Solar Convection and The Solar Dynamo

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Outline

I) Convection
   ‣ Fundamental Aspects
   ‣ Solar Convection

II) Mean Flows
   ‣ Differential Rotation
   ‣ Meridional Circulation

III) The Solar Dynamo
   ‣ Convective Dynamos
   ‣ Models of the Solar Cycle

Heliophysics Summer School, July-August, 2011
I) Solar Convection

Fundamental Aspects
- Plumes & Lanes
- Boundary Layers
- Rotation
- Stratification
- Magnetism
- Spherical Geometry

Application to the Sun
- Granulation
- Mesogranulation
- Supergranulation
- Giant Cells
Rayleigh-Bénard Convection

\[ \text{Ra} = \frac{\alpha \Delta g D^3}{\nu \kappa} \quad \text{Pr} = \frac{\nu}{\kappa} \]

Bénard (1900)  
Rayleigh (1916)  
Chandrasekhar (1961)  
Ahlers, Grossman & Lohse (2009, Rev. Mod. Phys, 81, 503)

Question: What happens as you decrease \( \nu, \kappa \) while keeping everything else the same, including \( \text{Pr} \)?

Ra = \( 10^8 \), Pr = 0.7, 6.4  

See Lab Exercise!
Rayleigh-Bénard Convection

\[ Ra = \frac{\alpha \Delta g D^3}{\nu \kappa} \]

\[ Pr = \frac{\nu}{\kappa} \]

Question: What happens as you decrease \( \nu, \kappa \) while keeping everything else the same, including \( Pr \)?

Answer: \( Re, Nu \) increase

\[ Re = \frac{UD}{\nu} \]

\[ Nu = \frac{H}{k \Delta D^{-1}} \]


Ra = 10^8, Pr = 0.7, 6.4

See Lab Exercise!
Plumes and Boundary Layers!

Ahlers et al (2009)


For \( L \sim 7 \text{m} \)
(Barrel of Ilmenau)

\[ \lambda \sim 1 \text{ mm} \]
for \( \text{Ra} \sim 10^{14} \)
What is a boundary layer?

\[
\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \kappa \nabla^2 T
\]

\[
\frac{UT}{L} \sim \kappa \frac{T}{\delta^2_T} \quad \text{Pe} = \frac{UL}{\kappa}
\]

\[
\delta_T \sim L \text{ Pe}^{-1/2}
\]

<table>
<thead>
<tr>
<th>Regime</th>
<th>Dominance of</th>
<th>BLs</th>
<th>Nu</th>
<th>Re</th>
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<td>(\epsilon_u,\text{BL}, \ \epsilon_\theta,\text{BL})</td>
<td>(\lambda_u &lt; \lambda_\theta)</td>
<td>(Ra^{1/4}Pr^{1/8})</td>
<td>(Ra^{1/2}Pr^{-3/4})</td>
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<td>(Ra^{4/7}Pr^{-6/7})</td>
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<td>(\lambda_u &gt; \lambda_\theta)</td>
<td>(Ra^{1/3})</td>
<td>(Ra^{4/9}Pr^{-2/3})</td>
</tr>
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For \(L \sim 7m\)

(Barrel of Ilmenau)

\(\lambda \sim 1 \text{ mm}\)

for \(Ra \sim 10^{14}\)
Rotation: Helical plumes and Even more Boundary Layers!

\[ \text{Ek} = \frac{\nu}{2\Omega R^2} \]

King et al (2009)
**Density Stratification: Downflow lanes and plumes**

- Top, Middle, Bottom images showing different regions of a stratified flow.
- Fast, turbulent downflows.
- Slower, more laminar upflows.
- Downward KE flux.

Brummell et al (1996)

Miesch et al (2008)

(Porter & Woodward 2000)
Rotation + Density Stratification: Helical Downflows

Turbulent Alignment

Rossby Number

\[ Ro = \frac{\omega_{rms}}{2\Omega} \]
Magnetism:
Field Amplification and Advection

Flux Expulsion
Flux Separation
Turbulent Diamagnetism
Magnetic Pumping
Subcritical instability


Tao et al. (1998)

$B_0 \approx u^2 / R_m$
**Magnetism: Magnetic Pumping**

*Why downward transport?*
Flow asymmetry (downflows are faster)
Topological connectivity (Moffatt 1978)

Tobias et al. (2001)
In convective shells, columnar convection modes only exist outside the tangent cylinder.

