Q: Why do the Sun and planets have magnetic fields?

Dana Longcope
Montana State University

w/ liberal “borrowing” from Bagenal, Stanley, Christensen, Schrijver, Charbonneau, ...
Q: Why do the Sun and planets have magnetic fields?

A: They all have dynamos
**Dynamo Ingredients**

(1) electrically conducting fluid
- Plasma (stars)
- Liquid iron (terrestrial planets)
- Metallic hydrogen (gas giants)
- Ionized water (ice giants)

(2) fluid must have complex motions
- Complex turbulent flows
- Rotation: breaks mirror-symmetry, not required, but needed for large-scale, organized fields

(3) motions must be vigorous enough
- Figure of merit: Magnetic Reynold’s #

\[ Rm = \text{velocity} \times \text{size} \times \text{conductivity} \]

From Stanley 2013
A Toy w/ all ingredients

- Conducting fluid
- Multi-part conductor
- V_disk = \int_0^\ell vB \, dr = \int_0^\ell (\Omega r) B \, dr = \frac{1}{2\pi} \Omega \Phi_{disk}

- No magnets
- No batteries
- No (net) charge

Lack of mirror symmetry
Complex motions
Differential motion of parts
A Toy w/ all ingredients

\[ V_{\text{disk}} = \int_0^\ell vB \, dr = \int_0^\ell (\Omega r) B \, dr = \frac{1}{2\pi} \Omega \Phi_{\text{disk}} \]

\[ IR = \frac{\Omega}{2\pi} \Phi_{\text{disk}} - L \frac{dI}{dt} = \frac{\Omega}{2\pi} M_{w,d} I - L \frac{dI}{dt} \]

\[ \frac{dI}{dt} = \left( \frac{\Omega}{2\pi} \frac{M_{w,d}}{L} - \frac{R}{L} \right) I = \gamma I \quad I(t) = I_0 e^{\gamma t} \]

Growth: \( \gamma > 0 \iff \Omega > 2\pi \frac{R}{M_{w,d}} \)

\[ \frac{\nu}{\ell} > 2\pi \frac{1}{\mu_0 \ell} = \frac{2\pi}{\mu_0 \sigma \ell^2} \]

Growth: \( Rm = \mu_0 \sigma \ell \nu > 2\pi \)

- conducting fluid
- multi-part conductor
- lack of mirror symmetry
- complex motions
- differential motion of parts
Toy dynamo amplifies fields of either sign: two attracting states

$I(t) = I_0 e^{\gamma t}$

- $I_0 > 0$
- $I_0 < 0$

- Reverse velocity AND reflect in mirror $\Rightarrow$ still amplifies
- Do one and not the other $\Rightarrow$ no amplification
Will I grow forever?

Torque on disk carrying current:
\[
\tau = \int_0^\ell Fr \, dr = \int_0^\ell IBr \, dr = \frac{1}{2\pi} I \Phi_{\text{disk}} = \frac{M_{w,d}}{2\pi} I^2
\]

Power needed to turn disk:
\[
P_\Omega = \Omega \tau = \frac{\Omega}{2\pi} M_{w,d} I^2
\]

Subtracting Ohmic losses
\[
P_\Omega - I^2 R = \left(\frac{\Omega}{2\pi} M_{w,d} - R\right) I^2 = \gamma LI^2
\]
\[
  = \frac{d}{dt} \left(\frac{1}{2} LI^2\right)
\]

Stored in energy of B
Reality: Conducting fluid – MHD

Fluid dynamics

\[ \begin{align*}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}) \\
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} &= -\nabla p + \rho \mathbf{g} + \nabla \cdot \mathbf{\tau} + \mathbf{J} \times \mathbf{B} \\
\rho c_v \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) &= -\frac{2}{3} T \nabla \cdot \mathbf{v} + \nabla \mathbf{v} : \mathbf{\tau} + \dot{\mathbf{Q}} + \frac{1}{\sigma} |\mathbf{J}|^2
\end{align*} \]

Faraday’s + Ohm’s laws

\[ \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times \left[ \mathbf{v} \times \mathbf{B} - \frac{1}{\sigma} \mathbf{J} \right] \]

\[ \nabla \times \left[ -\frac{1}{\sigma} \mathbf{J} \right] = \nabla \times \left[ -\frac{1}{\sigma \mu_0} \nabla \times \mathbf{B} \right] = \eta \nabla^2 \mathbf{B} \quad \eta = \frac{1}{\mu_0 \sigma} \]

