Q: Why do the Earth & planets have ionospheres? magnetospheres?

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w/ liberal “borrowing” from Fuller-Rowel, Solomon, Sojka, Lean, Vasylinunas, Bagenal, Luhman
Heliophysics chain

Q: Why do the Earth & planets have ionospheres?
A: Because of the Sun’s corona (its EUV & X-rays)

Q: Why does the Sun have a corona?
A: Because of its magnetic field (and its heating)

Q: Why does the Sun have a magnetic field?
A: Because of its dynamo
Earth's neutral atmosphere

<table>
<thead>
<tr>
<th>Gas</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>N₂</td>
<td>77%</td>
</tr>
<tr>
<td>O₂</td>
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<tr>
<td>H₂O</td>
<td>1%</td>
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<tr>
<td>CO₂</td>
<td>0.03%</td>
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</tbody>
</table>

Solomon (cf. vol I fig. 12.1)
Fate of a photon w/ absorption x-section $\sigma$

Prob. of survival: $P(x) = \exp\left[-\int \sigma n(\ell) d\ell\right]$  

optical path $\tau(x)$ = avg. # absorbers in cylinder w/ x-section $\sigma$ 

$\tau = 1 \Rightarrow 1$ absorber: mean-free path 

$\tau(z) = \int z \sigma n(z') \sec(\chi) \, dz'$ 

$= \sigma n_0 \sec(\chi) \int e^{-z'/H} \, dz'$ 

$\tau(z) = \sigma n_0 H \sec(\chi) e^{-z/H} = e^{-(z-z_{\tau_1})/H}$ 

height of $\tau = 1$: $z_{\tau_1} = H \ln[\sigma n_0 H \sec(\chi)]$ 

Prob. of survival: $P(z) = e^{-\tau(z)} = \exp\left[-e^{-(z-z_{\tau_1})/H}\right]$
\[ e^{z_{\tau_1}/H} = \sigma n_0 H \sec(\chi) = \frac{\sigma n_0 kT}{\bar{m}g} \sec(\chi) = \sigma \frac{p_0}{\bar{m}g} \sec(\chi) = \frac{\sigma}{\sigma_0} \sec(\chi) \]

\[ \sigma_0 = \frac{\bar{m}g}{p_0} = \frac{5 \times 10^{-23} \text{g} \cdot 980 \text{cm/s}^2}{10^6 \text{erg/cm}^3} = 5 \times 10^{-26} \text{cm}^2 \]

\[ P(z) = e^{-\tau(z)} = \exp\left[-e^{-(z-z_{\tau_1})/H}\right] \]
\( z_{\tau_1}(\lambda) = H \ln \left( \frac{\sigma(\lambda)}{5 \times 10^{-26} \text{ cm}^2} \right) \)
\[
P[z(\lambda)] = \exp \left[ -e^{-\left[ z-z_{\tau 1}(\lambda) \right]/H} \right]
\]

\[
z_{\tau 1}(\lambda) = H \ln \left[ \frac{\sigma(\lambda)}{5 \times 10^{-26} \text{ cm}^2} \right]
\]
Radial attenuation

Energy flux: \( I(z) = I_\infty P(z) = I_\infty \exp \left[ -e^{-(z-z_{\tau 1})/H} \right] \)

Energy deposition: \( \frac{dI}{dz} = \frac{I_\infty}{H} \exp \left[ -e^{-(z-z_{\tau 1})/H} - \frac{z-z_{\tau 1}}{H} \right] \)

Chapman layer
$T = 5770\, \text{K}$

$I_{\infty} = 300\, \text{mW m}^{-2}\, \text{nm}^{-1}$

$I_{\infty}/h = 10^{-2}\, \text{mW m}^{-3}\, \text{nm}^{-1}$

$\sigma = 10^{-23}\, \text{cm}^{2}$

$T = 5770\, \text{K}$

$I_{\infty} = 300\, \text{mW m}^{-2}\, \text{nm}^{-1}$
Absorption via ionization creates ion/electron pairs.