Delineates two distinct convection regimes:

- Equatorial modes
- Polar Modes
**Spherical Geometry: Thermal Rossby Waves**

Can be driven either by the spherical curvature of the outer boundary or by the density stratification.

Simplest example: Boussinesq fluid, centrifugal gravity, local, linear perturbations, small boundary curvature

(Busse 2002)

\[
Q = \frac{\omega_z + 2\Omega}{H \rho}
\]

\[
\frac{DQ}{Dt} = 0
\]

anelastic, adiabatic motions, inviscid, non-magnetic, Ro << 1, \( \Omega \cdot \nabla \rho = 0 \)

(Glatzmaier & Gilman 1981)

\[
v_p = \frac{4\Omega}{L} \left( 1 + Pr \right) \frac{\tan \chi}{(k_y^2 + k_x^2)}
\]
What does all this have to do with the Sun?

Solar Granulation

$L \sim 1-2 \text{ Mm}$

$U \sim 1 \text{ km s}^{-1}$

$\tau \sim 10-15 \text{ min}$
What does all this have to do with the Sun?

Solar Granulation

$L \sim 1-2 \text{ Mm}$

$U \sim 1 \text{ km s}^{-1}$

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Radiative MHD Simulations of Solar Granulation

**Upflows**
- warm, bright

**Downflows**
- cool, (dark?)

Vertical magnetic fields swept to downflow lanes by converging horizontal flows

Bright spots in downflow lanes attributed to magnetism

Vogler et al. (2005)
Cool doesn’t necessarily mean dark

**Channelling of radiation in magnetic flux concentrations** \((B_z > 1 \text{ kG})\)

*Viewed at an angle they look brighter still*

**Faculae**

Keller et al. (2004)

Vogler et al. (2005)
Granulation is driven by strong radiative cooling in the photosphere.

Downflows dominate buoyancy work.

Upflows are largely a passive response induced by horizontal pressure gradients; peak velocities occur adjacent to downflows.

When granules get too wide, radiative cooling overcomes the convective flux coming up from below, reversing the buoyancy driving in the center of the granule.

Upflow becomes downflow and the granule bisects (exploding granules).

\[
\rho v_z y N_A \chi H \gtrsim \sigma T^4
\]

\[
L \sim D \frac{v_h}{v_z} \quad v_h \lesssim c_s
\]

\[
D \sim H \rho
\]
The Magnetic Network

CalIIK
narrow-band core filter
PSPT/MLSO

Supergranulation
$L \sim 30-35 \text{ Mm}$
$U \sim 500 \text{ m s}^{-1}$
$\tau \sim 20 \text{ hr}$
Most prominent in horizontal velocities near the limb

D. Hathaway
(NASA MSFC)
Mesogranulation

Most readily seen in horizontal velocity divergence maps obtained from local correlation tracking (LCT)

Vertical velocity and temperature signatures of mesogranulation and supergranulation are still elusive

hard to verify that they are convection per se

$L \sim 5 \text{ Mm}$

$\tau \sim 3-4 \text{ hr}$
A hierarchy of convective motions

- Kinematic advection
  - Converging flows cause plumes to cluster

- Density stratification
  - Promotes mergers by squeezing plumes together as they penetrate deeper

Discrete or Continuous??

Cattaneo, Lenz & Weiss (2001)

Spruit, Nordlund & Title (1990)

“toy model” Rast (2003)
Beyond Solar Dermatology
But still stops at 0.97R!
what lies deeper still?

Size, time scales of convection cells increases with depth
Eventually the hierarchy must culminate in motions large enough to sense the spherical geometry and rotation.

