Planetary Dynamos

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I) What is a Dynamo?
  ‣ Magnetic field creation vs dissipation
  ‣ Conditions for a planetary dynamo

II) A whirlwind tour of the Solar System
  ‣ Observed magnetic fields
  ‣ Internal structure

III) Convection in Rotating Spheres
  ‣ Dynamical balances
  ‣ Columns and waves

IV) Numerical Models
  ‣ General trends
  ‣ Case Studies (Earth, Jupiter…)

Outward heat (energy) flux $F_c$
What is a (hydromagnetic) Dynamo?

An object (such as a star or a planet or a lab experiment) that converts the kinetic energy of fluid motions into magnetic energy

Glatzmaier & Roberts (1995)
MHD Magnetic Induction equation

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})
\]

Comes from Maxwell’s equations (Faraday’s Law and Ampere’s Law)

\[
\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
\]

\[
\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} \quad \text{(Assumes } v \ll c\text{)}
\]

And Ohm’s Law

\[
\mathbf{J} = \sigma \mathbf{E}
\]

Magnetic diffusivity

\[
\eta = \frac{c^2}{4\pi \sigma}
\]

electrical conductivity
Creation and destruction of magnetic fields

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \]

**Source of Magnetic Energy**

**Sink of Magnetic Energy**

How would you demonstrate this?

\[ E_m = \frac{B^2}{8\pi} \]

(Hint: have a sheet handy with lots of vector identities!)
Creation and destruction of magnetic fields

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \]

Source of Magnetic Energy

\[ \frac{\partial E_m}{\partial t} = -\nabla \cdot \mathbf{F}_P - \frac{\mathbf{v}}{c} \cdot (\mathbf{J} \times \mathbf{B}) - \Phi_o \]

Poynting Flux

\[ \mathbf{F}_P = \mathbf{E} \times \mathbf{B} = \left[ \frac{\eta}{c} \mathbf{J} - \frac{1}{4\pi} (\mathbf{v} \times \mathbf{B}) \right] \times \mathbf{B} \]

Sink of Magnetic Energy

Ohmic Heating

\[ \Phi_o = \frac{4\pi \eta}{c^2} J^2 \]
Creation and destruction of magnetic fields

\[ \frac{\partial B}{\partial t} = \nabla \times (v \times B - \eta \nabla \times B) \]

Source of Magnetic Energy
\[ \sim U B / D \]

Sink of Magnetic Energy
\[ \sim \eta B / D^2 \]

\[ Rm = \frac{UD}{\eta} \]

If \( Rm >> 1 \) the source term is much bigger than the sink term

....Or is it???
Creation and destruction of magnetic fields

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B - \eta \nabla \times B)
\]

\(\delta\) can get so small that the two terms are comparable

It’s not obvious which term will “win” - it depends on the subtleties of the flow, including geometry & boundary conditions
Creation and destruction of magnetic fields

\[ \frac{\partial B}{\partial t} = \nabla \times (v \times B - \eta \nabla \times B) \]

**Source of Magnetic Energy**
\[ \sim UB / D \]

**Sink of Magnetic Energy**
\[ \sim \eta B / \delta^2 \]

**What is a Dynamo? (A corollary)**

A dynamo must sustain the magnetic energy (through the conversion of kinetic energy) against **Ohmic dissipation**
The need for a Dynamo

If \( v = 0 \) and \( \eta = \text{constant} \) then the induction equation becomes

\[
\frac{\partial B}{\partial t} = -\eta \nabla \times \nabla \times B = \eta \nabla^2 B
\]

The field will diffuse away (dissipation of magnetic energy) on a time scale of \( \tau_d \approx \frac{D^2}{\eta} \)

A more careful calculation for a planet gives \( \tau_d \approx \frac{R^2}{\pi^2 \eta} \)

Earth: \( \tau_d \sim 80,000 \) yrs

Jupiter: \( \tau_d \sim 30 \) million yrs

Planetary fields must be maintained by a dynamo or they would have decayed by now!
Conditions for a Planetary (or Stellar) Dynamo