Effect of \( \mathbf{B} \) on conducting fluid

\[ \mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B} \]

Lorentz force

Ohmic heat

Magnetic diffusivity
Reality: Conducting fluid – MHD

If $B$ is weak: kinematic equations

Fluid dynamics

\[
\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}) \\
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} &= - \nabla p + \rho \mathbf{g} + \nabla \cdot \mathbf{\sigma} \\
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\end{aligned}
\]

Faraday’s + Ohm’s laws

\[
\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{B} = (\mathbf{B} \cdot \nabla) \mathbf{v} - \mathbf{B} (\nabla \cdot \mathbf{v}) + \eta \nabla^2 \mathbf{B}
\]

Linear equation for $\mathbf{B}(\mathbf{x},t)$ – solve w/ known $\mathbf{v}(\mathbf{x},t)$
Dynamo action in MHD

\[ \frac{DB}{Dt} \quad \text{B} \cdot [\nabla v - I(\nabla \cdot v)] = B \cdot M \]

\[ \frac{\partial B}{\partial t} + (v \cdot \nabla)B = (B \cdot \nabla)v - B(\nabla \cdot v) + \eta \nabla^2 B \]

If \( M \) has a positive eigenvalue \( \lambda > 0 \)

\( B \) can grow exponentially: **DYNAMO ACTION**

- \( B \rightarrow -B \): same e-vector \( \Rightarrow \) same \( \lambda \)
- Reverse velocity AND reflect in mirror \( \Rightarrow \lambda \rightarrow \lambda \)
- Do one and not the other \( \Rightarrow \lambda \rightarrow -\lambda \)

\[ \gamma \sim \frac{v}{\ell} - \frac{\eta}{\ell^2} = \frac{v}{\ell} \left( 1 - \frac{\eta}{lv} \right) \]

Growth:

\[ Rm = \frac{lv}{\eta} = \mu_0 lv\sigma > 1 \]
Q: What kind of flow has $\lambda > 0$?

\[ \mathbf{v}(x, y) = \frac{v_0}{\ell} (x \hat{x} - y \hat{y}) \]

\[ M = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} \end{bmatrix} = \frac{v_0}{\ell} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]

\[ M \cdot \begin{bmatrix} B_0 \\ 0 \end{bmatrix} = \frac{v_0}{\ell} \cdot \begin{bmatrix} B_0 \\ 0 \end{bmatrix} \]

\[ \lambda = +\frac{v_0}{\ell} \]
Q: What kind of flow has $\lambda > 0$?

$$v(x, y) = \frac{v_0}{\ell} (x\hat{x} - y\hat{y})$$

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$$\lambda = +\frac{v_0}{\ell}$$

A: stretching flow
Q: What kind of flow has $\lambda > 0$?

$$v(x, y) = \frac{v_0}{\ell} (x\hat{x} - y\hat{y})$$

$$M = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} \end{bmatrix} = \frac{v_0}{\ell} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$M \cdot \begin{bmatrix} B_0 \\ 0 \end{bmatrix} = \frac{v_0}{\ell} \cdot \begin{bmatrix} B_0 \\ 0 \end{bmatrix}$$

$$\lambda = +\frac{v_0}{\ell}$$

A: stretching flow

Aspect ratio growth

$\sim$ Lyapunov exponent
Q: What kind of flow has $\lambda > 0$?

- Turbulent flows have pos. Lyapunov exponent: $\lambda > 0$
  - tend to stretch balls into strands
  - tend to amplify fields
- Conditions for turbulence:
  - driving: e.g. Rayleigh-Taylor instability
  - viscosity fights driving – must be small
- Rotation can organize turbulence:
  - align stretching direction $\rightarrow$ azimuthal (toroidal) – known as $\Omega$-effect
  - must be significant w.r.t. fluid motion