Giant Cells

(Loosely, anything bigger than supergranulation)

radial velocity, \( r = 0.98R \)

Miesch et al (2008)
Structure of Giant Cells

Solar Cyclones at high latitudes (cool, helical downflows)

Convective columns at low latitudes (thermal Rossby waves: prograde propagation)
Plumes and Boundary Layers
- Characteristic feature of turbulent convection (lab, simulations, stars...)
- Strong influence on dynamics throughout the domain despite their small extent
- Granulation driven by strong radiative cooling in the photosphere
- Merging of downflow plumes produces hierarchy of convective motions (granulation, mesogranulation, supergranulation, [giant cells] ~ 1-100+ Mm)

Stratification and Rotation
- Density stratification introduces asymmetry: downflows stronger
- Rotation imparts helicity (sign = \( \hat{\Omega} \cdot \hat{j} \)): solar cyclones
- Rotation imparts tilt: Turbulent alignment

Magnetism
- Weak fields amplified by convection: dynamo action
- Intermediate fields pushed aside by convection: flux separation, magnetic pumping
- Strong fields suppress convection: sunspots

Spherical Geometry
- Tangent Cylinder
- Convective Columns/Thermal Rossby waves
- Giant Cells!
Solar Differential Rotation

Sunspots closer to the equator rotate faster!

The rotation rate determined from spots has not changed by more than a few % since Carrington’s measurements spanning 1853-1861 (published in 1863)
The Internal Rotation of the Sun

\[ P \sim 35 \text{ days} \]

\[ P \sim 27 \text{ days} \]

\[ P \sim 25 \text{ days} \]

Thompson et al. (2003)
The Internal Rotation of the Sun

**Differential Rotation (DR)**

Monotonic decrease in $\Omega$ of $\sim 30\%$ from equator to high latitudes in CZ

Thompson et al. (2003)
The Internal Rotation of the Sun

**Differential Rotation (DR)**
Monotonic decrease in $\Omega$ of ~30% from equator to high latitudes in CZ

**Nearly uniform rotation in radiative interior**
The Internal Rotation of the Sun

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Monotonic decrease in $\Omega$ of $\sim 30\%$ from equator to high latitudes in CZ

**Nearly uniform rotation in radiative interior**

**Convection Implicated as source of DR**

Thompson et al. (2003)
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**Interior rate intermediate relative to CZ**

Thompson et al. (2003)
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**Conical isosurfaces at mid-latitudes**
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**Near-surface shear layer** ($0.95R < r < R$)

Thompson et al. (2003)
The Internal Rotation of the Sun

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**Conical isosurfaces at mid-latitudes**

**Near-surface shear layer** (0.95$R < r < R$)

**Tachocline** (0.69$R < r < 0.72R$; CZ base = 0.713$R \pm 0.003$)

- Toroidal field generation by rotational shear (critical for global dynamo)
- Penetrative convection, internal gravity waves
- Instabilities (magnetic buoyancy, magneto-shear)
- Confinement

Meridional Flow

**Photospheric Doppler measurements**


**Local Helioseismology**

![Images showing meridional circulation cells for the years 1999, 1997, and 2001.]
Meridional Flow

**Photospheric Doppler measurements**

**Local Helioseismology**

**Poleward near surface ($r > 0.97R$) at latitudes < 60° (unknown elsewhere)**
Meridional Flow

Photospheric Doppler measurements

Poleward near surface ($r > 0.97R$) at latitudes < 60° (unknown elsewhere)

Amplitude ~ 10-20 m s$^{-1}$ but highly variable (much weaker than DR)
Meridional Flow

**Photospheric Doppler measurements**

Poleward near surface ($r > 0.97R$) at latitudes $< 60^\circ$ (unknown elsewhere)

Amplitude $\sim 10$-$20$ m s$^{-1}$ but highly variable (much weaker than DR)

Possible evidence for multiple cells at high latitudes, deeper levels
Meridional Flow

**Photospheric Doppler measurements**

Poleward near surface ($r > 0.97R$) at latitudes < 60° (unknown elsewhere)

Amplitude ~ 10-20 m s$^{-1}$ but highly variable (much weaker than DR)

Possible evidence for multiple cells at high latitudes, deeper levels

Solar cycle variations; convergence into activity bands (near surface)

**Local Helioseismology**
Dynamical Balances

(1) Meridional Circulation = Reynolds stress

\[ \langle \rho v_m \rangle \cdot \nabla \mathcal{L} = -\nabla \cdot \left[ \rho \lambda \langle v'_\phi v'_m \rangle \right] \]

\[ \mathcal{L} = \lambda^2 \Omega \]

\[ \lambda = r \sin \theta \]

