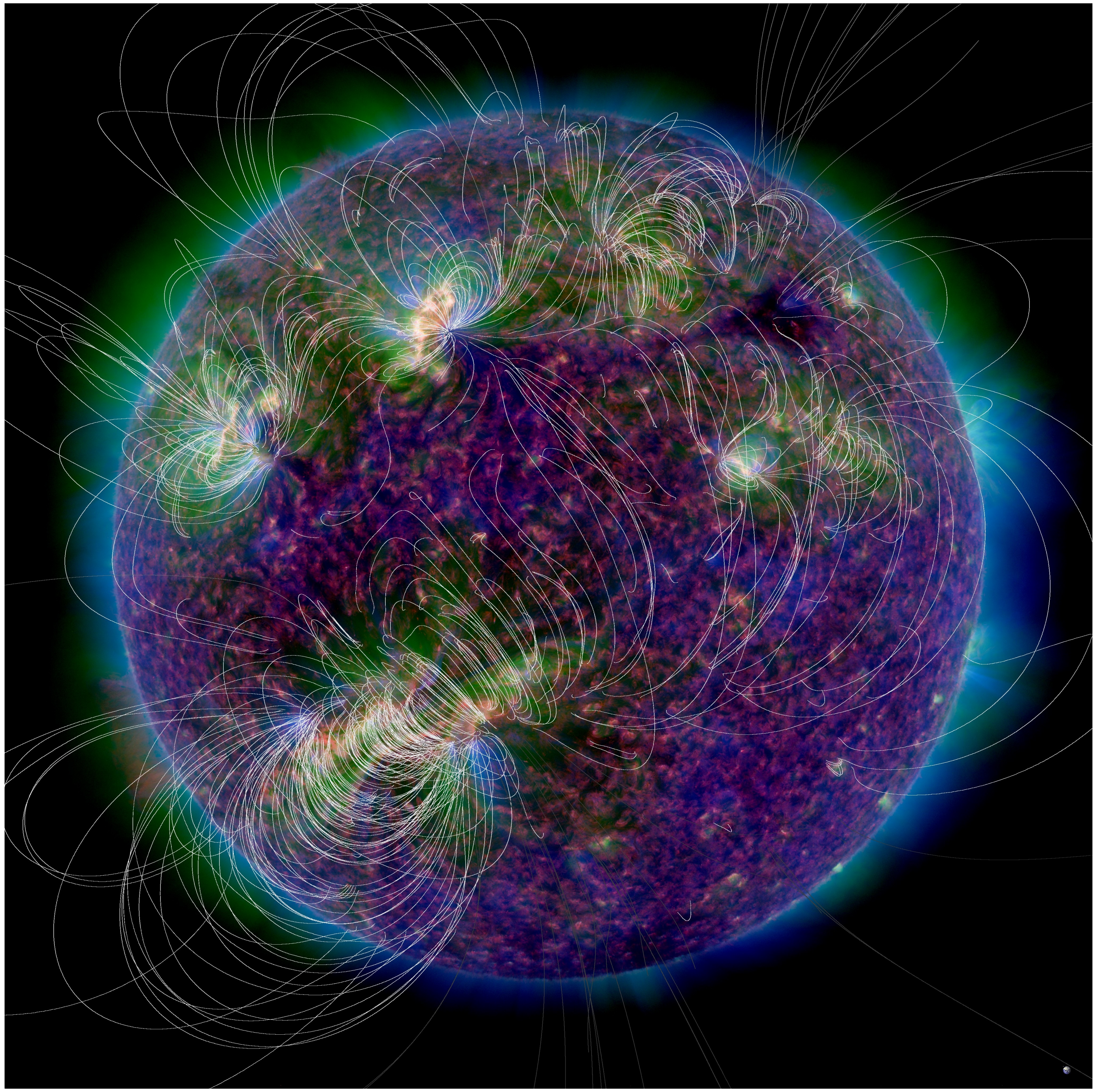
Exploring the Sun and its effects on the Earth's atmosphere and physical environment...

HIGH ALTITUDE OBSERVATORY

The Solar Dynamo

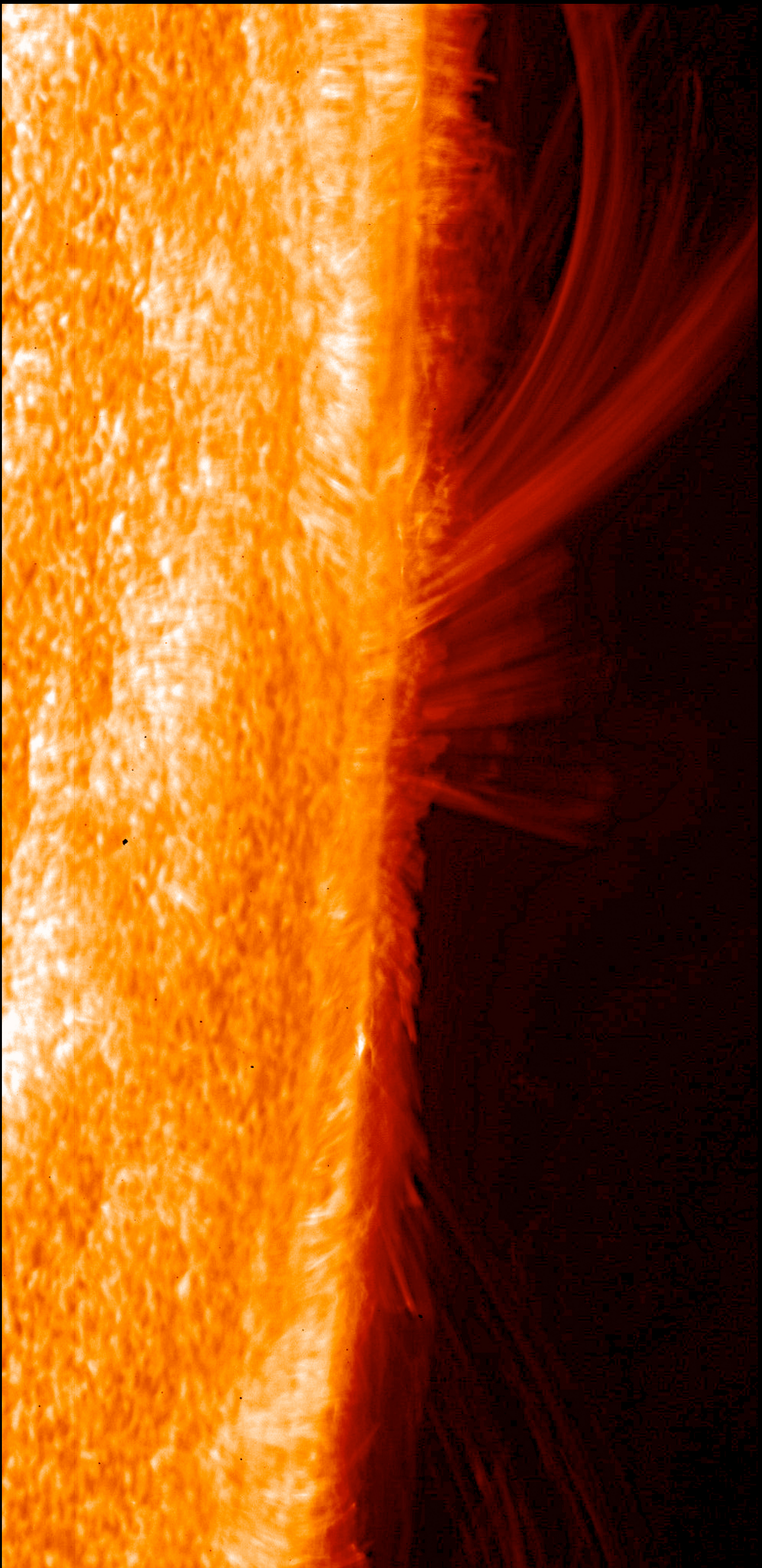


Mark Miesch

HAO/NCAR

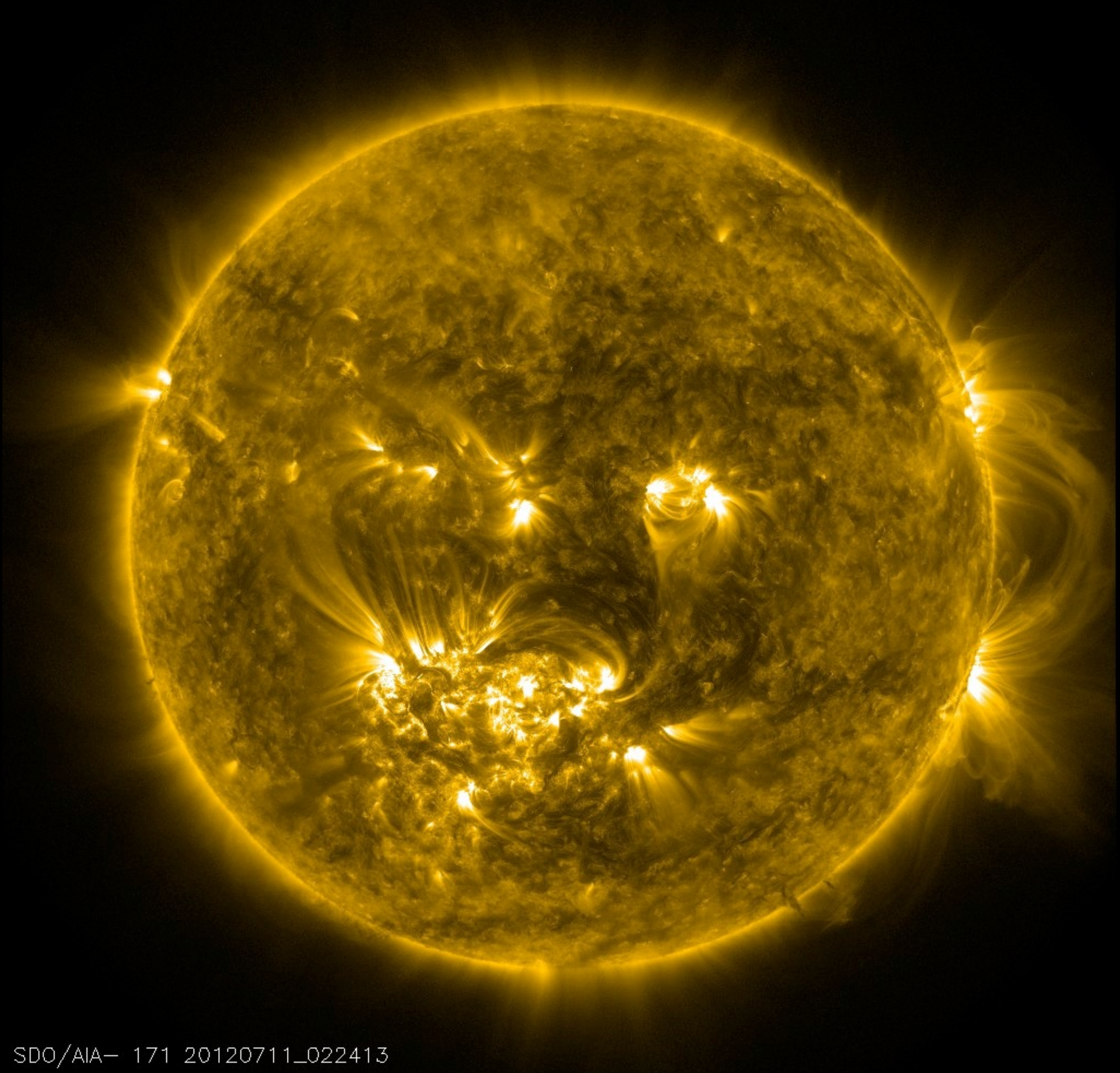
NASA Heliophysics Summer School
Boulder, Colorado