$$Re = \frac{\ell \nu}{\nu} \gg 1$$

$$Ro = \frac{\nu}{\ell \Omega} << 1$$

<table>
<thead>
<tr>
<th></th>
<th>$\eta$ [m$^2$/s]</th>
<th>$\nu$ [m$^2$/s]</th>
<th>$L$ [m]</th>
<th>$V$ [m/s]</th>
<th>$\Omega$ [rad/s]</th>
<th>$Rm$</th>
<th>$Re$</th>
<th>$Ro$</th>
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</thead>
<tbody>
<tr>
<td>Sun (CZ)</td>
<td>1</td>
<td>$10^{-2}$</td>
<td>$10^8$</td>
<td>1</td>
<td>$10^{-6}$</td>
<td>$10^8$</td>
<td>$10^{10}$</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>Earth (core)</td>
<td>1</td>
<td>$10^{-5}$</td>
<td>$10^6$</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$10^2$</td>
<td>$10^7$</td>
<td>$10^{-6}$</td>
</tr>
</tbody>
</table>
**Dynamo Ingredients**

1. **Electrically conducting fluid**
   - Plasma (stars)
   - Liquid iron (terrestrial planets)
   - Metallic hydrogen (gas giants)
   - Ionized water (ice giants)

2. **Fluid must have complex motions**
   - Complex turbulent flows
   - Rotation: breaks mirror-symmetry not required, but needed for large-scale, organized fields

3. **Motions must be vigorous enough**
   - Figure of merit: Magnetic Reynold’s #

   \[ Rm = \text{velocity} \times \text{size} \times \text{conductivity} \]

From Stanley 2013
How this works for Earth

Non-conducting mantle

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} = 0 \]

\[ \mathbf{B} = -\nabla \chi \quad \nabla \cdot \mathbf{B} = -\nabla^2 \chi = 0 \]

\[ \chi(r, \theta, \phi) = \sum_{\ell, m} \tilde{g}_{\ell, m} Y^m_{\ell}(\theta, \phi) \left( \frac{R_\oplus}{r} \right)^{\ell+1} \]

\[ B_r(r, \theta, \phi) = -\frac{\partial \chi}{\partial r} = \sum_{\ell, m} (\ell + 1) \tilde{g}_{\ell, m} Y^m_{\ell}(\theta, \phi) \left( \frac{R_\oplus}{r} \right)^{\ell+2} \]

simplifies w/ increasing \( r \)

Turbulent conducting fluid:

DYNAMO

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \neq 0 \]

Complex flows – complex field

- Fe + 10% light element
- Fe + 2 - 4% light element
A Spherical Harmonic Refresher

\( \ell = 1 \)
dipole

\[ Y_1^0(\theta, \varphi) \sim \cos \theta \quad Y_1^{\pm 1}(\theta, \varphi) \sim \sin \theta \, e^{\pm i \varphi} \]

\[ \tilde{g}_{1,\pm 1} \sim g_{1,1} \mp ih_{1,1} \quad (g_{1,0}, g_{1,1}, h_{1,1}) \leftrightarrow \tilde{\mu} \quad \text{dipole moment} \]

\( \ell = 2 \)
quadrupole

\[ Y_2^0(\theta, \varphi) \sim \frac{3}{4} \cos 2\theta + \frac{1}{4} \quad Y_2^{\pm 2}(\theta, \varphi) \sim \sin \theta \, e^{\pm 2i \varphi} \]

\[ (g_{2,0}, g_{2,1}, h_{2,1}, g_{2,2}, h_{2,2}) \leftrightarrow \tilde{Q} \quad \text{quadrupole tensor} \]

higher \( \ell \):

\[ Y_\ell^0(\theta, \varphi) \sim \cos \ell \theta + \cdots \quad Y_\ell^{\pm \ell}(\theta, \varphi) \sim \sin \theta \, e^{\pm \ell i \varphi} \]

Finer scale: \( \ell \) periods around circle

More components: \( 2\ell + 1 \) real coefficients

\( \ell = 5 \)

\( \ell = 2 \)

\( \ell = 9 \)
\[ B_r(r, \theta, \phi) = \sum_{\ell,m} (\ell + 1) \tilde{g}_{\ell,m} Y^m_\ell(\theta, \phi) \left( \frac{R^+}{r} \right)^{\ell+2} \]

Simplifies w/ radius

More even distribution over \( \ell \)

@ surface

Dominated by low \( \ell \)

@ core-mantle boundary

Dipole

\[ \tilde{g}_{\ell,m} \sim e^{-1.26 \ell} \]
Observation: what $B_r$ looks like today