Diagrams:

- **Convection**
- **Differential Rotation**
- **Thermal Gradients**
- **Meridional Circulation**

Key Points:

- **Steady State**
- **Neglect LF, VD**
- **Rapid Rotation RS << CF**
- **ideal gas**
- **hydrostatic, adiabatic background**

Conservation of momentum in a rotating fluid:

\[ \rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P - \rho \mathbf{g} \]

Coriolis-induced tilting of convective structures

See Homework Problem I
Summary: Mean Flows

**Mean = averaged over longitude and time**

- **Differential Rotation**
  - zonal (east-west) Mean Flow
  - Known throughout most of the convection zone
  - fast equator, slower poles

- **Meridional Circulation**
  - Mean Flow in the radius-latitude plane
  - Only known above about 0.97R, low-mid latitudes (poleward)

*Inferred from Surface observations, Helioseismology*

*Maintained via momentum and energy transport by Giant Cells*

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Convection Breeds Magnetism

Charbonneau (2009)


Linsky (1985)
Generation of Magnetic Fields: The MHD Magnetic Induction Equation

$$\frac{\partial B}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Follows from Faraday’s Law of Induction

$$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

And Ohm’s Law (with a Galilean transformation)

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

With the usual MHD assumptions

- Highly ionized, Quasi-neutral
- High collision frequency/short mean-free paths (high density, temperature)
- sub-relativistic bulk velocity
**Lagrangian Chaos**

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B - \eta \nabla \times B)
\]

\[
\frac{DB}{Dt} = \frac{\partial B}{\partial t} + (v \cdot \nabla) B = (B \cdot \nabla) v - B (\nabla \cdot v) - \nabla \times (\eta \nabla \times B)
\]

If \( \nabla \cdot v = \eta = 0 \) then

\[
\frac{DB}{Dt} = (B \cdot \nabla) v
\]

\[
\frac{d\delta}{dt} = (\delta \cdot \nabla) v
\]

\[
\frac{d\delta_i(x_0, t)}{dt} = \mathcal{J}_{ij}(x_0, t) \delta_j(x_0, t)
\]

(Chaotic fluid trajectories amplify magnetic fields)

(provided that chaotic stretching wins the battle against ohmic diffusion)

\( \lambda = \text{Local Lyapunov exponents} \)

\( L_{ij} = \exp [\lambda_i(x_{0j})t] \)

\( \lambda_1 + \lambda_2 + \lambda_3 = 0 \)

Ott (1998)
Spatially smooth, temporally chaotic flows work best

$$R_m = \frac{UL}{\eta}$$  $$P_m = \frac{\nu}{\eta}$$

If $P_m > 1$ then turbulent dynamos build fields on sub-viscous scales (near resistive scale)

Folded field topologies
sheets and filaments

Turbulent flows beget turbulent fields!
Spatially smooth, temporally chaotic flows work best

\[ R_m = \frac{UL}{\eta} \quad P_m = \frac{\nu}{\eta} \]

If \( P_m > 1 \) then turbulent dynamos build fields on sub-viscous scales (near resistive scale)

Folded field topologies
sheets and filaments

Turbulent flows beget turbulent fields!

Like pulling taffy!


Do we see anything like this in stars?
The Magnetic Carpet

$B_{\text{app}}^L$ (+/- 50 G)

$B_{\text{app}}^T$ (200 G)

Lites et al (2008)
Types of Dynamos

**define**

**Small-scale dynamo**

Generates magnetic fields on scales smaller than the velocity field

\[ l_B \leq l_v \]

---

**define**

**Large-scale dynamo**

Generates magnetic fields on scales larger than the velocity field

\[ l_B \gg l_v \]

---

Are local solar/stellar dynamos small-scale dynamos?

**Probably - but intimately coupled to deep CZ**

---

Are global solar/stellar dynamos large-scale dynamos?