July, 2017



Outline

- ★ ***Solar Magnetism***
  - ***Order amid chaos***
- ★ ***Solar Convection and Mean Flows***
  - ***Heirarchy of convective motions***
  - ***Differential Rotation***
  - ***Meridional Circulation***
- ★ ***Solar Dynamo Models***
  - ***Small-Scale and Large-Scale Dynamos***
  - ***The Solar Cycle***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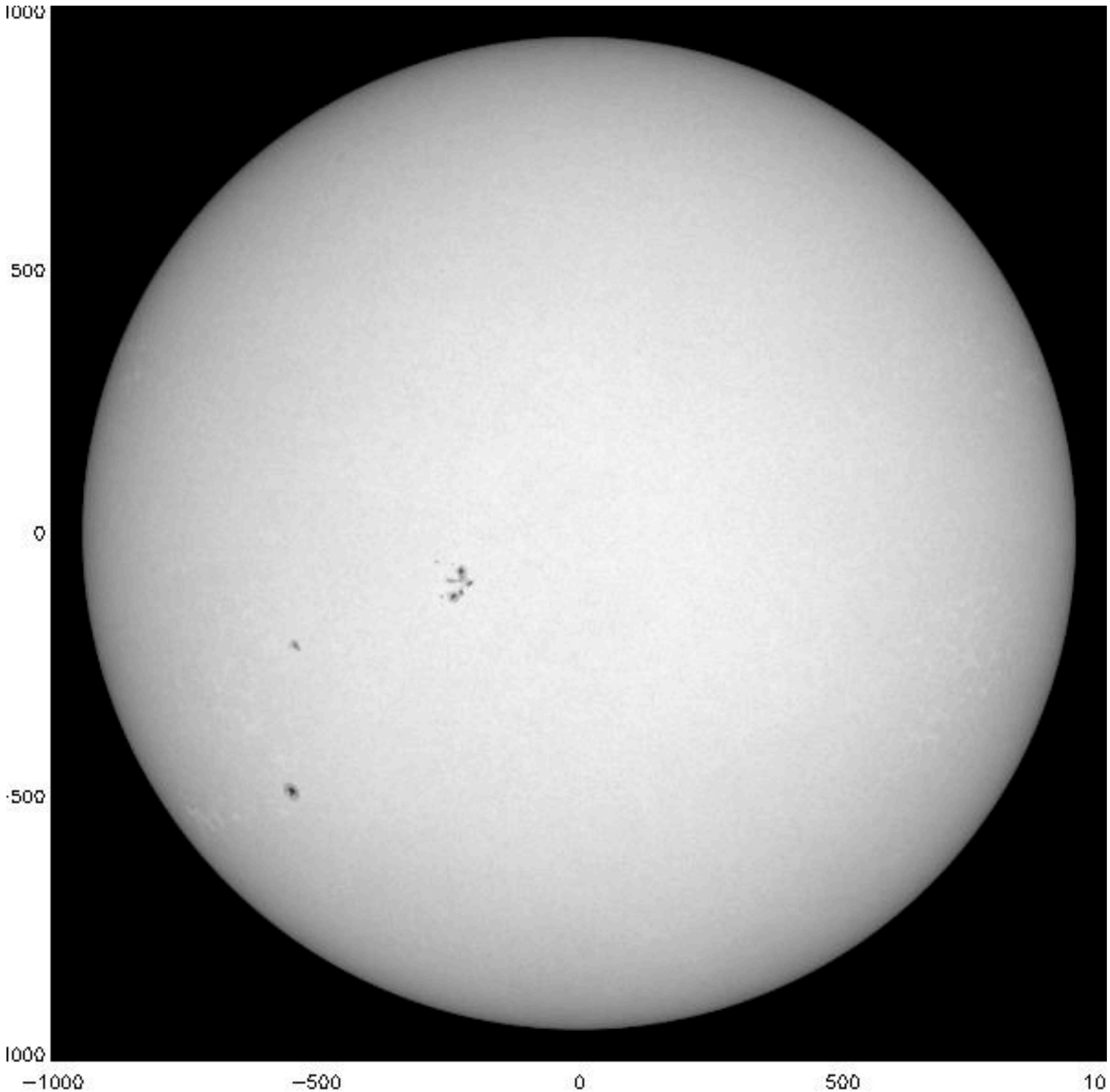

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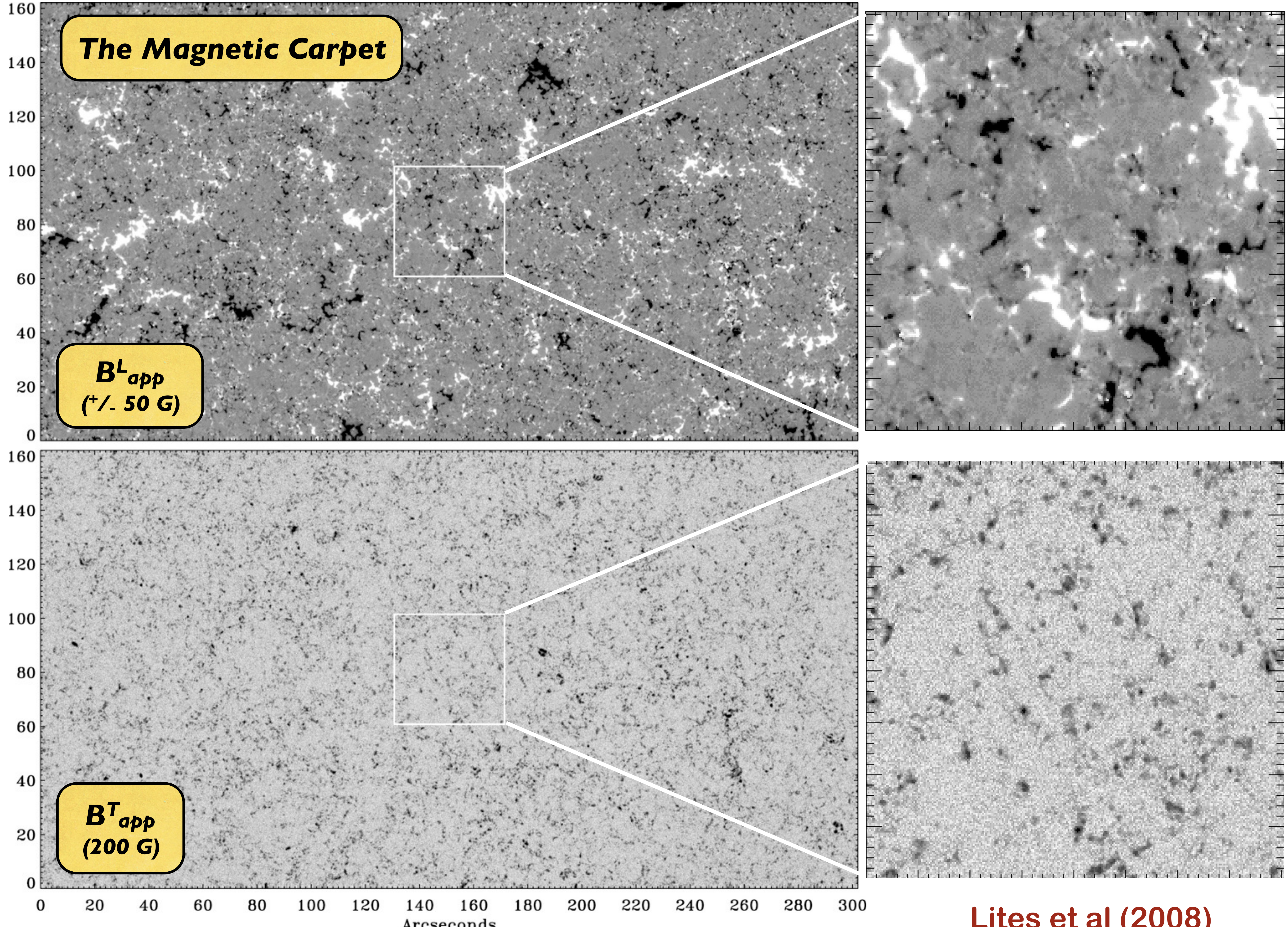
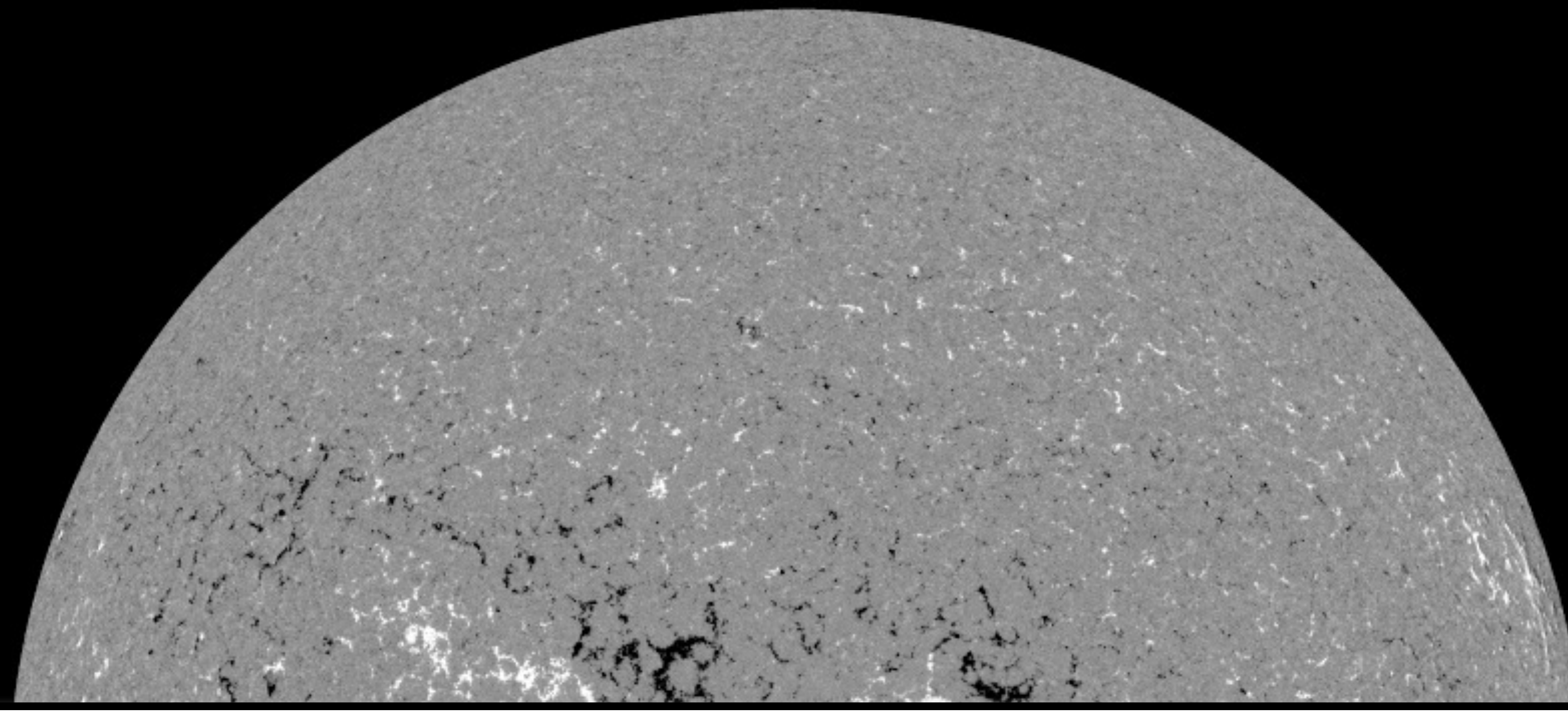
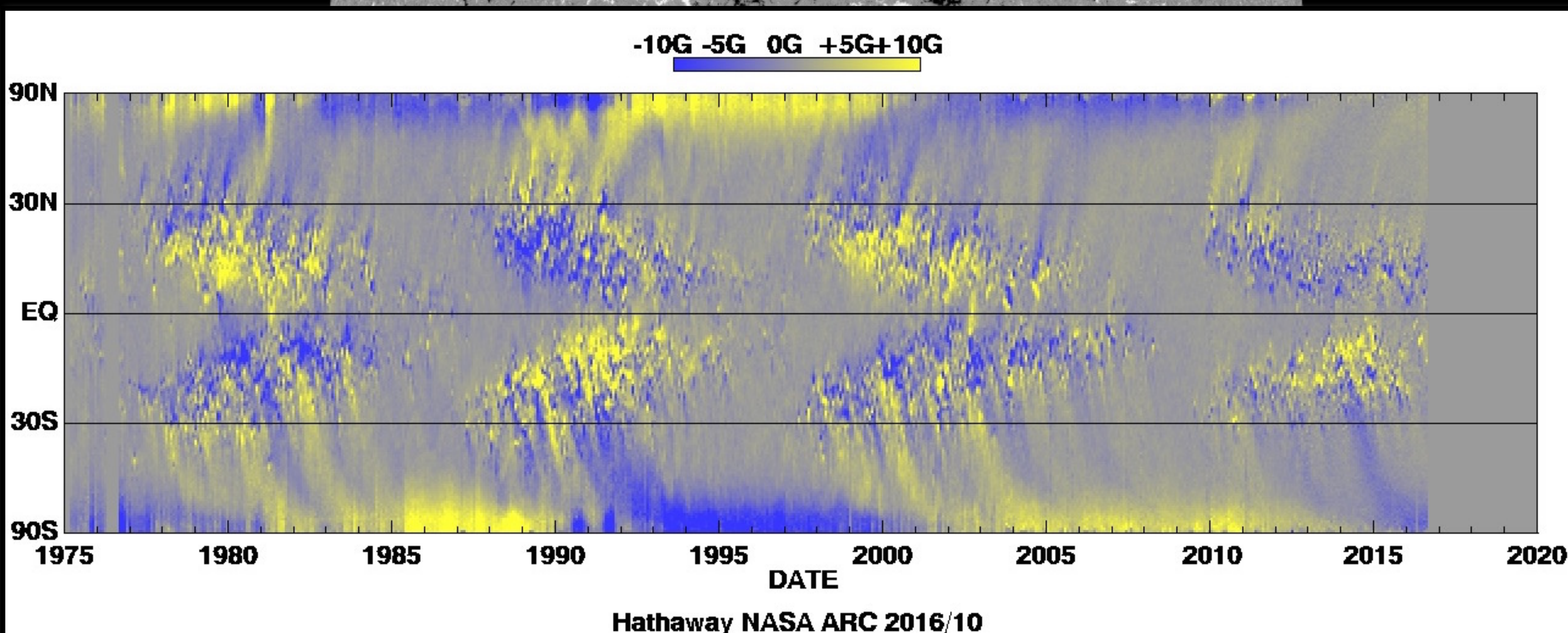


