot \frac{\partial f_s}{\partial \mathbf{A}} = 0.
\]

Split every dependent variable into a mean and a fluctuation

\[
f_s = \langle f_s \rangle + f_{s1}, \quad \langle f_{s1} \rangle = 0
\]
Quasilinear Diffusion

It follows after some algebra that the mean or average distribution function obeys a diffusion equation:

$$\frac{\partial}{\partial t} \langle f_s \rangle = \frac{\partial}{\partial \mathbf{v}} \left( \mathbf{D} \cdot \frac{\partial}{\partial \mathbf{v}} \langle f_s \rangle \right)$$

Here $\mathbf{D}$ is a diffusion tensor, dependent on wave fluctuations (pertinent to Lee, Liemohn, and Opher lectures).
Fluid Models

The primary fluid model of focus in this summer school is **Magnetohydrodynamics (MHD)**

It treats the plasma as a single fluid, without distinguishing between electrons or protons, moving under the influence of self-consistent electric and magnetic fields.

It can be derived from kinetic theory by taking moments (integrating over velocity space), and making some drastic approximations.
Frozen Flux/Field Theorem (Alfven’s Theorem)

Fig. 1.6. Magnetic flux conservation: if a curve $C_1$ is distorted into $C_2$ by plasma motion, the flux through $C_1$ at $t_1$ equals the flux through $C_2$ at $t_2$.

Fig. 1.7. Magnetic field-line conservation: if plasma elements $P_1$ and $P_2$ lie on a field line at time $t_1$, then they will lie on the same line at a later time $t_2$. 
Magnetic Reconnection: Working Definition

If a plasma is perfectly conducting, that is, it obeys the ideal Ohm’s law,

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

$\mathbf{B}$-lines are frozen in the plasma. Departures from ideal behavior, represented by

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{R}, \quad \nabla \times \mathbf{R} \neq 0$$

break ideal topological invariants, allowing field lines to break and reconnect. In generalized Ohm’s law for collisionless plasmas, $\mathbf{R}$ contains resistivity, Hall current, electron inertia, and pressure. (More in lecture by Longcope, Forbes, Vasyliunas, and Kozyra.)