@ core-mantle boundary: lower boundary of potential region

$\ell < 14$
Simplifies w/ increasing r

\[ B_r(r, \theta, \phi) = -\frac{\partial \chi}{\partial r} = \sum_{\ell, m}(\ell + 1)\tilde{g}_{\ell, m} Y^m_{\ell}(\theta, \phi) \left( \frac{R_\oplus}{r} \right)^{\ell+2} \]

2010: \( B_r \otimes r = 0.55 \)
Evolution of field

@ surface for 100 years

B, 1900
Evolution of field

@ core-mantle boundary for 100 years
Use evolution to infer fluid velocity

\[ \frac{\partial B}{\partial t} + (\nabla \cdot \mathbf{v}) B = (B \cdot \nabla) \mathbf{v} - B (\nabla \cdot \mathbf{v}) + \eta \nabla^2 B \]

Subsonic flow, Ignore stratification
\[ \nabla \cdot \mathbf{v} = 0 \]
Ignore diffusion

\[ \mathbf{v} \sim 3 \times 10^{-4} \text{ m/s} \]
\[ \Rightarrow \Delta x = 1,000 \text{ km} \]
\[ \Rightarrow \text{in 100 years} \]
Longer-term evolution

Like toy dynamo, Earth works in 2 modes. Flips between them seemingly at random.
Model geodynamo

- Glatzmaier & Roberts 1995
- Numerical solution of MHD
- Toroidal structure inside convecting core

http://www.es.ucsc.edu/~glatz/geodynamo.html
Other planets

- Mercury
  - rock
  - liquid Fe
  - solid Fe

- Earth
  - rock
  - liquid Fe

- Venus / Mars
  - rock
  - liquid Fe

- Ganymede
  - H₂O
  - rock
  - liquid Fe

- Moon
  - rock

- Jupiter
  - H₂ + He
  - H (metallic) + He

- Saturn
  - H₂ + He
  - inhomog.
  - H (met.) + He

- Uranus / Neptune
  - H₂ + He
  - H₂O + NH₃ + CH₄
  - rock
## Other planets

<table>
<thead>
<tr>
<th>Planet</th>
<th>Dynamo</th>
<th>$R_c/R_p$</th>
<th>$B_s$ [$\mu$T]</th>
<th>Dip. tilt</th>
<th>Quadr Dipole</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>Yes (?)</td>
<td>0.75</td>
<td>0.35</td>
<td>$&lt;5^\circ$</td>
<td>0.1-0.5</td>
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<tr>
<td>Venus</td>
<td>No</td>
<td>0.55</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Earth</td>
<td>Yes</td>
<td>0.55</td>
<td>44</td>
<td>$10.4^\circ$</td>
<td>0.04</td>
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<tr>
<td>Moon</td>
<td>No</td>
<td>0.2 ?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mars</td>
<td>No, but in past</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Jupiter</td>
<td>Yes</td>
<td>0.84</td>
<td>640</td>
<td>$9.4^\circ$</td>
<td>0.10</td>
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<tr>
<td>Saturn</td>
<td>Yes</td>
<td>0.6</td>
<td>31</td>
<td>$0^\circ$</td>
<td>0.02</td>
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<tr>
<td>Uranus</td>
<td>Yes</td>
<td>0.75</td>
<td>48</td>
<td>$59^\circ$</td>
<td>1.3</td>
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<tr>
<td>Neptune</td>
<td>Yes</td>
<td>0.75</td>
<td>47</td>
<td>$45^\circ$</td>
<td>2.7</td>
</tr>
<tr>
<td>Ganymede</td>
<td>Yes</td>
<td>0.3 ?</td>
<td>1.0</td>
<td>$&lt;5^\circ$</td>
<td>?</td>
</tr>
</tbody>
</table>
What that means

Jupiter

Saturn

Neptune

Mars

Christensen
Gas Giants

- Large pressure range $\rightarrow$ radially variable properties can be important
ICE GIANTS

Stanley & Bloxham 2004, 2006

- work in a geometry suggested by low heat flow observations
Level of saturation

\[ B^2/2\mu_0 \begin{array}{c} \text{[Jm}^{-3}\text{]} \\
\text{[Jm}^{-3}\text{]} \end{array} \]

from Christensen

B saturates (exp growth ends) when driving power – thermal conduction \( q_0 \) – balances Ohmic dissipation

\[ R_c = 0.4 R \]

\[ R_c = 0.6 R \]