**Probably - but v-B correlations induced by large-scale convective modes or instabilities may contribute to global field generation**
Recipe for a Large-Scale Dynamo

- Lagrangian Chaos
  - Builds magnetic energy

- Rotational Shear
  - Builds large-scale toroidal flux (Ω-effect)
  - Enhances dissipation of small-scale fields
  - Promotes magnetic helicity flux

- Helicity
  - Rotation and stratification generate kinetic helicity
  - Kinetic helicity generates magnetic helicity
  - Upscale spectral transfer of magnetic helicity generates large-scale fields
    ✦ Local transfer: inverse cascade of magnetic helicity
    ✦ Nonlocal transfer: α-effect

\[
\begin{align*}
H_k &= \langle \omega \cdot v \rangle \\
H_m &= \langle A \cdot B \rangle \\
H_c &= \langle J \cdot B \rangle \\
\omega &= \nabla \times v \\
B &= \nabla \times A \\
J &= \frac{c}{4\pi} \nabla \times B
\end{align*}
\]
Recipe for a Large-Scale Dynamo

- Local, small-scale dynamo may be churning away in the surface layers (growth rate ~ 5 min) while the global dynamo plods along deeper down (activity cycle ~ 22 years)
Building Mean Fields: Rotation Helps!

$P = 28 \text{ days, } 9.3 \text{ days, } 5.6 \text{ days}$

Miesch et al (2010)

Magnetic Cycles in Convective Dynamos

Racine et al (2011)

Coexistence of turbulent and mean fields

\[ \langle B_\phi \rangle \]

Toroidal field generated near base of the convection zone

cf. “Butterfly Diagram”
Summary: Convective Dynamos

Local Dynamo
- Lagrangian Chaos
- Small-scale fields
- Magnetic carpet

Global Dynamo
- Rotational Shear
- Helicity
- Spherical Geometry
- Meridional Circulation
- Boundary Layers
- MHD Instabilities
- Activity cycle

Solar Activity Cycle still the most pressing and formidable challenge
Most solar cycle models still employ Mean-Field Dynamo Theory
Mean-Field Dynamo Theory: Reynolds Decomposition

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B - \eta \nabla \times B)
\]

\[
B = \bar{B} + b
\]

\[
v = \bar{v} + u
\]

\[
\frac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{v} \times \bar{B} + \mathcal{E} - \eta \nabla \times \bar{B})
\]

\[
\frac{\partial b}{\partial t} = \nabla \times (\bar{v} \times b + u \times \bar{B} + \mathcal{G} - \eta \nabla \times b)
\]

Define

Turbulent emf
\[
\mathcal{E} = \bar{u} \times b
\]

G-current
\[
\mathcal{G} = u \times b - \bar{u} \times b
\]

No assumptions so far...
Mean-Field Dynamo Theory: Underlying Assumptions

Now... The two fundamental assumptions central to (traditional) MFT

(I) Kinematic
Velocity field \( v \) is independent of \( B \)

Cannot be true in any saturated dynamo!

(II) Locality
Spatial scale on which fluctuations operate is small relative to that of mean field (scale separation)
\( (\ell \ll L) \)

Unlikely to be true in a turbulent dynamo!

\[ \mathcal{E}_i = \alpha_{ij} B_j + \beta_{ijk} \frac{\partial B_j}{\partial x_k} + \epsilon_{ijkl} \frac{\partial^2 B_j}{\partial x_k \partial x_l} + \ldots \]

Mean-Field Dynamo Theory: The Mean-Field Equation

Now write the mean velocity field as (spherical coordinates)

\[ \mathbf{V} = \mathbf{V}_m + r \sin \theta \Omega \hat{\phi} \]

meridional circulation differential rotation

And (finally!) obtain the mean-field dynamo equation as it is typically solved

\[ \frac{\partial \mathbf{B}}{\partial t} = \lambda (\mathbf{B}_p \cdot \nabla \Omega) \hat{\phi} + \nabla \times \left( \alpha \mathbf{B} + (\mathbf{v}_m + \gamma) \times \mathbf{B} + (\eta + \beta) \mathbf{J} \right) \]

This is the basis for virtually all current models of the solar activity cycle!
α-Ω Models for the Solar Cycle

In spherical geometry, dynamo waves can produce activity cycles!

