Q: Why do the Earth & planets have ionospheres? magnetospheres?

Dana Longcope
Montana State University

w/ liberal “borrowing” from Fuller-Rowel, Solomon, Sojka, Lean, Vasylinunas, Bagenal, Luhman
Heliophysics chain

Q: Why do the Earth & planets have ionospheres?
A: Because of the Sun’s corona (its EUV & X-rays)

Q: Why does the Sun have a corona?
A: Because of its magnetic field (and its heating)

Q: Why does the Sun have a magnetic field?
A: Because of its dynamo
Earth’s neutral atmosphere

Earth

<table>
<thead>
<tr>
<th>Gas</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>N₂</td>
<td>77%</td>
</tr>
<tr>
<td>O₂</td>
<td>21%</td>
</tr>
<tr>
<td>H₂O</td>
<td>1%</td>
</tr>
<tr>
<td>CO₂</td>
<td>0.03%</td>
</tr>
</tbody>
</table>

Solomon (cf. vol I fig. 12.1)
Solar Energy Deposition ($\log_{10}{\text{mW m}^{-3}\text{ nm}^{-1}}$)

T = 5770 K

from vol III fig. 13.3

Solomon

171 Å

from vol III fig. 10.1
Fate of a photon  
\( w/ \) absorption x-section \( \sigma \)

Prob. of survival:  
\[ P(x) = \exp \left[ - \int \sigma n(\ell) d\ell \right] \]

Optical path \( \tau(x) \) = avg. # absorbers in cylinder \( w/ \) x-section \( \sigma \)

\( \tau = 1 \Rightarrow 1 \) absorber:  
mean-free path

\[ n(z) = n_0 e^{-z/H} \]

height of \( \tau = 1 \):  
\[ z_{\tau_1} = H \ln \left[ \sigma n_0 H \sec(\chi) \right] \]

Prob. of survival:  
\[ P(z) = e^{-\tau(z)} = \exp \left[ - e^{-z}/H \right] \]
\[ e^{z_{\tau_1}/H} = \sigma n_0 H \sec(\chi) = \frac{\sigma n_0 kT}{\bar{m}g} \sec(\chi) = \sigma \frac{p_0}{\bar{m}g} \sec(\chi) = \frac{\sigma}{\sigma_0} \sec(\chi) \]

\[ \sigma_0 = \frac{\bar{m}g}{p_0} = \frac{5 \times 10^{-23} \text{g} \cdot 980 \text{cm/s}^2}{10^6 \text{erg/cm}^3} = 5 \times 10^{-26} \text{cm}^2 \]

\[ P(z) = e^{-\tau(z)} = \exp \left[ -e^{-(z-z_{\tau_1})/H} \right] \]
\[
\sigma(\lambda) = H \ln \left( \frac{\sigma(\lambda)}{5 \times 10^{-26} \text{cm}^2} \right)
\]
\[
P[z(\lambda)] = \exp\left[-e^{-\frac{z-z_{\tau_1}(\lambda)}{H}}\right]
\]
\[
z_{\tau_1}(\lambda) = H \ln\left[\frac{\sigma(\lambda)}{5 \times 10^{-26} \text{ cm}^2}\right]
\]
Radiation intensity & heating

Energy flux: \[ I(z) = I_\infty P(z) = I_\infty \exp \left[ -e^{-(z-z_{\tau_1})/H} \right] \]

Energy deposition: \[ \frac{dI}{dz} = \frac{I_\infty}{H} \exp \left[ -e^{-(z-z_{\tau_1})/H} - \frac{z-z_{\tau_1}}{H} \right] \]

Chapman layer
\[ I_\infty/h = 10^{-2} \text{ mW m}^{-3} \text{ nm}^{-1} \]

\[ I_\infty = 300 \text{ mW m}^{-2} \text{ nm}^{-1} \]
Absorption via ionization creates ion/electron pairs.
Rate of photo-ionization (per volume)

\[ q(z) = \sigma_{ion} n(z) F(z) = \sigma_{ion} n(z) F_\infty P(z) \]

\[ = \sigma_{ion} n_0 F_\infty \exp \left[ -e^{-(z-z\tau_1)/H} - \frac{z}{H} \right] \]