Lites et al (2008)





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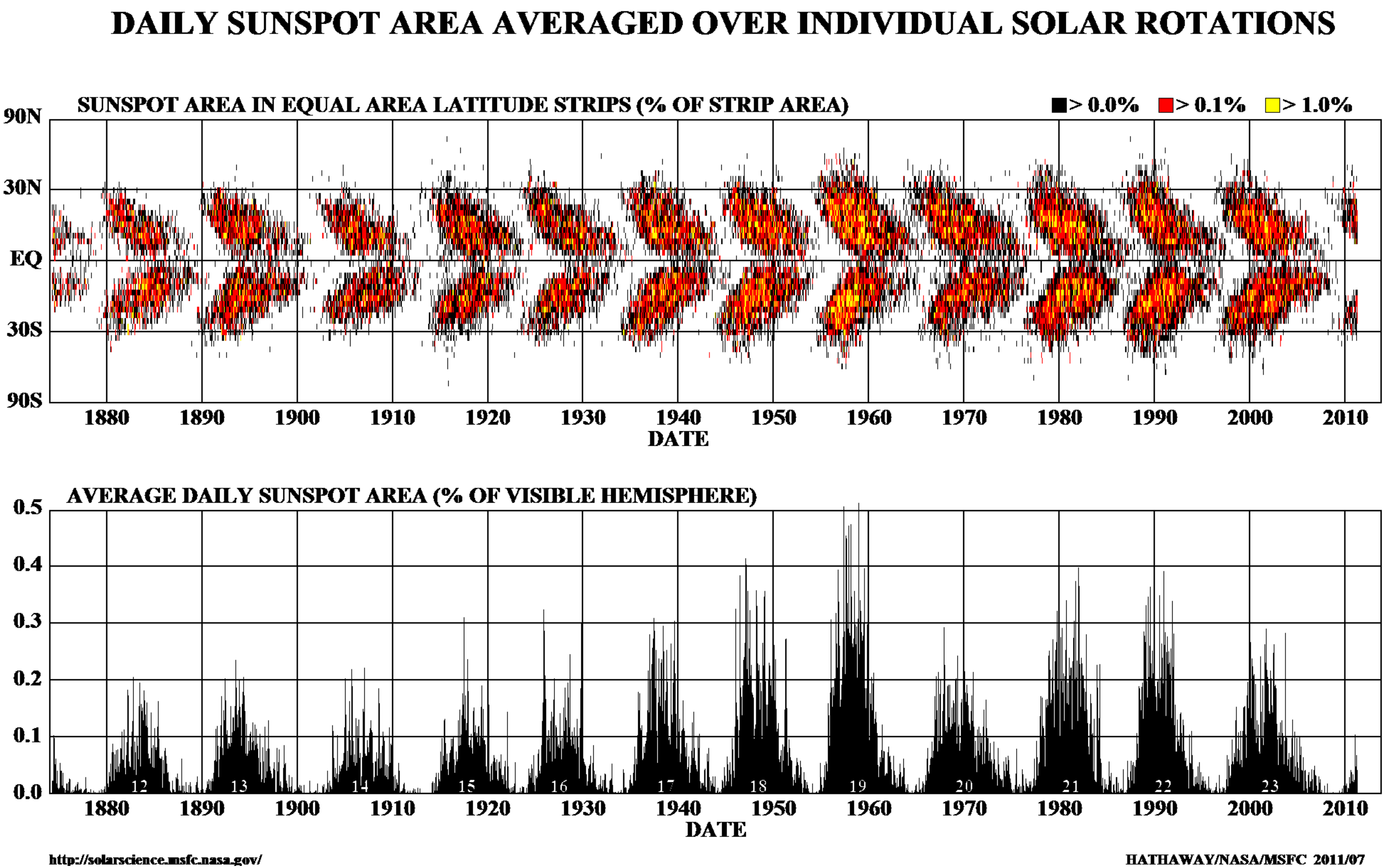
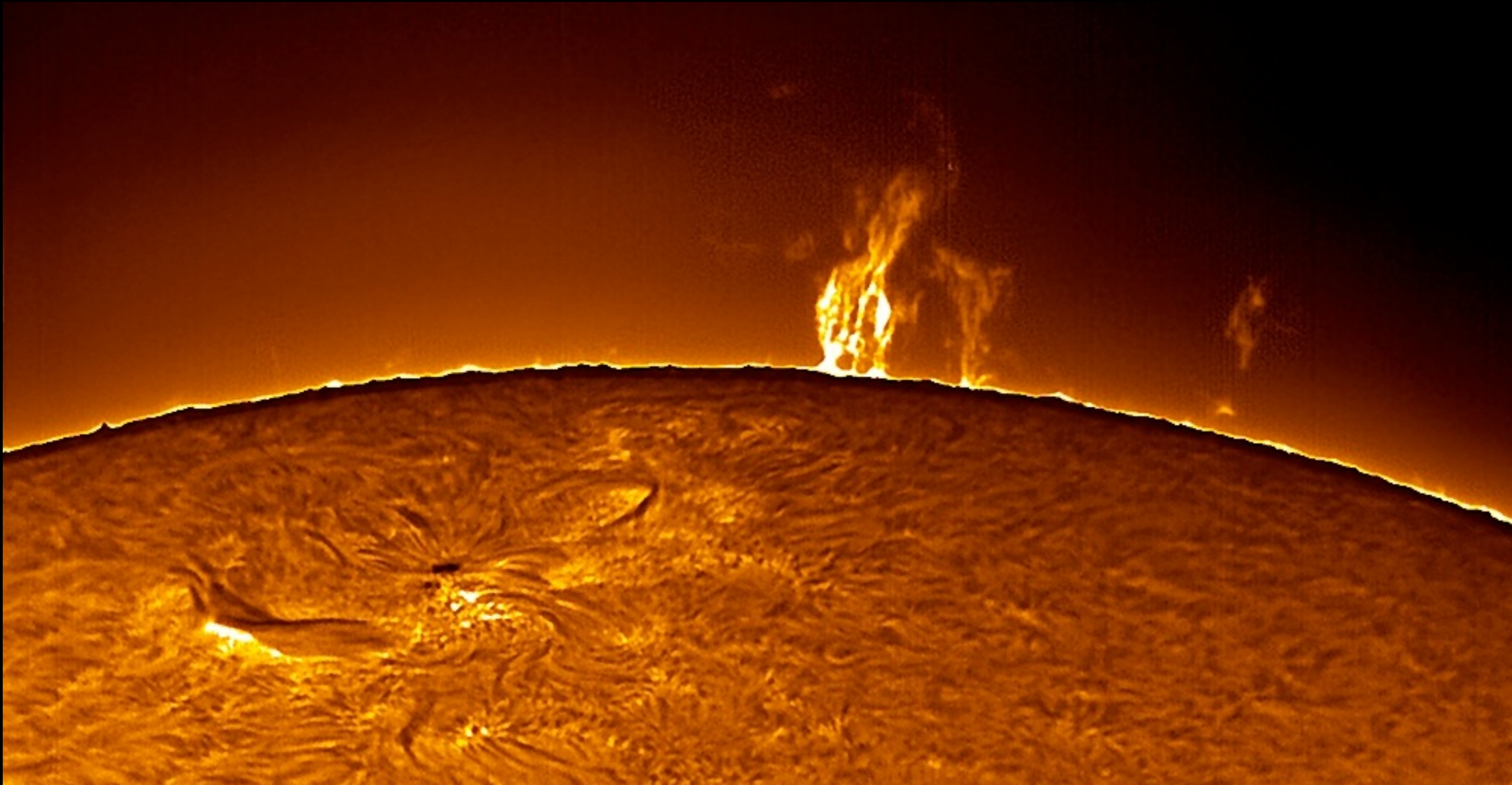
The Solar Cycle: Order Amid chaos



Solar dynamo research

Where does this magnetism come from?

The Solar Dynamo



Question

Where does the energy in the solar magnetic field come from?

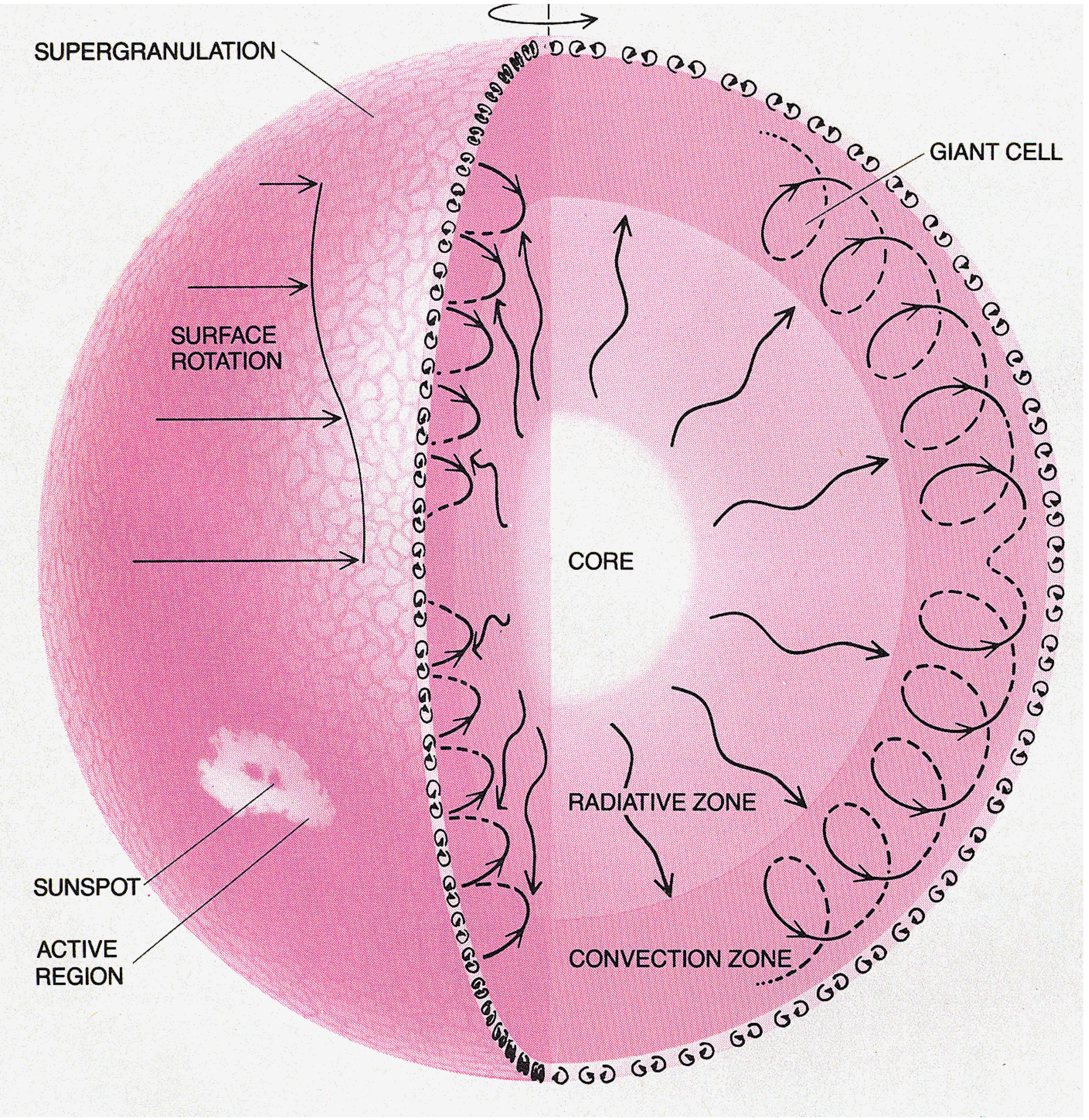
The Solar Dynamo generates magnetic fields from flows