Dynamo Waves: propagating solutions to the mean-field dynamo equations

Growth rate, propagation speed, and propagation direction determined by the product of $\alpha$ and $\nabla \Omega$

$$\alpha \propto -H_k$$

Parker-Yoshimura sign rule

$$\alpha \frac{\partial \Omega}{\partial r} < 0$$

in NH gives equatorward propagation

Expect equatorward propagation at low latitudes near the base of the convection zone

For an assessment of how these and other models do, see Ossendrijver (2003), Charbonneau (2010)
Mean-Field Dynamo Theory: Interface Dynamos

<table>
<thead>
<tr>
<th>Convection Zone</th>
<th>Large turbulent diffusion</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>alpha effect</td>
</tr>
<tr>
<td></td>
<td>weak radial shear</td>
</tr>
</tbody>
</table>

| Overshoot Region/ Tachocline                        | Small turbulent diffusion |
|                                                      | no alpha effect           |
|                                                      | strong radial shear       |

Proposed by Parker (1993)

Still follow Parker-Yoshimura rule for dynamo wave propagation but now the cycle period, dynamo growth rate also depend on transport via $\beta$

$$\frac{B_{or}}{B_{cz}} \sim \frac{\eta_{cz}}{\eta_{or}}$$

Can help alleviate $\alpha$-quenching & promote storage of toroidal flux

Charbonneau & Macgregor (1996)
The Babcock-Leighton Mechanism

Arises from Coriolis-induced tilts in emerging flux tubes followed by dispersal of poloidal flux in surface layers by turbulent diffusion, meridional flow

Often implemented as a non-local $\alpha$-effect

$$\frac{\partial B}{\partial t} = \nabla \times \left( S \hat{\phi} \right) + \ldots$$

$$S(r, \theta) = \alpha f(r) g(\theta) B_{\phi}^{bcz}(\theta)$$

with $f(r)$ confined to surface layers

One of several alternatives to the conventional turbulent $\alpha$-effect (Charbonneau 2010)
Flux-Transport Dynamo Models

Equatorward meridional flow near the base of the convection zone largely responsible for equatorward migration of active bands (butterfly diagram)

Most current Flux-Transport Models are also Babcock-Leighton Models

BLFT Models

May operate in the advection-dominated or diffusion/pumping-dominated regime

Dikpati & Gilman (2006)
The (Global) Solar Dynamo:

**Miesch & Toomre (2009)**

<table>
<thead>
<tr>
<th></th>
<th>Toroidal field generation</th>
<th>Poloidal field generation</th>
<th>Principal coupling mechanisms</th>
<th>Cycle period determined by</th>
</tr>
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<tbody>
<tr>
<td>BLFT models</td>
<td>Region III</td>
<td>Region I</td>
<td>MC, MB</td>
<td>Meridional flow</td>
</tr>
<tr>
<td>Interface models</td>
<td>Region III</td>
<td>Region II</td>
<td>CT</td>
<td>Dynamo waves(^a)</td>
</tr>
</tbody>
</table>

\(^a\) Dispersion relation involving \(\alpha, \Delta \Omega,\) and \(\eta_t\).

**Boundary Layers**

*Makes numerical modeling more challenging*

**Time Delays**

*Promotes chaotic modulation of cycle periods/amplitudes*
Mean-field Models: Current Challenges

General Issues
- Turbulent transport/diffusion not well understood
- Lorentz force back-reactions not well understood
- Meridional flow not well known
- Parity selection (dipole/quadropole)
- What is the dominant source of poloidal field?
- Where do active regions originate?

BLFT Dynamos
- Advection-dominated regime not well justified
- Flux emergence not well understood (links toroidal field at base to poloidal source)
- Strong polar fields, self-excitation, etc

Interface/Distributed Dynamos
- Turbulent \( \alpha \)-effect not well justified/understood
- Tend to produce overlapping cycles with small latitudinal extents

For much more on this see Ossendrijver (2003), Charbonneau (2010)
Dynamos are complex!

Solar magnetism
- Multiple scales (seconds to centuries, km to Gm)
- Magnetic Carpet
- Solar Cycle
  - Convection
  - Differential Rotation
  - Meridional Circulation
  - MHD Instabilities

Tools of the Trade
- Solar Observations
- Stellar Observations
- Numerical Models
- Theoretical Insights