Electron destruction by recombination with +’ve ions @ rate

\[ L = \alpha n_e n_i \approx \alpha n_e^2 \quad \text{Assuming neutrality} \]

Production balances destruction: \( q = L \)

\[ n_e(z) = \sqrt{\frac{q(z)}{\alpha(z)}} \]
Production shut off
– recombination removes electrons

\[ \frac{dn_e}{dt} = -L \approx -\alpha n_e^2 \]

Mar. 21
45° lat.
99.9% neutral!
Why do different species decrease differently with height?

\[ H = \frac{kT}{mg} \]

\( (T=1450 \text{ K}) \)

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<table>
<thead>
<tr>
<th>Species</th>
<th>Mass</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>He(^+)</td>
<td>(28 , m_p)</td>
<td>43 km</td>
</tr>
<tr>
<td>H(^+)</td>
<td>(32 , m_p)</td>
<td>38 km</td>
</tr>
<tr>
<td>O(^+)</td>
<td>(4 , m_p)</td>
<td>300 km</td>
</tr>
<tr>
<td>NO(^+)</td>
<td>(16 , m_p)</td>
<td>76 km</td>
</tr>
</tbody>
</table>
Ionospheric plasma

- Ions/e\(^{-}\) form plasma – conducting fluid
- Neutrals: separate fluid
- Continual creation/destruction couples fluids – created "drag force" between them

A plasma with electron density \(n_e\) (cm\(^{-3}\)) screens out E fields w/ \(f < \) its plasma frequency

\[
f_p = \sqrt{\frac{e^2 n_e}{\pi m_e}} = 10^4 \text{Hz } n_e^{1/2}
\]

Q: what is the lowest freq. solar radio emission we can observe from the ground?
Corona varies — ionosphere varies

\[ n_e = \sqrt{\frac{q}{\alpha}} \sim \sqrt{F_\infty} \]
Increasing coronal flux

vol. III 14.4
Other planets... other atmospheres

Why is Earth’s thermosphere so hot?

<table>
<thead>
<tr>
<th></th>
<th>Venus</th>
<th>Earth</th>
<th>Mars</th>
</tr>
</thead>
<tbody>
<tr>
<td>N₂</td>
<td>3%</td>
<td>77%</td>
<td>3%</td>
</tr>
<tr>
<td>CO₂</td>
<td>96%</td>
<td>0.03%</td>
<td>95%</td>
</tr>
<tr>
<td>O₂</td>
<td>-</td>
<td>21%</td>
<td>-</td>
</tr>
<tr>
<td>H₂O</td>
<td>0.01%</td>
<td>1%</td>
<td>-</td>
</tr>
<tr>
<td>log(σ₀)</td>
<td>-27.1</td>
<td>-25.3</td>
<td>-23.4</td>
</tr>
</tbody>
</table>
Principal Ionization Processes on Venus & Mars

Venus Vol III fig. 13.4

Vol III fig. 13.4

Venus

\[ \chi = 60^\circ \]

\[ N_2 \]

\[ 3\% \]

\[ CO_2 \]

\[ 96\% \]

\[ O_2 \]

\[ \text{--} \]

\[ H_2O \]

\[ 0.01\% \]
Principal Ionization Processes on Venus & Mars

Mars

\[ \text{N}_2 \quad 3\% \]
\[ \text{CO}_2 \quad 95\% \]
\[ \text{O}_2 \quad - \]
\[ \text{H}_2\text{O} \quad - \]

vol III fig. 13.4

vol III fig. 13.6
Say I discover an exoplanet – Dana-I
It is a Jupiter-size gas giant orbiting fairly close (1 AU) to its star: a B-type star
(M = 10 M⊙, \(T_s = 20,000\text{K}\), \(L = 10^4 L_\odot\))

Q: Is it likely that Dana-I has an ionosphere?