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

convection

differential rotation

meridional circulation



magnetic energy ultimately comes from the Sun's own mass

Fusion

mass ⇒ radiation & thermal energy

Convection

thermal energy ⇒ kinetic energy

Dynamo

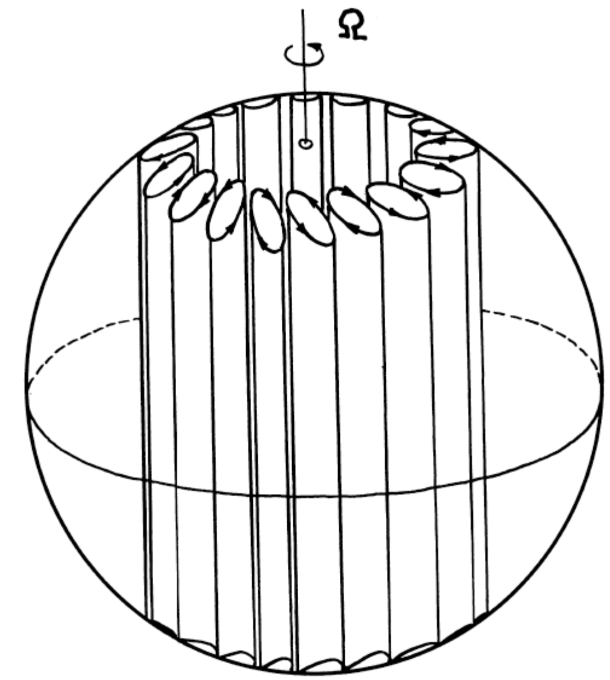
kinetic energy ⇒ magnetic energy

Part 2 (of 3)

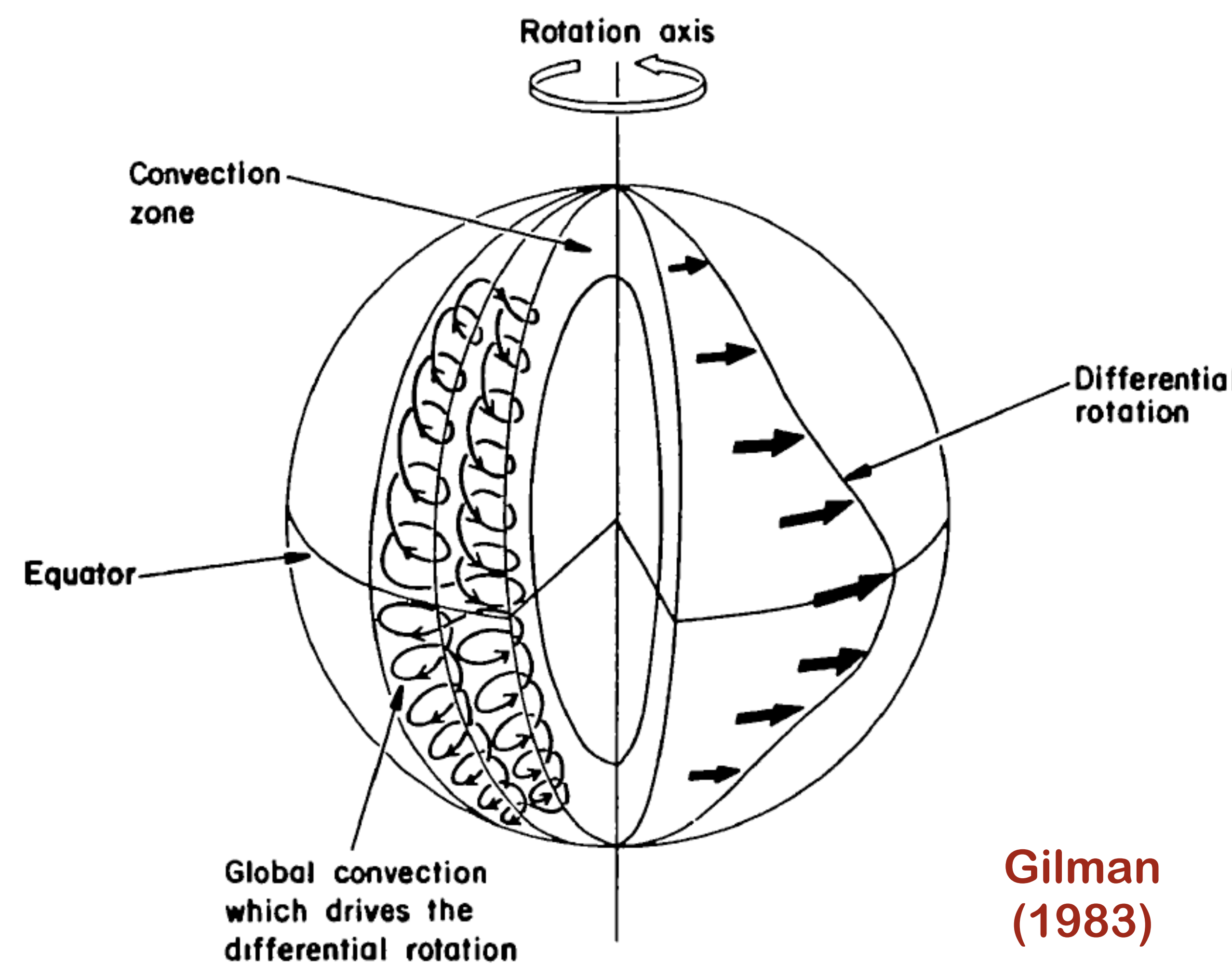
- ★ Solar Magnetism
- ★ Solar Convection and Mean Flows
- ★ Solar Dynamo Models

Differences compared to planetary convection

- **Not in the rapid rotation limit**
  - $\tau_c \sim P_{rot} \sim$ 1 month
  - Ro ~ 0.1
    (increasing to 180 in the surface layers!)
  - No MAC balance:
    $(v \cdot \nabla)\, v$ term is (very!) important
  - Stronger differential rotation
    (smaller ME/KE ~ 1?)



- **Large Density stratification**
  - Hierarchy of convective motions
    (granulation ⇒ giant cells)
  - Boussinesq approximation
    out of the question!
  - But anelastic still ok (Ma still << 1)

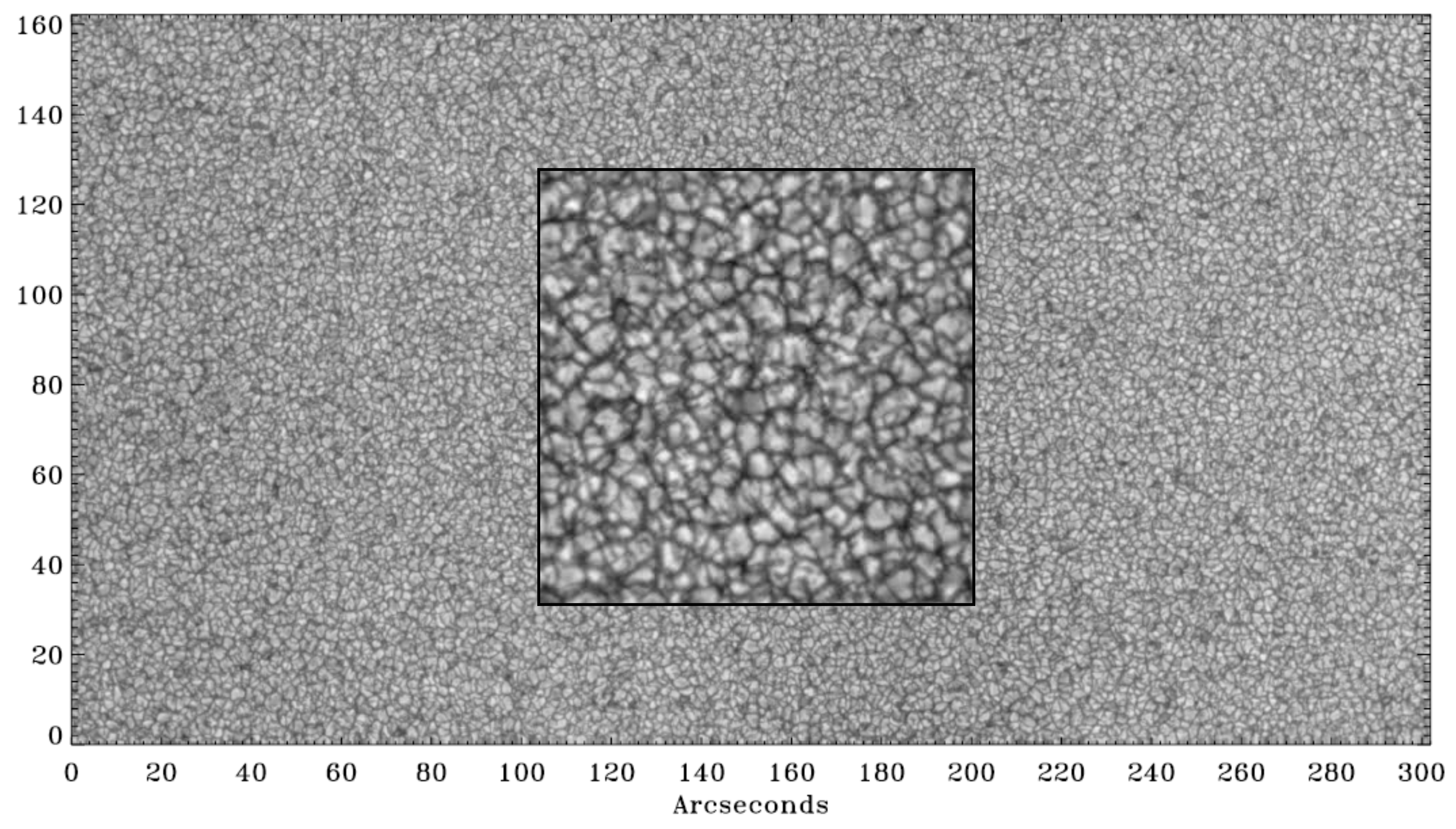


Gilman (1983)

Banana cells look more like bananas!

Granulation in the Quiet Sun

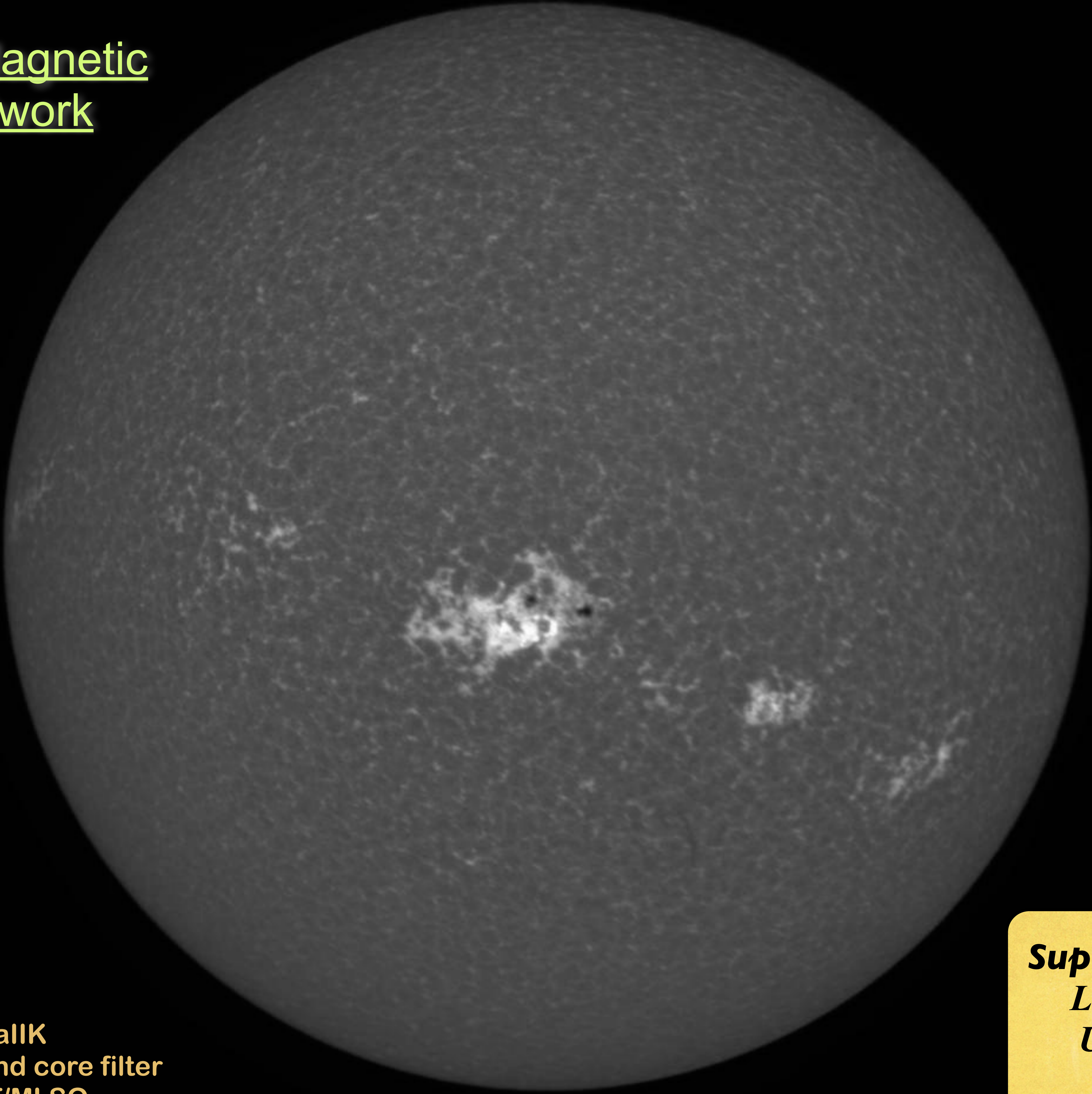
Lites et al (2008)