Earth

Uranus

Jupiter

Neptune

Saturn 1

Saturn 2

dimensionless factors
How it works in the Sun

• Entire Star: H/He plasma
• Convection Zone (CZ)
  • Outer 200,000 km
  • Turbulence:
    Re = 10^{10}
  • Thermally driven
  • Good conductor
    Rm = 10^8
  • Rotation effective
    Ro = 10^{-2}
• Corona – conductive but tenuous:
  J smaller (~0?)
Evidence of the dynamo

- Magnetic field where there are sunspots
- Field outside sunspots and elsewhere too
Evidence of the dynamo

Field is fibril

[Diagram of a magnetic field with fibril structure]

[Image of a solar disk with highlighted regions]

SDO/HMI 2011-07-19T22:03:41.500
Evidence of the dynamo

Field orientation: mostly toroidal
Synoptic plot: unwrapped view built up over time
Dynamo comparison: Sun vs. Earth

\[ Rm = 10^8 \]
\[ Ro = 10^{-2} \]

\[ Rm = 10^2 \]
\[ Ro = 10^{-6} \]
Assume corona has small (negligible) current:

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} = 0 \]

\[ \mathbf{B} = -\nabla \chi \]

\[ \nabla \cdot \mathbf{B} = -\nabla^2 \chi = 0 \]

\[ \chi(r, \theta, \phi) = \sum_{\ell,m} \tilde{g}_{\ell,m} Y_{\ell}^{m}(\theta, \phi) \left( \frac{R_{\oplus}}{r} \right)^{\ell+1} \]
\[ r = R_\odot \]

\[ r = 2.5 \, R_\odot \]
DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS

SUNSPOT AREA IN EQUAL AREA LATITUDE STRIPS (% OF STRIP AREA)

DATE

-10G -5G 0G +5G+10G

DATE

Hathaway NASA ARC 2016/07
Comparison of Wolf and Group Sunspot Number Series

Wolf $R_i$
Group $R_G$

AVERAGE DAILY SUNSPOT AREA (% OF VISIBLE HEMISPHERE)

http://solarscience.msfc.nasa.gov/

HATHAWAY NASA/ARC 2016.07
**Stellar Magnetic Fields**

- correlations between stellar types and magnetic field properties, probably due to geometry of convection zones

- stars with outer convection zones (late-type stars) have observed magnetic fields whose strength tends to increase with their angular velocity

- Cyclic variations are known to exist only for spectral types between G0 and K7).
Convection zones

\[ R_\odot \]

\[ 2 R_\odot \]

\[ R^* \]

\[ d_{ce} \]

Fully radiative
Other stars

Evidence of magnetic activity

Activity on main sequence:
types $F \rightarrow M$

$B-V > 0.4$

(From Linsky 1985)
Explaining Activity Levels

Individual Variation

Variance within class
The Dynamo Number

Parker’s Dynamo #

\[ N_D = \frac{\alpha_{\text{dyn}} \Omega' d^4}{\eta^2} \]

Dynamo is linear instability for \( N_D > N_{\text{crit}} \)

Dynamo \( \alpha \)-effect:

\[ \alpha_{\text{dyn}} \equiv \tau \langle \mathbf{v} \cdot (\nabla \times \mathbf{v}) \rangle \sim \Omega d \]

\[ \eta = \eta_{\text{turb}} \sim \frac{d^2}{\tau_c} \]

\[ \Omega' = \frac{d \Omega}{d r} \sim \frac{\Omega}{d} \]

\[ N_D \sim (\Omega \tau_c)^2 \sim \left( \frac{P_{\text{obs}}}{\tau_c} \right)^{-2} = Ro^{-2} \]
Activity vs. Rossby Number

41 Local stars
$P_{\text{obs}}$ from $S(t)$

(from Noyes et al. 1984)
Activity vs. Rossby Number

(from Patten and Simon 1996)

- Stars in open cluster 2391 (30My old)
- $R_X$ from ROSAT observations
- Rotation periods $P_{\text{obs}}$ from optical photometry
- $N_{R} = \text{Ro} = P_{\text{obs}}/t_c$
Summary

• Magnetic fields – all from dynamos
  – Conducting fluid
  – Complex motions w/ enough *umph*

• Create complex fields

• Fields evolve in time – reverse occasionally

• Differences from different parameters:
  Rm, Re, Ro