Q: What if the host star were a much cooler M-type star?
Venus or Mars

- No dynamo – no $B$
- Ionosphere $\rightarrow$ conducting bdry
- SW– w/ $B$ – can’t penetrate
- Supersonic flow deflected by obstacle
- Bow shock forms

Spreiter & Stahara 1980
Simple picture of bow shock

- Ignore pressure from SW
- SW: \( u_\infty / c_{s,\infty} , \rho_\infty , M_\infty \gg 1 \)
- Standing shock \( \sim \) sphere radius= \( R_s \)
- Post-shock flow
  - v. ubsonic – \( M \ll 1 \)
  - \( \sim \) incompressible w/ \( u_r(R) = 0 \)
  \[ u = \nabla \psi \times \nabla \phi \]

\[ \psi(r,\theta) = C \left( \frac{r^4}{R^4} - \frac{R^2}{r^2} \right) \sin^2 \theta \quad \text{Lighthill 1957} \]

- \( u_{n,2} = u_{n,1}/4 \) , \( u_{t,2} = u_{t,1} \)

\[ \frac{u_{r,2}}{\cos \theta} = 2 \frac{CR_s^2}{R^4} \left( 1 - \frac{R^5}{R_s^5} \right) = -\frac{1}{4} u_\infty \]
\[ \frac{u_{\theta,2}}{\sin \theta} = - \frac{CR_s^2}{R^4} \left( 4 + \frac{R^5}{R_s^5} \right) = u_\infty \]

\[ R_s = \left( \frac{3}{2} \right)^{2/5} R = 1.18 R \]
Numerical solution from Spreiter et al. 1966

\[ M_{\infty,n} = 1 \]

\[ \alpha = \sin^{-1}(1/M_{\infty}) \]

Weak shock far down stream
Shock partially thermalizes flow KE of SW:

- Nose point (normal)

\[ T_N = \frac{3}{8} \cdot \frac{1}{2} m u_\infty^2 \]

- Stagnation point

\[ T_s = \frac{16}{15} T_N = \frac{2}{5} \cdot \frac{1}{2} m u_\infty^2 \]

\( u_\infty = 400 \text{ km/s} \)

\( \Rightarrow T_N = 3.6 \text{ MK} \)

\( \Rightarrow T_s = 3.8 \text{ MK} \)

- pressure

\[ p_s = \frac{4}{5} \rho_\infty u_\infty^2 \]

Spreiter et al. 1966
Wind @ Magnetized Planets
Earth, Jupiter, Saturn, ...

- Planetary B prevents SW from reaching ionosphere
- SW deflected by magnetosphere
- “squishy” obstacle

Hughes (cf. vol. I fig. 10.1)
Shock & sheath: similar to before

- Stagnation point (SP) @ $r=R_{mp}$
- Plasma pressure:
  $$p_s = \frac{4}{5} \rho_\infty u_\infty^2$$
- Inside ($r < R_{mp}$):
  $$\mathbf{B} = -\nabla \chi$$
  $$\chi(r, \theta) = \frac{B_\odot R_\odot^3}{R_{mp}^2} \left( \frac{R_{mp}^2}{r^2} + \frac{2r}{R_{mp}} \right) \cos \theta$$
- Magnetic pressure @ SP
  $$\frac{1}{8\pi} |\mathbf{B}(R_{mp}, 0)|^2 = \frac{1}{8\pi} \left( \frac{1}{R_{mp}} \frac{\partial \chi}{\partial \theta} \right)^2 = \frac{9R_\odot^6}{8\pi R_{mp}^6} B_\odot^2$$
- Ignore inner plasma – balance

Chapman-Ferraro Distance

$$R_{mp} = \left( \frac{45}{32\pi} \right)^{1/6} \left( \frac{B_\odot^2}{\rho_\infty u_\infty^2} \right)^{1/6} R_\odot$$
Intuition break

\[ R_{mp} = \left( \frac{45}{32\pi} \right)^{1/6} \left( \frac{B_\oplus^2}{\rho_\infty u_\infty^2} \right)^{1/6} R_\oplus \sim 12 R_\oplus \]

\[ \rho_{sw} = 10^{-23} \text{ g/cm} \]
\[ u_{sw} = 400 \text{ km/s} \]

• At what distance do geostationary satellites orbit?

• Is the moon inside or outside the magnetopause?