$L \sim$ 1-2 Mm
$U \sim$ 1 km s^{-1}
$\tau_c \sim$ 10-15 min

Dominant size scale of solar convection

The Magnetic Network

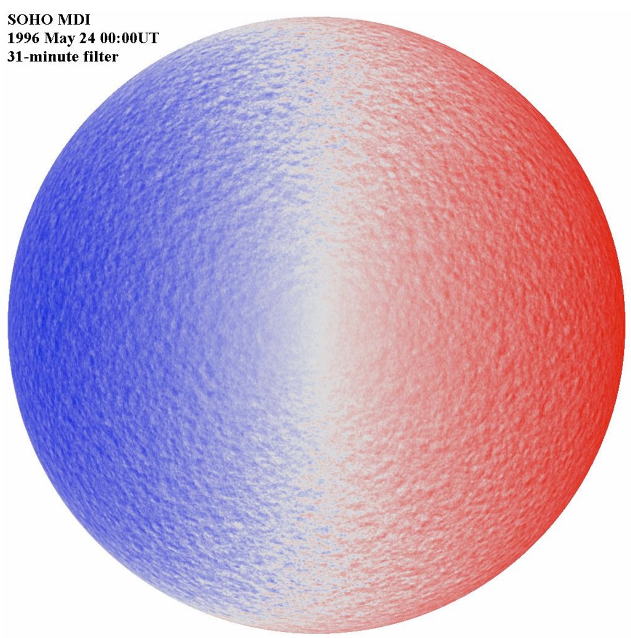


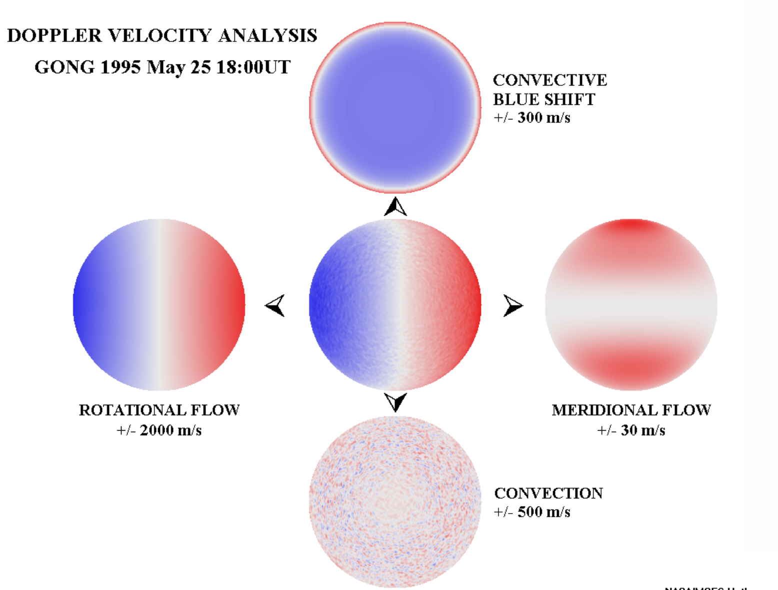
CaIIK
narrow-band core filter
PSPT/MLSO

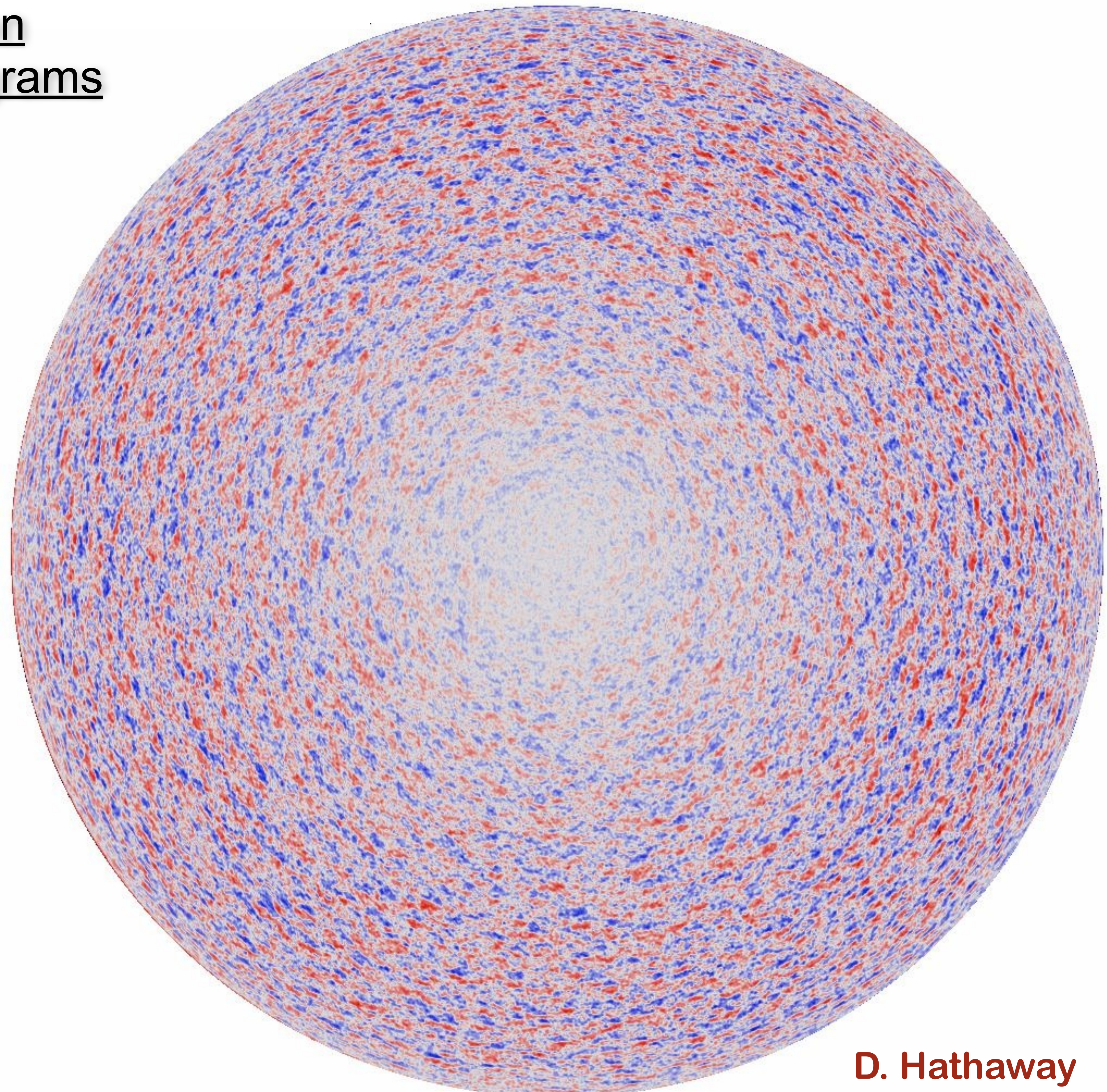
Supergranulation
$L \sim$ 30-35 Mm
$U \sim 500$ m s^{-1}
$\tau_c \sim 20$ hr