**Figure 13.4**

Ionization edges:
- \( N_2 \) and \( O_2 \)
- \( O \) and \( O^+ \)
- \( N \) and \( NO \)

**Solar Energy Deposition**

- Log, 10 mW m\(^{-2}\) nm\(^{-1}\)
- Altitude (km)

**Electrons/m\(^3\)**

- \( F_2 \) and \( F_1 \)
- \( D \) and \( E \)

**TR + Corona**
Rate of photo-ionization (per volume)

\[ q(z) = \sigma_{\text{ion}} n(z) F(z) = \sigma_{\text{ion}} n(z) F_\infty P(z) \]

\[ = \sigma_{\text{ion}} n_0 F_\infty \exp \left[ -e^{-\left(z-z_{\tau1}\right)/H} - \frac{z}{H} \right] \]

Electron destruction by recombination with +’ve ions @ rate

\[ L = \alpha n_e n_i \approx \alpha n_e^2 \quad \text{Assuming neutrality} \]

Production balances destruction: \( q=L \)

\[ n_e(z) = \sqrt{\frac{q(z)}{\alpha(z)}} \]
Production shut off – recombination removes electrons

\[ \frac{dn_e}{dt} = -L \approx -\alpha n_e^2 \]
Why do different species decrease differently with height?

\[ H = \frac{kT}{mg} \]

\( (T=1450 \text{ K}) \)
Ionospheric plasma

- Ions/e⁻ form plasma – conducting fluid
- Neutrals: separate fluid
- Continual creation/destruction couples fluids – created "drag force" between them

A plasma with electron density \( n_e \) (cm\(^{-3}\)) screens out E fields w/ \( f < \) its plasma frequency

\[
f_p = \sqrt{\frac{e^2 n_e}{\pi m_e}} = 10^4 \text{Hz} \ n_e^{1/2}
\]

Q: what is the lowest freq. solar radio emission we can observe from the ground?
Corona varies — ionosphere varies

\[ n_e = \sqrt{\frac{q}{\alpha}} \sim \sqrt{F_\infty} \]
increasing coronal flux

vol. III 14.4
Other planets... other atmospheres

Why is Earth’s thermosphere so hot?

<table>
<thead>
<tr>
<th></th>
<th>Venus</th>
<th>Earth</th>
<th>Mars</th>
</tr>
</thead>
<tbody>
<tr>
<td>N₂</td>
<td>3%</td>
<td>77%</td>
<td>3%</td>
</tr>
<tr>
<td>CO₂</td>
<td>96%</td>
<td>0.03%</td>
<td>95%</td>
</tr>
<tr>
<td>O₂</td>
<td>-</td>
<td>21%</td>
<td>-</td>
</tr>
<tr>
<td>H₂O</td>
<td>0.01%</td>
<td>1%</td>
<td>-</td>
</tr>
</tbody>
</table>

\[
\log(\sigma_0) = -27.1, -25.3, -23.4
\]
Principal Ionization Processes on Venus & Mars

\[ \chi = 60^\circ \]

Venus

\[ \text{N}_2 \quad 3\% \]
\[ \text{CO}_2 \quad 96\% \]
\[ \text{O}_2 \quad - \]
\[ \text{H}_2\text{O} \quad 0.01\% \]

vol III fig. 13.4

Solomon
Principal Ionization Processes on Venus & Mars

Mars

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Vol III fig. 13.6

Vol III fig. 13.4

- N₂: 3%
- CO₂: 95%
- O₂: -
- H₂O: -
Venus or Mars

- No dynamo – no $B$
- Ionosphere $\Rightarrow$ conducting bdry
- SW– w/ $B$ – can’t penetrate
- Supersonic flow deflected by obstacle
- Bow shock forms

Spreiter & Stahara 1980
Simple picture of bow shock

- Ignore pressure from SW
- SW: \( u_\infty / c_{s,\infty}, \rho_\infty, M_\infty \gg 1 \)
- Standing shock ~ sphere radius= \( R_s \)
- Post-shock flow
  - \( v. \) ubsonic – \( M \ll 1 \)
  - \( \sim \) incompressible w/ \( u_r(R) = 0 \)

\[
u = \nabla \psi \times \nabla \phi
\]

\[
\psi(r, \theta) = C \left( \frac{r^4}{R^4} - \frac{R^2}{r^2} \right) \sin^2 \theta \quad \text{Lighthill 1957}
\]