• What happens to \( R_{mp} \) during fast SW: \( u_{sw} = 800 \text{ km/s} \)
Similar picture from high-powered codes

vol. I fig. 11.2
Other planets... same story

\[ B_J \sim 15 \quad B_\oplus \sim 5 \ \text{G} ; \quad \rho_\infty \sim 0.04 \ \rho_\infty, \oplus \]

\[ \Rightarrow \text{Jupiter's magnetopause:} \]

\[ R_{mp,J} \sim 50 \ R_J = 3.5 \times 10^{11} \ \text{cm} \]

vol. I fig. 13.6
But not all of Earth’s field stays confined to m-sphere

**Reconnection** with SW field (consider southward IMF)
- Creates “open” flux connected to poles @ $\dot{\Phi}_{ds}$
- SW sweeps flux downstream – into **magnetotail**
- Steady state only when reconnection in tail “closes” flux at rate $\dot{\Phi}_n = -\dot{\Phi}_{ds}$
- Requires long & strong **neutral sheet** in magnetotail

vol. I fig. 10.3
But not all of Earth’s field stays confined to m-sphere

Hughes (cf. vol. I fig. 6.3)

“closes” flux at rate \( \dot{\Phi}_n = -\dot{\Phi}_{ds} \)

- Requires long & strong neutral sheet in magnetotail

When balance occurs, tail...

- ... has some length \( L_t \gg R_{mp} \)
- ... has some open flux \( \Phi_t \)
vol. I fig. 13.8
vol. I fig. 13.10
closed/open boundary maps down to “auroral oval”
\[ \Phi_t = \Phi_{pc} = \pi \left( R_\oplus \sin \theta_{pc} \right)^2 B_{np} \sim \pi R_\oplus^2 \theta_{pc}^2 B_{np} \sim 10^{17} \text{Mx} \]

\[ \Phi_t = \frac{\pi}{2} R_t^2 B_t \]

Mag. pressure

In tail:

\[ \frac{1}{8\pi} B_t^2 = \frac{1}{2\pi^3} \frac{\Phi_t^2}{R_t^4} = \frac{1}{2\pi} \left( \frac{R_\oplus}{R_t} \right)^4 \theta_{pc}^4 B_{np}^2 \]

Pressure balance

@ m-pause:

\[ \frac{R_t}{R_\oplus} = (2\pi)^{-1/4} \frac{B_{np}^{1/2}}{p_{sw}^{1/4}} \theta_{pc} \sim 25 \]

\[ B_t \sim 10^{-4} \text{G} \sim 10 \text{nT} \]
Other auroral ovals

vol. I fig. 2.9
Convection: magnetosphere meets ionosphere

Field lines are frozen to M-spheric plasma. Motion sweeps filed lines back.

Objection: field lines are also frozen into liquid core – ends cannot be moved.

**BUT:** atmosphere & solid crust are insulators – field lines are imaginary there.
Example of how the motions meet

\[ E = -v \times B \]
Example of how the motions meet

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\[ E = \frac{J}{\sigma} \]
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Example of how the motions meet
Example of how the motions meet

$E = -v \times B$

$E = J / \sigma$
Example of how the motions meet

Field-aligned currents in MHD region

\[ E = -v \times B \]

\[ E = \frac{J}{\sigma} \]

MHD Field line motion creates current in ionosphere – accompanied by \( E \)
Convection: magnetosphere meets ionosphere

MHD motions drag footpoints across polar caps and back around to day side

Integrate* $\mathbf{E}$ across polar cap:

$$\int_A^B \mathbf{E} \cdot d\mathbf{l} = \varphi_{pc} = \Phi_{ds}$$

Really an EMF – but called “cross polar cap potential”

* use MKS here
\[
\int_{A}^{B} \mathbf{E} \cdot d\mathbf{l} = \varphi_{pc} = \dot{\Phi}_{ds}
\]

\[
\Phi_{pc} = 50 \text{ kV} \\
= 5 \times 10^{12} \text{ Mx/s}
\]

Field-aligned currents in MHD region

Convection flow

recycle in \(\Phi_t\) in \(\sim 5\) hours

vol. I fig. 10.5
Summary

• Ionospheres created by EUV & X-rays from Sun’s TR and corona
• Diminish during night – lower during solar minimum
• SW deflected by ionospheres of unmagnetized planets (Venus & Mars)
• SW deflected by magnetospheres
• Magnetotail created by reconnection with solar wind magnetic field