Supergranulation in Filtered Dopplergrams

Most prominent in horizontal velocities near the limb







D. Hathaway
(NASA MSFC)

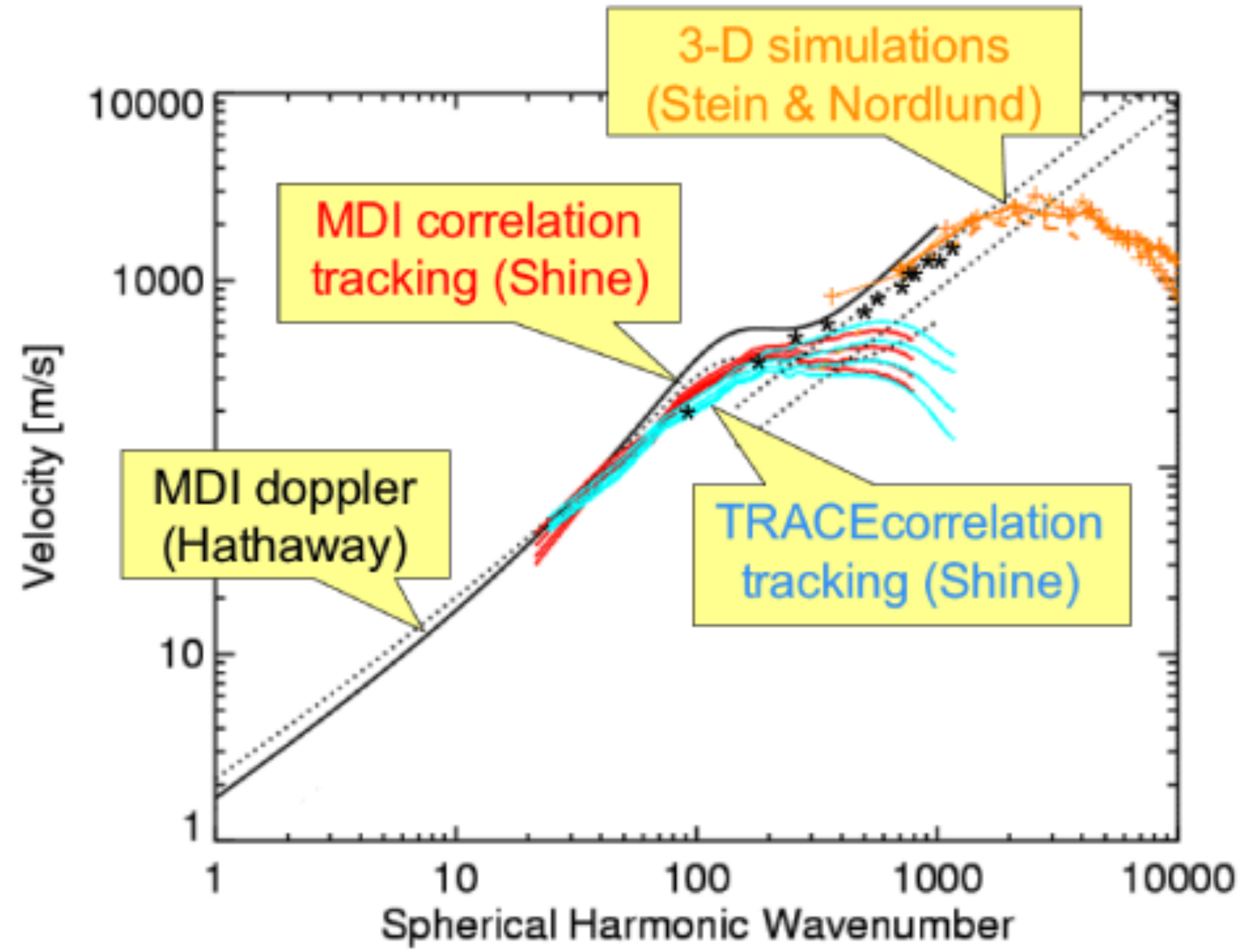
A hierarchy of convective scales

Density increases dramatically with depth below the solar surface

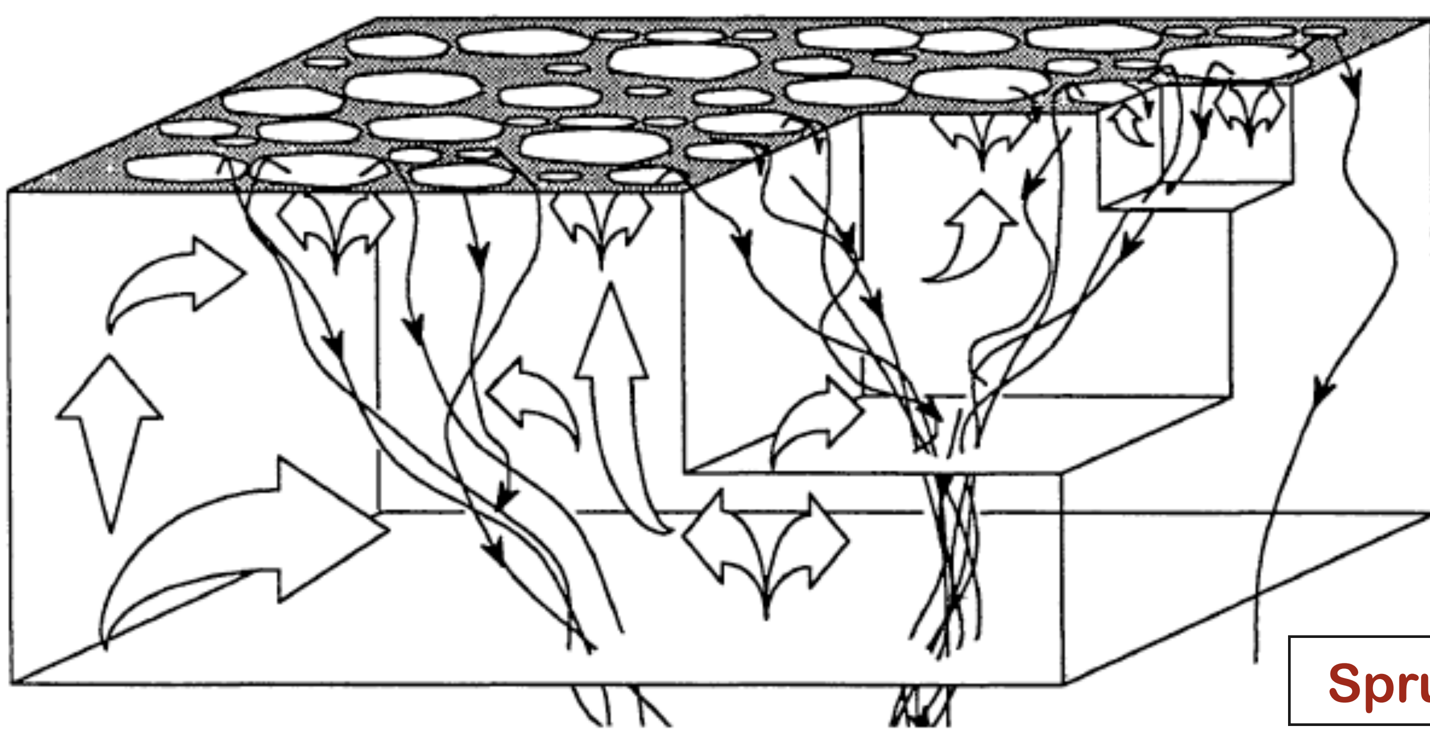
Fast, narrow down flows (plumes) Slow, broad upflows

Most of the mass flowing upward does not make it to the surface

Downward plumes merge into superplumes that penetrate deeper



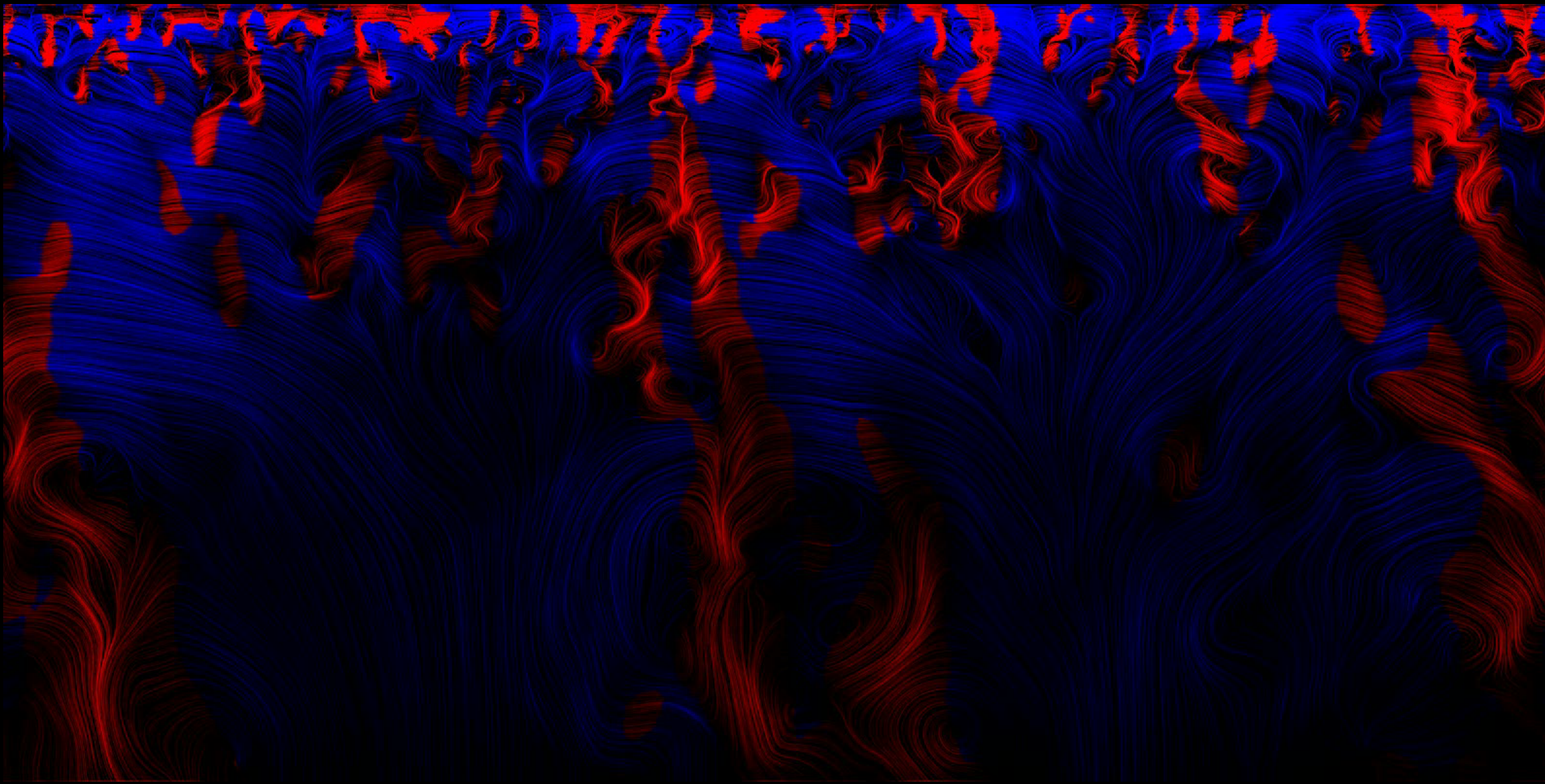
Nordlund, Stein & Asplund (2009)



Spruit, Nordlund & Title (1990)

Supergranulation and mesogranulation are part of a continuous (self-similar?) spectrum of convective motions

simulation by Stein et al (2006), visualization by Henze (2008)



Size, time scales of convection cells increases with depth

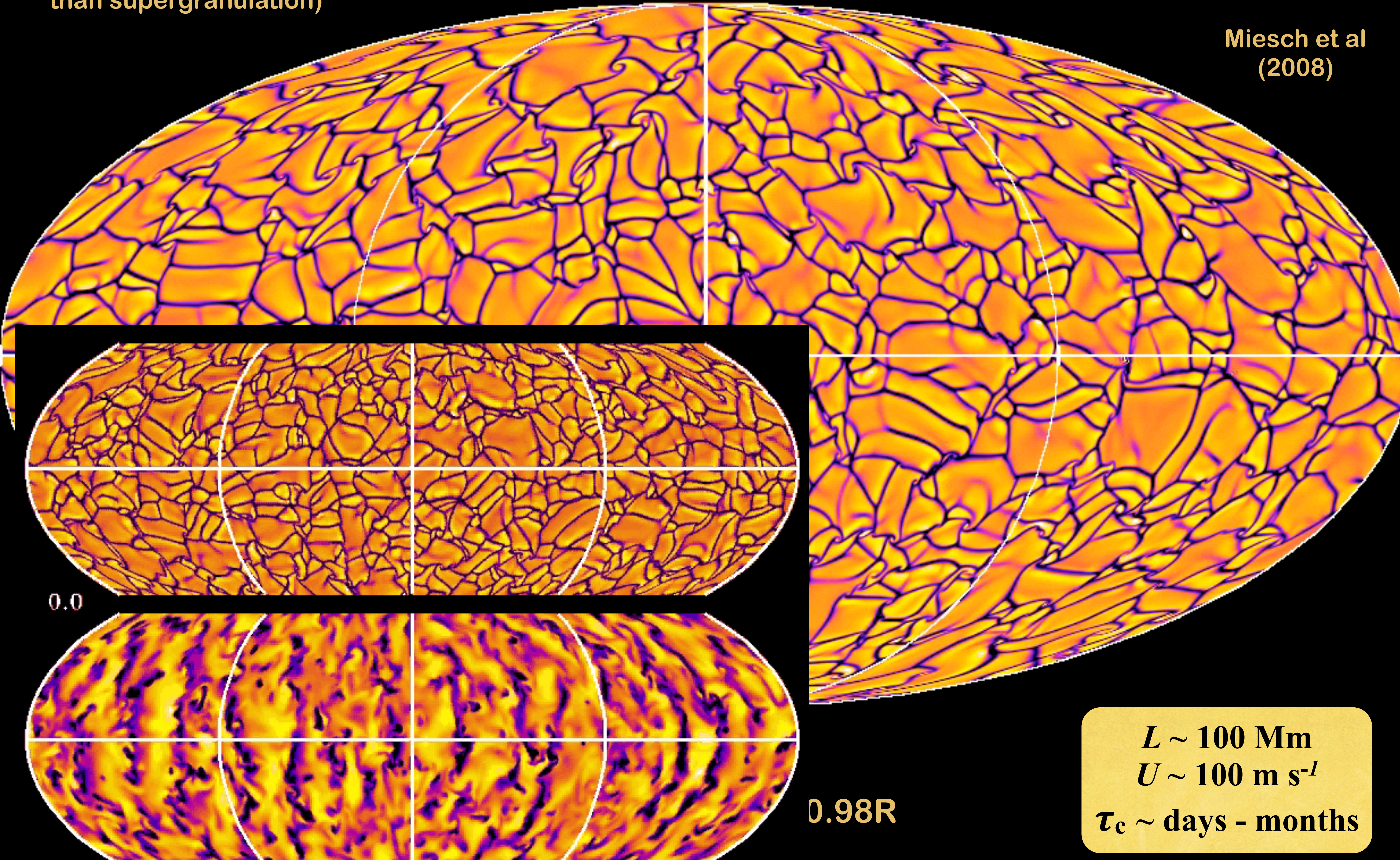
Beyond Solar Dermatology
But still stops at 0.97R!
what lies deeper still?