Absolutely necessary

- An electrically conducting fluid
  - Stars: plasma
  - Terrestrial planets: molten metal (mostly iron)
  - Jovian planets: metallic hydrogen (maybe molecular H)
  - Ice Giants: water/methane/ammonia mixture
  - Icy moons: salty water

- Fluid motions
  - Usually generated by buoyancy (convection)

- $R_m \gg 1$
  - Too much ohmic diffusion will kill a dynamo
Conditions for a Planetary (or Stellar) Dynamo

- Not strictly necessary but it (usually) helps

- **Rotation**
  - **Good**: helps to build strong, large-scale fields (promotes magnetic self-organization)
  - **Bad**: can suppress convection (though this is usually not a problem for planets)

- **Turbulence** (low viscosity / Re >> 1)
  - **Good**: Chaotic fluid trajectories good at amplifying magnetic fields (chaotic stretching)
  - **Bad**: can increase ohmic dissipation
Earth

Dynamo!

*Field strength*
~ 0.4 G

*Dipolarity*
~ 0.61

*Tilt*
~ 10°

Archetype of a terrestrial planet!
Direct measurements of Earth’s magnetic field date back to the early 1500’s, with a boost in the early 1800’s with the **Magnetic Crusade** led by Sabine in England and Gauss and Weber in Germany.

Today we also have satellite measurements.

Longer time history can be inferred from measurements of magnetic signatures in crustal rocks.
Magnetic poles flip every ~ 200,000 years on average, but randomly.

Irregular reversals!

Heirtzler et al (1960’s)
Mantle convection responsible for plate tectonics but not the geodynamo.
Earth

Mantle
non-conducting, slow

Overturning time
~100 million years

Outer Core
conducting, fast

Overturning time
~500 years
Earth

Rotational influence quantified by Rossby number

\[ Ro = \frac{U}{2\Omega D} = \frac{1}{4\pi} \frac{P_{\text{rot}}}{\tau_c} \]

\[ Ro \sim 4 \times 10^{-7} \]

Outer Core conducting, fast

Overturning time

\(~500\) years
Earth

Spherical Harmonic expansion of the surface field allows for a backward extrapolation to the core-mantle boundary (CMB)

Assuming no currents in the non-conducting mantle & crust

\[ B_r \propto r^{-(\ell+2)} \]

R. Townshend (Wisconsin)
Earth

\[ B_r \propto r^{-(\ell+2)} \]

Dipole dominates at large distances from the dynamo region \( \sim r^{-3} \)

Time evolution of surface field can be used to infer flows at the CMB

Jones (2011)
Earth

Energy sources for convective motions

- **Outward heat transport by conduction**
  - Cooling of the core over time
  - Proportional to the heat capacity

- **Latent heat**
  - Associated with the freezing (phase change) of iron onto the solid core

- **Gravitational Differentiation**
  - Redistribution of light and heavy elements, releasing gravitational potential energy

- **Radioactive Heating**
  - Energy released by the decay of heavy elements
Venus

No Dynamo

No field detected

Why?

Core may be liquid and conducting, but it may not be convecting (rigid top may inhibit cooling)

Also - slow rotation
Mars

No Dynamo

Fields patchy, reaching \( \sim 0.01 \) G in spots but no dipole

Why?

It had a dynamo in the past (remnant crustal magnetism) but it cooled off fast, freezing out its molten core

Arkani-Hamed (2007)
Mercury

Dynamo!

Field strength
\(~ 0.003 \, G\)

Dipolarity
\(~ 0.71 \, G\)

Tilt \(\sim 3^\circ\)

Huge iron core relative to size of planet that is still partially molten

Schubert & Soderlund (2011)
But we’re still not really sure what’s going on!

Stanley & Glatzmaier (2009)
Ganymede!

Dynamo!