- \( u_{n,2} = u_{n,1}/4 \), \( u_{t,2} = u_{t,1} \)

\[
\frac{u_{r,2}}{\cos \theta} = 2 \frac{CR_s^2}{R^4} \left( 1 - \frac{R^5}{R_s^5} \right) = -\frac{1}{4} u_\infty \quad \frac{u_{\theta,2}}{\sin \theta} = - \frac{CR_s^2}{R^4} \left( 4 + \frac{R^5}{R_s^5} \right) = u_\infty
\]

\[
R_s = \left( \frac{3}{2} \right)^{2/5} \quad R = 1.18 R
\]
Numerical solution from Spreiter et al. 1966

\[ M_{\infty,n} = 1 \]

\[ \alpha = \sin^{-1}(1/M_{\infty}) \]

Weak shock far down stream
Shock partially thermalizes flow KE of SW:

- Nose point (normal)

\[ T_N = \frac{3}{8} \cdot \frac{1}{2} m u_\infty^2 k_B \]

- Stagnation point

\[ T_s = \frac{16}{15} T_N = \frac{2}{5} \cdot \frac{1}{2} m u_\infty^2 k_B \]

\[ u_\infty = 400 \text{ km/s} \]

\[ \Rightarrow T_N = 3.6 \text{ MK} \]

\[ \Rightarrow T_s = 3.8 \text{ MK} \]

- pressure

\[ p_s = \frac{4}{5} \rho_\infty u_\infty^2 \]
Venus

Bow Shock

Ionopause - Magnetic Barrier

Magnetosheath

Magnetotail

Solar Wind

vol. I fig. 13.12
Wind @ Magnetized Planets
Earth, Jupiter, Saturn, ...

- Planetary $\mathbf{B}$ prevents SW from reaching ionosphere
- SW deflected by magnetosphere
- “squishy” obstacle

Hughes (cf. vol. I fig. 10.1)
Shock & sheath: similar to before

- Stagnation point (SP) @ \( r=R_{mp} \)
- Plasma pressure: \( p_s = \frac{4}{5} \rho \infty u_\infty^2 \)
- Inside \( (r < R_{mp}) \): \( B = -\nabla \chi \)

\[
\chi(r, \theta) = \frac{B_\oplus R_\oplus^3}{R_{mp}^2} \left( \frac{R_{mp}^2}{r^2} + \frac{2r}{R_{mp}} \right) \cos \theta \]

- Magnetic pressure @ SP

\[
\frac{1}{8\pi} |B(R_{mp}, 0)|^2 = \frac{1}{8\pi} \left( \frac{1}{R_{mp}} \frac{\partial \chi}{\partial \theta} \right)^2 = \frac{9R_\oplus^6}{8\pi R_{mp}^6} B_\oplus^2
\]

- Ignore inner plasma – balance

\[
R_{mp} = \left( \frac{45}{32\pi} \right)^{1/6} \left( \frac{B_\oplus^2}{\rho_\infty u_\infty^2} \right)^{1/6} R_\oplus
\]

Chapman-Ferraro Distance
Intuition break

\[ R_{mp} = \left( \frac{45}{32\pi} \right)^{1/6} \left( \frac{B_+^2}{\rho\infty u^2} \right)^{1/6} R_+ \sim 12 R_+ \]

\( \rho_{sw} = 10^{-23} \text{ g/cm} \)

\( u_{sw} = 400 \text{ km/s} \)

• At what distance do geostationary satellites orbit?

• Is the moon inside or outside the magnetopause?