Giant Cells

Eventually the heirarchy must culminate in motions large enough to sense the spherical geometry and rotation

(Loosely, anything bigger than supergranulation)

Miesch et al (2008)

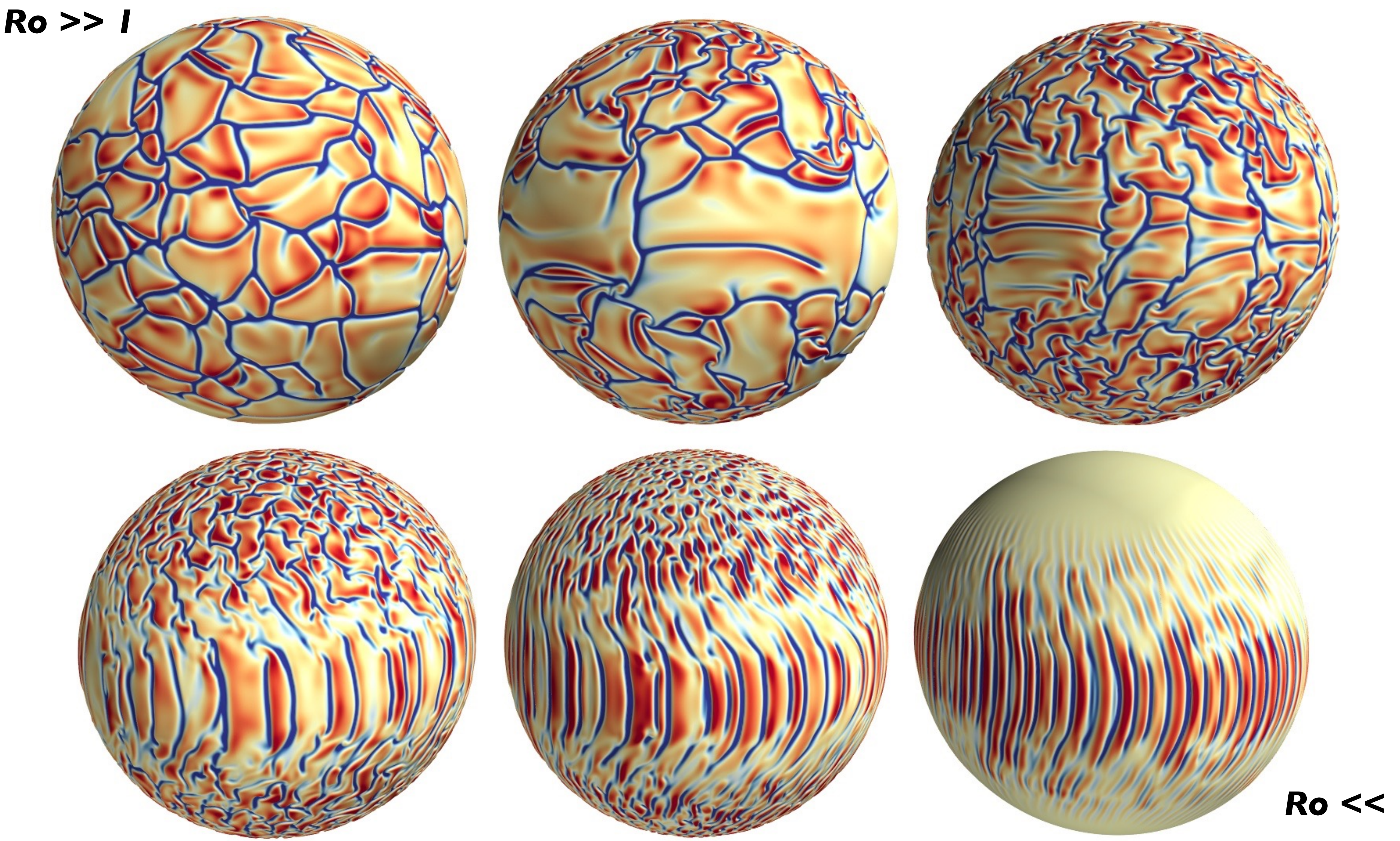


$L \sim 100$ Mm

$U \sim 100$ m s^{-1}

$\tau_c \sim$ days - months

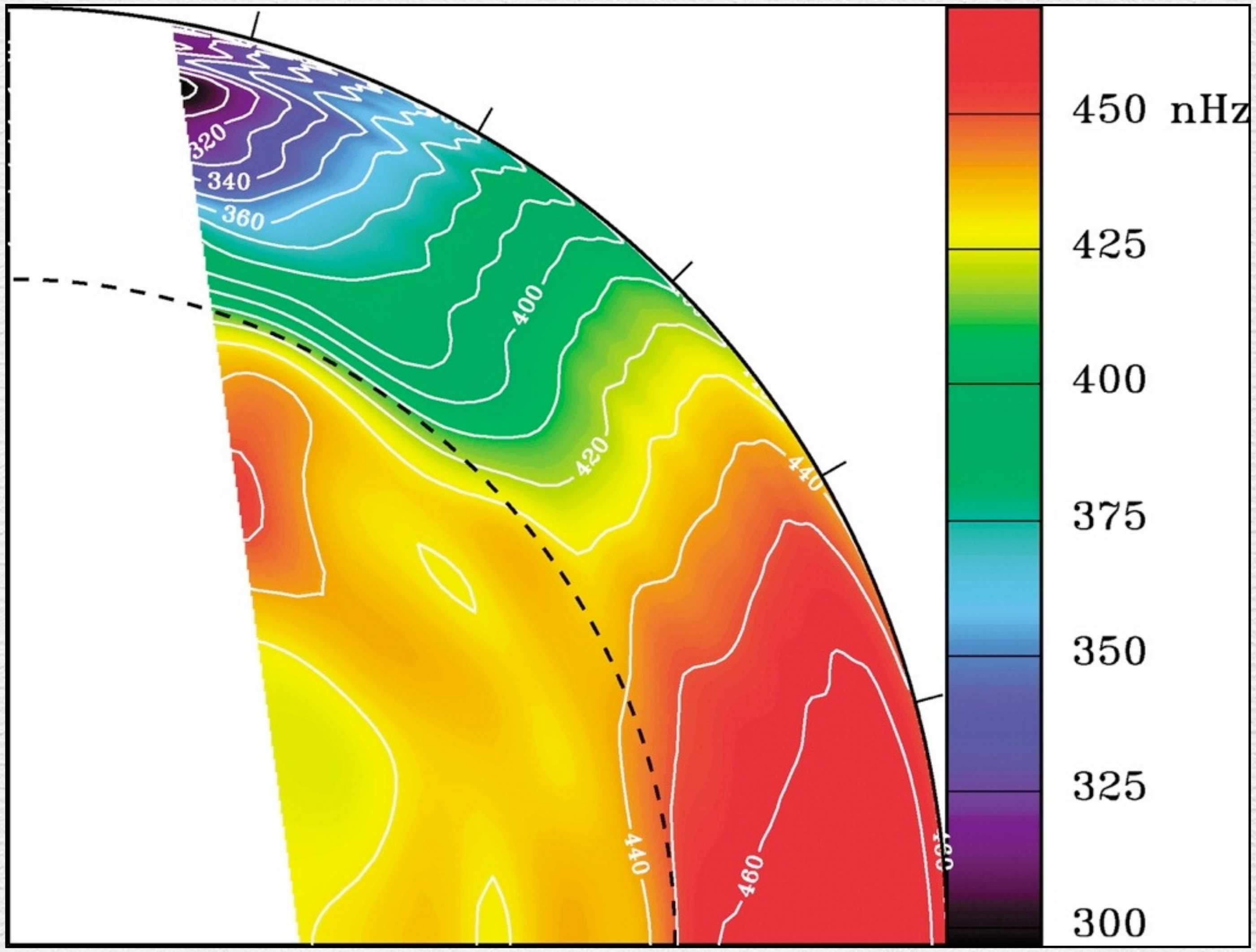
N. Featherstone



Giant cells are notoriously difficult to detect (masked by more vigorous surface convection)

How do we know they are there?

Giant Cells carry energy and redistribute angular momentum



That's how the Sun shines
(Carrying energy from 0.7R to surface)

That's why
the equator spins faster than the poles
(Only giant cells are big and slow enough to sense the rotation and spherical geometry)

Differential Rotation

Monotonic decrease in Ω of ~30% from equator to high latitudes in CZ

Nearly uniform rotation in radiative zone

Convection Implicated as source of DR

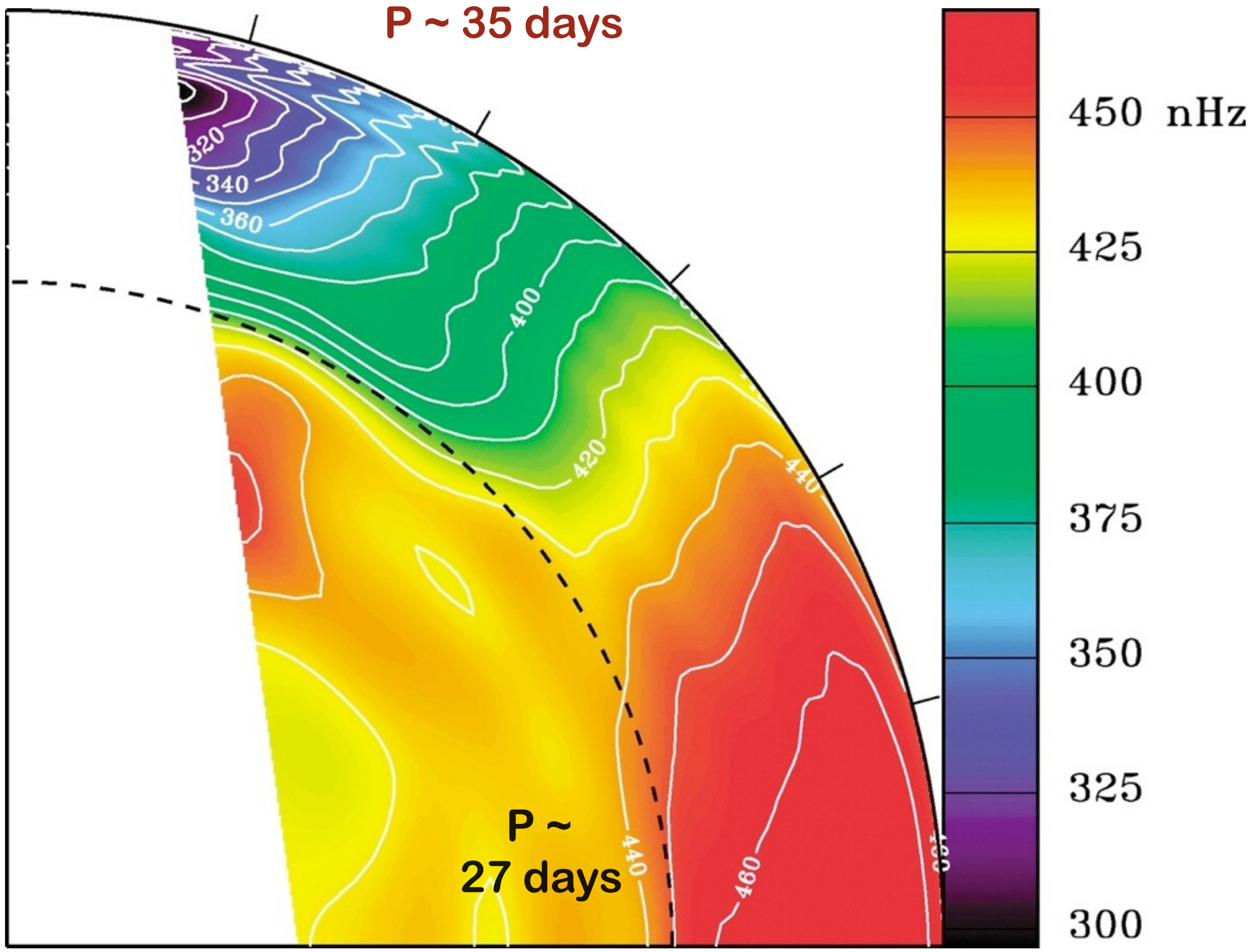
Nearly radial contours at mid-latitudes in CZ