Field strength
~ 0.01 G

Dipolarity
~ 0.95 G

Tilt
~ 4°

Other icy satellites have induced magnetic fields from passing through the magnetospheres of their planets

Schubert & Soderlund (2011)
Jupiter

Field strength \(\sim 7\, \text{G}\)

Dipolarity \(\sim 0.61\)

Tilt \(\sim 10°\)

Archetypical Jovian planet!
Jupiter

Layer composed of Hydrogen and Helium in liquid state

Layer composed of metallic Hydrogen

The core made of rock, metallic Hydrogen and Helium

Hydrogen and Helium in gas state
French et al (2012)

Jupiter: Internal Structure

![Graph showing the internal structure of Jupiter with temperature (T) and pressure (P) on the axes. The graph includes various lines and markers representing different conditions and data sets, such as nonmetallic fluid, metallic fluid, solid, and the Jupiter adiabat.](image-url)
Jupiter: Internal Structure

\[ R_m = \frac{UD}{\eta} \]

Transition from metallic to molecular (liquid) H

French et al. (2012)

\[ \eta \quad (m^2 \text{ s}^{-1}) \]

\[ 10^0 \quad 10^2 \quad 10^4 \quad 10^6 \quad 10^8 \quad 10^{10} \quad 10^{12} \]

\[ 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1 \]

\[ R \quad [R_J] \]
Jupiter: Magnetic Field (Pre-Juno)

Ridley & Holme (2016)
Initial results from Juno

Stronger and more patchy than expected (higher-order multipoles)

This suggests that dynamo action might exist closer to the surface than previously thought

\[ B_r \propto r^{-(l+2)} \]

Moore et al (2017)
Saturn

**Dynamo!**

*Field strength*
\~ 0.6 G

*Dipolarity*
\~ 0.85 G

*Tilt*
< 0.5°

Remarkably axisymmetric!

A surprise!

Connerney (1993)
Cowling's Theorem

Why is this a surprise?

Assume $B$ is axisymmetric and consider the longitudinally-averaged MHD induction equation:

$$\frac{\partial B}{\partial t} = \nabla \times (\langle v \rangle \times B - \langle \eta \rangle \nabla \times B)$$

Express $B$ as $B = \nabla \times \left( A \hat{\phi} \right) + B \hat{\phi}$

Evolution eqn for $A$ (after some manipulation)

$$\frac{\partial}{\partial t} \left( \lambda A \right) = -\mathbf{v} \cdot \nabla (\lambda A) + \eta \lambda \left( \nabla^2 A - \lambda^{-2} \right) A$$

Multiply by $\lambda A$ and integrate over volume: if $\nabla \cdot \mathbf{v} = 0$ then the first term on the RHS is zero and the second term is negative
Cowling's Theorem (cont.)

\[ \frac{\partial}{\partial t} (\lambda A) = -\mathbf{v} \cdot \nabla (\lambda A) + \eta \lambda (\nabla^2 A - \lambda^{-2}) A \]

A decays with time

If A decays with time, then B will decay with time too (Work it out!)

Even if \( \nabla \cdot \mathbf{v} \neq 0 \) you can show that a steady field \( (\partial A/\partial t = 0) \) cannot be maintained

Conclusion: it is not possible to sustain a steady axisymmetric B field against ohmic dissipation

Corollary: It is not possible for a dynamo to produce a steady axisymmetric field!!
Uranus & Neptune

Dynamos!

Field strength ~ 0.3 G

Dipolarity ~ 0.42, 0.31

Tilt ~ 59°, 45°
Understanding the Dynamics

Conservation of momentum in MHD

\[ \rho \frac{\partial \mathbf{v}}{\partial t} = - (\rho \mathbf{v} \cdot \nabla) \mathbf{v} - 2\rho (\mathbf{\Omega} \times \mathbf{v}) - \nabla P + \rho \mathbf{g} + c^{-1} \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{D} \]