• What happens to \( R_{mp} \) during fast SW: \( u_{sw} = 800 \text{ km/s} \)
Similar picture from high-powered codes

vol. I fig. 11.2
Other planets... same story

\[ B_J \sim 15 \quad B_\oplus \sim 5 \, \text{G} \quad ; \quad \rho_\infty \sim 0.04 \quad \rho_\infty, \oplus \]

\[ \Rightarrow \text{Jupiter’s magnetopause:} \]

\[ R_{mp,J} \sim 50 \, R_J = 3.5 \times 10^{11} \, \text{cm} \]
But not all of Earth’s field stays confined to m-sphere

**Reconnection** with SW field (consider southward IMF)
- Creates “open” flux connected to poles @ \( \dot{\Phi}_{ds} \)
- SW sweeps flux downstream – into **magnetotail**
- Steady state only when reconnection in tail “closes” flux at rate \( \dot{\Phi}_n = -\dot{\Phi}_{ds} \)
- Requires long & strong **neutral sheet** in magnetotail

vol. I fig. 10.3
But not all of Earth’s field stays confined to m-sphere

Hughes (cf. vol. I fig. 6.3)

“closes” flux at rate $\dot{\Phi}_n = -\dot{\Phi}_{ds}$

- Requires long & strong neutral sheet in magnetotail

When balance occurs, tail...

- ... has some length $L_t \gg R_{mp}$
- ... has some open flux $\Phi_t$
vol. I fig. 13.10
closed/open boundary maps down to "auroral oval"
\[
\Phi_t = \Phi_{pc} = \pi \left( R_\oplus \sin \theta_{pc} \right)^2 B_{np} \sim \pi R_\oplus^2 \theta_{pc}^2 B_{np} \sim 10^{17} \text{Mx}
\]

\[
\Phi_t = \frac{\pi}{2} R_t^2 B_t
\]

mag. pressure

In tail:

\[
\frac{1}{8\pi} B_t^2 = \frac{1}{2\pi^3} \frac{\Phi_t^2}{R_t^4} = \frac{1}{2\pi} \left( \frac{R_\oplus}{R_t} \right)^4 \theta_{pc}^4 B_{np}^2
\]

Pressure balance

@ m-pause:

\[
\frac{R_t}{R_\oplus} = \left( 2\pi \right)^{-1/4} \frac{B_{np}^{1/2}}{p_{sw}^{1/4}} \theta_{pc} \sim 25
\]

\[
B_t \sim 10^{-4} \text{G} \sim 10 \text{nT}
\]

\[p_{ms} \sim p_{sw} \sim 10^{-9} \text{erg/cm}^3\]
Other auroral ovals

vol. I fig. 2.9
Convection: magnetosphere meets ionosphere

Field lines are frozen to M-spheric plasma. Motion sweeps filed lines back.

**Objection:** Field lines are also frozen into liquid core – ends cannot be moved.

**But:** Atmosphere & solid crust are insulators – field lines are imaginary there.
Example of how the motions meet

\[ E = -v \times B \]
Example of how the motions meet

\[ E = -v \times B \]

Slide upper boundary

slab

ionosphere

atmosphere
Example of how the motions meet

\[ E = \frac{J}{\sigma} \]

\[ E = -v \times B \]

Slide upper boundary
Example of how the motions meet

\[ E = -v \times B \]

\[ E = J / \sigma \]
Example of how the motions meet

Slide upper boundary

$E = -v \times B$

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Example of how the motions meet

\[ E = -v \times B \]

\[ E = J/\sigma \]

MHD Field line motion creates current in ionosphere — accompanied by \( E \)
Convection: magnetosphere meets ionosphere

MHD motions drag footpoints across polar caps and back around to day side

Integrate* $\mathbf{E}$ across polar cap:

$$\int_{A}^{B} \mathbf{E} \cdot d\mathbf{l} = \varphi_{pc} = \Phi_{ds}$$

Really an EMF – but called “cross polar cap potential”

* use MKS here
\[ \oint_{A} \mathbf{E} \cdot d\mathbf{l} = \varphi_{pc} = \dot{\Phi}_{ds} \]

Field-aligned currents in MHD region

\[ \Phi_{pc} = 50 \text{ kV} = 5 \times 10^{12} \text{ Mx/s} \]

recycle in \( \Phi_t \) in \( \sim 5 \) hours

Convection flow

vol. I fig. 10.5
Summary

• Ionospheres created by EUV & X-rays from Sun’s TR and corona
• Diminish during night – lower during solar minimum
• SW deflected by ionospheres of unmagnetized planets (Venus & Mars)
• SW deflected by magnetospheres
• Magnetotail created by reconnection with solar wind magnetic field