Convection established by buoyancy

But rotation exerts an overwhelming influence
Coriolis accelerations happen quickly (days) compared to convection and dynamo time scales (hundreds to thousands of years)

\[ \text{Ro} = \frac{U}{2\Omega D} \ll 1 \quad \text{Ek} = \frac{\nu}{2\Omega D^2} \ll 1 \]
Dynamical Balances

\[
\rho \frac{\partial \mathbf{v}}{\partial t} = - (\rho \mathbf{v} \cdot \nabla) \mathbf{v} - 2\rho (\Omega \times \mathbf{v}) - \nabla P + \rho g + c^{-1} \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{D}
\]

Result: Flows evolve quasi-statically in so-called Magnetostrophic (MAC) Balance

\[
c^{-1} \mathbf{J} \times \mathbf{B} \approx 2\rho (\Omega \times \mathbf{v}) + \nabla P - \rho g
\]

Conservation of mass

Anelastic approximation (valid for small \( Ma \))

\[
\nabla \cdot (\hat{\rho} \mathbf{v}) = 0
\]

Boussinesq approximation (valid for small \( Ma, H_\rho >> D \))

\[
\nabla \cdot \mathbf{v} = 0
\]

hydrostatic background
Dynamical Balances

\[ \mathbf{c}^{-1} \mathbf{J} \times \mathbf{B} \approx 2\rho (\Omega \times \mathbf{v}) + \nabla P - \rho g \]

Now set \( B = 0 \) and assume that \( \nabla \rho \) is mainly radial.

Then the \( \phi \) component of the curl gives (anelastic approximation):

\[ \Omega \cdot \nabla (\rho \mathbf{v}) = \frac{\partial}{\partial z} (\rho \mathbf{v}) = 0 \]

**Taylor-Proudman Theorem**

Boussinesq version:

\[ \frac{\partial \mathbf{v}}{\partial z} = 0 \]

Rapidly rotating flows tend to align with the rotation axis.
Work with a partner to draw what you think **convective** motions might look like in a rapidly-rotating spherical shell.

How can you get the heat out while still satisfying the Taylor-Proudman theorem?

$$\frac{\partial v}{\partial z} = 0$$

Can you satisfy it everywhere?
The most unstable convective modes in a rapidly-rotating, weakly-stratified shell are Busse columns aka Banana Cells

The preferred longitudinal wavenumber \( (m) \) scales as \( \text{Ek}^{-1/3} \)

Coriolis vs viscous diffusion
Linear Theory

The Tangent Cylinder

Delineates two distinct dynamical regimes

Implication of the Taylor-Proudman theorem
Linear Theory: Traveling Waves

Prograde propagation
*(thermal Rossby waves)*

*Induced by curvature of outer boundary and/or density stratification*

\[ \frac{\omega_z}{H} = \text{constant} \]

Simplest example: Boussinesq fluid, centrifugal gravity, local, linear perturbations, small boundary curvature *(Busse 2002)*

\[ v_p = \frac{4\Omega}{L} \frac{\tan \chi}{(1 + Pr)(k_y^2 + k_x^2)} \]
Nonlinear Regimes require Numerical Models

Solve the MHD equations in a rotating spherical shell
Anelastic or Boussinesq approximation
\( \rho, T, P, S \) are linear perturbations about a hydrostatic, adiabatic background state

Convection simulations: heating from below, cooling from above
Axial alignment persists even in turbulent parameter regimes

Busse columns give way to vortex sheets but the flow is still approximately 2D

$Ek = 2.3 \times 10^{-7}$

$Ek = 2.6 \times 10^{-6}$

$Ek = \frac{\nu}{2\Omega R^2}$
...and in MHD

\[ \text{Ra} = \frac{GMDF}{\nu \kappa C_P} = \frac{\text{buoyancy driving}}{\text{dissipation}} \]

\[ \text{Ra} = 8 \times 10^5 \]

Jones et al (2011)
Busse columns are really good at making roughly dipolar fields

Kageyama & Sato (1997)
Complexity of magnetic field depends mainly on the rotational influence.

Rapid rotators tend to be more dipolar.

Christensen & Aubert (2006)
Assuming MAC balance, compute the ratio of ME/KE
How does it scale with Ro?

\[ \nabla \times B = \frac{4\pi}{c} J \]

\[ \text{ME} = \frac{B^2}{8\pi} \]

\[ \text{KE} = \frac{1}{2} \rho u^2 \]
Assuming MAC balance, compute the ratio of $ME/KE$

How does it scale with $Ro$?

\[
\frac{ME}{KE} \sim Ro^{-1}
\]

$>>1$ if $Ro \ll 1$!

But how do $KE$ and $Ro$ (and thus, $ME$) depend on observable global parameters like $\Omega$ and $F_c$?

*in principle*
General trends

Field strength scales with the heat flux through the shell (independent of $\Omega$!)

Rapid rotators seem to operate at maximum efficiency, tapping all the energy they can

Christensen et al (2009)
General trends

Rapid rotators seem to operate at maximum efficiency, tapping all the energy they can

This may apply to rapidly-rotating stars as well as planets!

Christensen et al (2009)
### Numerical Models: The Challenge

\[
P_m = \frac{\nu}{\eta}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Earth</th>
<th>Jupiter</th>
<th>Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ra</td>
<td>$10^{31}$</td>
<td>$10^{37}$</td>
<td>$10^6$-$10^7$</td>
</tr>
<tr>
<td>Ek</td>
<td>$3 \times 10^{-15}$</td>
<td>$10^{-9}$</td>
<td>$10^{-6}$-$10^{-7}$</td>
</tr>
<tr>
<td>Rm</td>
<td>300-1000</td>
<td>400-$3 \times 10^4$</td>
<td>50-3000</td>
</tr>
<tr>
<td>Pm</td>
<td>$5-6 \times 10^{-7}$</td>
<td>$6 \times 10^{-7}$</td>
<td>0.1-0.01</td>
</tr>
</tbody>
</table>
Numerical Models: The Hope

Realistic simulations might be possible if you can achieve the right dynamical balances (e.g. MAC balance)

- The most important parameters to get right (or as right as possible)
  - **Ro**
    - Appropriate rotational influence on the convection
  - **Rm**
    - Reasonable estimate of the ohmic dissipation
  - **Ek**
    - At least get it small enough that viscosity isn’t part of the force balance
Example: The Geodynamo

Points of comparison: Field strength, morphology (spectrum, symmetry, etc), Reversal timescale

Inferred from observations

Christensen et al (2010)
Best matches are those with $E_k < 10^{-4}$ and $Rm$ “large enough”
Example: The Geodynamo

Inferred from observations

But be careful! They could be right for the wrong reasons!
For example, both c and d have a higher Ra and lower Ek than b they *should* be more realistic, right?
Example: The Geodynamo

Coupling to inner core needed to get the reversal time scale right (Glatzmaier & Roberts 1995; Glatzmaier et al 1999)
Example: Jupiter

Inferred from observations

But “dipole solutions are not easy to find” for the “best” parameters

Jones (2014)
Another example of a Gas Giant dynamo highlighting banded zonal flows

Stanley & Glatzmaier (2009)
So what’s going on with Saturn?

Maybe the field is “axisymmeterized” by an overlying stable layer that has differential rotation but no convection (Stevenson 1982, Stanley 2010)

Or, maybe it’s running a different type of dynamo, driven more by shear than buoyancy (Cao et al 2012)
Numerical Models: Summary

Lessons Learned

- *Rapid Rotation has a profound influence on the dynamics*
- *Success attributed to correct dynamical balances and (when possible) realistic Rm*

Future challenges

- What happens at really low Ek (tiny $v$)?
- *Peculiarities of particular planets* (Saturn, Mercury, Uranus, Neptune…)
  - Boundary conditions (adjacent layers)
  - Rapid variations of $\eta$
  - Energy sources
  - Compositional convection
- Moving to more realistic parameters doesn’t always improve the fidelity of the model
- Exoplanets!
Juno!