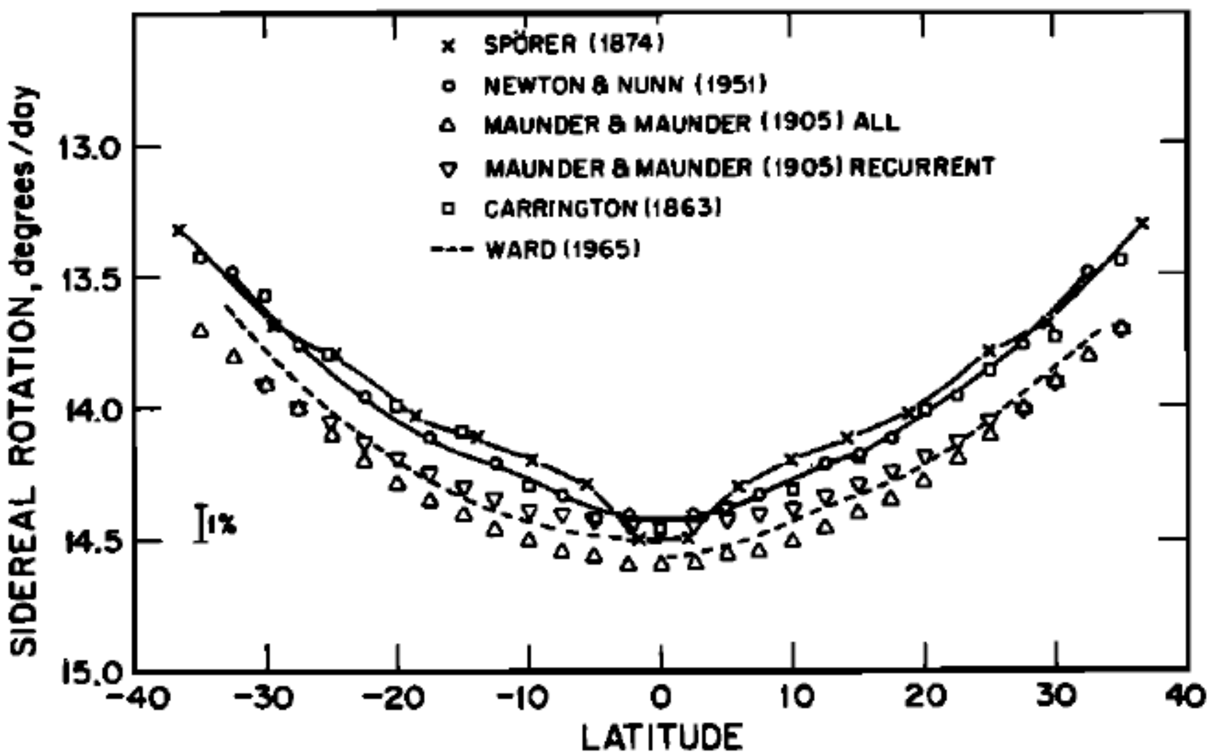
Radial Ω gradients near top & bottom:

Tachocline
Near-surface shear layer

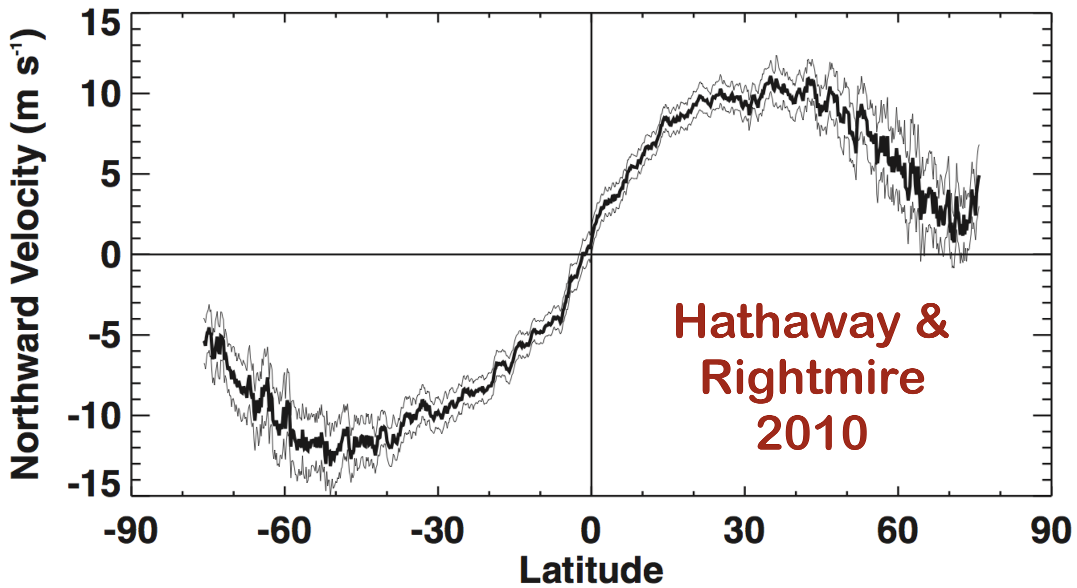
Interior rate intermediate between equator & poles in CZ

Persistent in time





Meridional Circulation



Systematically poleward at mid latitudes near surface (r > 0.95R)

Much weaker that differential rotation (~ 20 m/s)

Variable in time

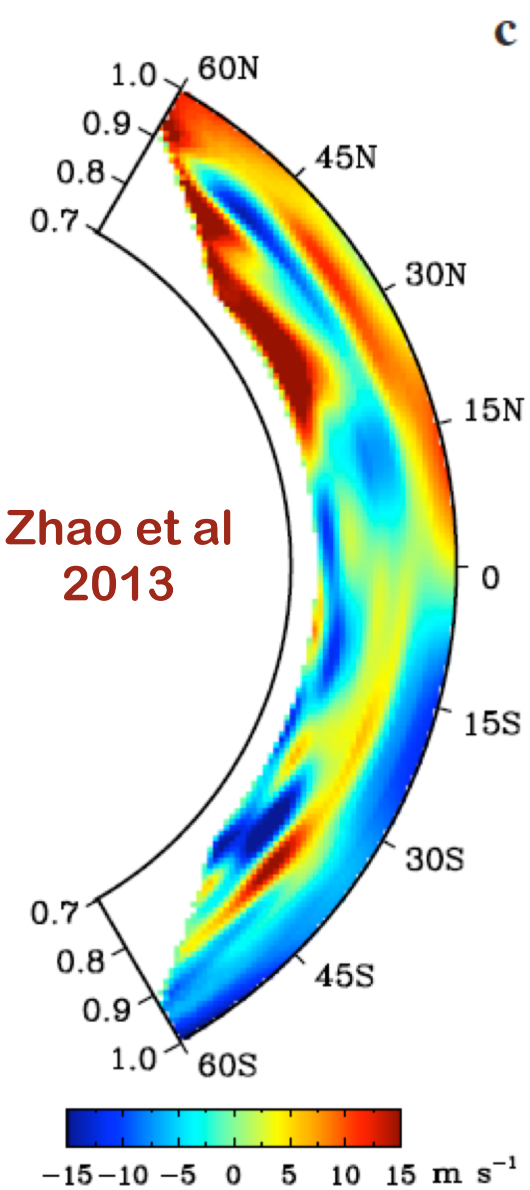
...and that's about all we know!

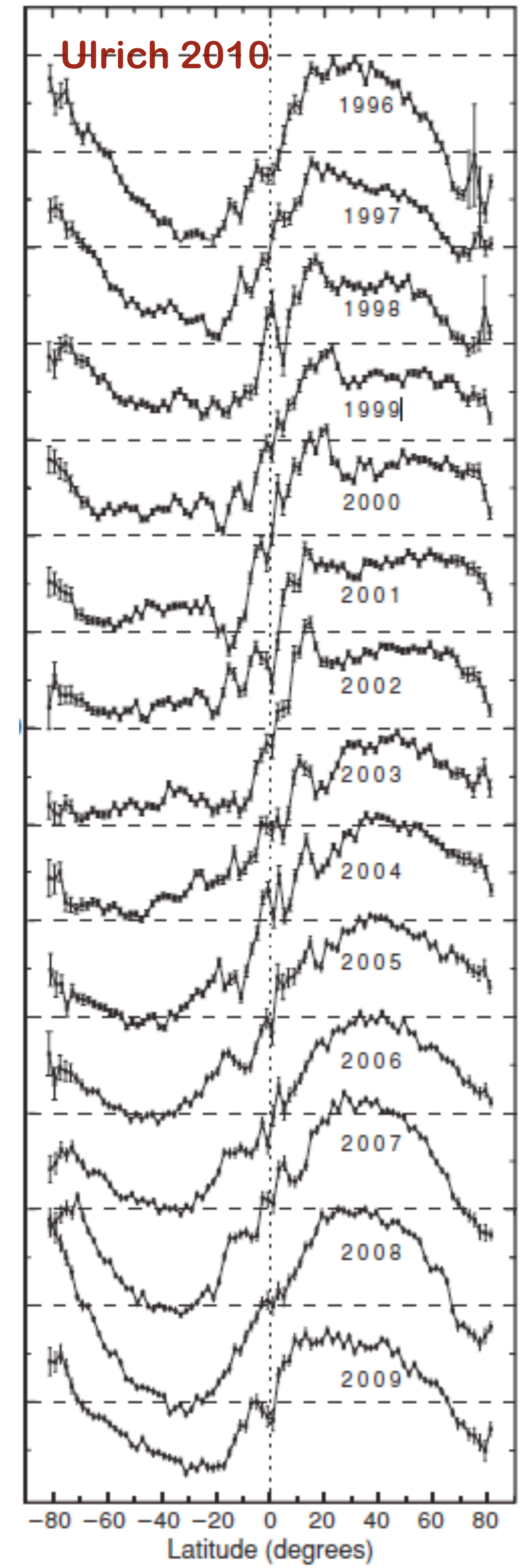
Observational techniques

Local helioseismology ***(left and below)***

Surface Doppler measurements ***(right)***

Feature Tracking





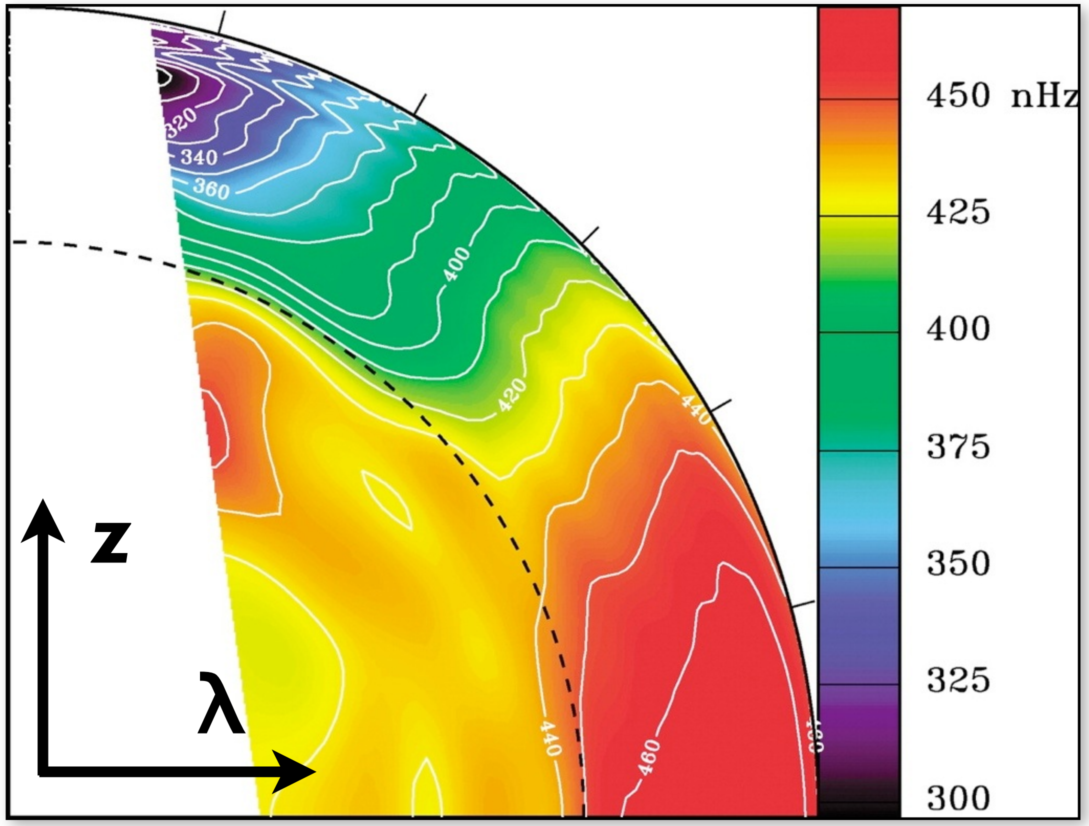
Angular Momentum Transport

Angular momentum per unit mass

$$\mathcal{L}^* = \lambda v_\phi$$

Average over longitude

$$\mathcal{L} = \langle \lambda v_\phi \rangle$$



Conservation of ϕ momentum

$$\frac{\partial}{\partial t}\left(\rho \mathcal{L}^*\right) = -\boldsymbol{\nabla}\cdot\left(\rho \mathbf{v} \mathcal{L}^*\right) - \frac{1}{r\sin\theta}\frac{\partial P}{\partial \phi}$$

Now average over longitude and write it as follows

$$\mathcal{F}_{mc} = \langle \rho \mathbf{v}_m \rangle \mathcal{L}$$

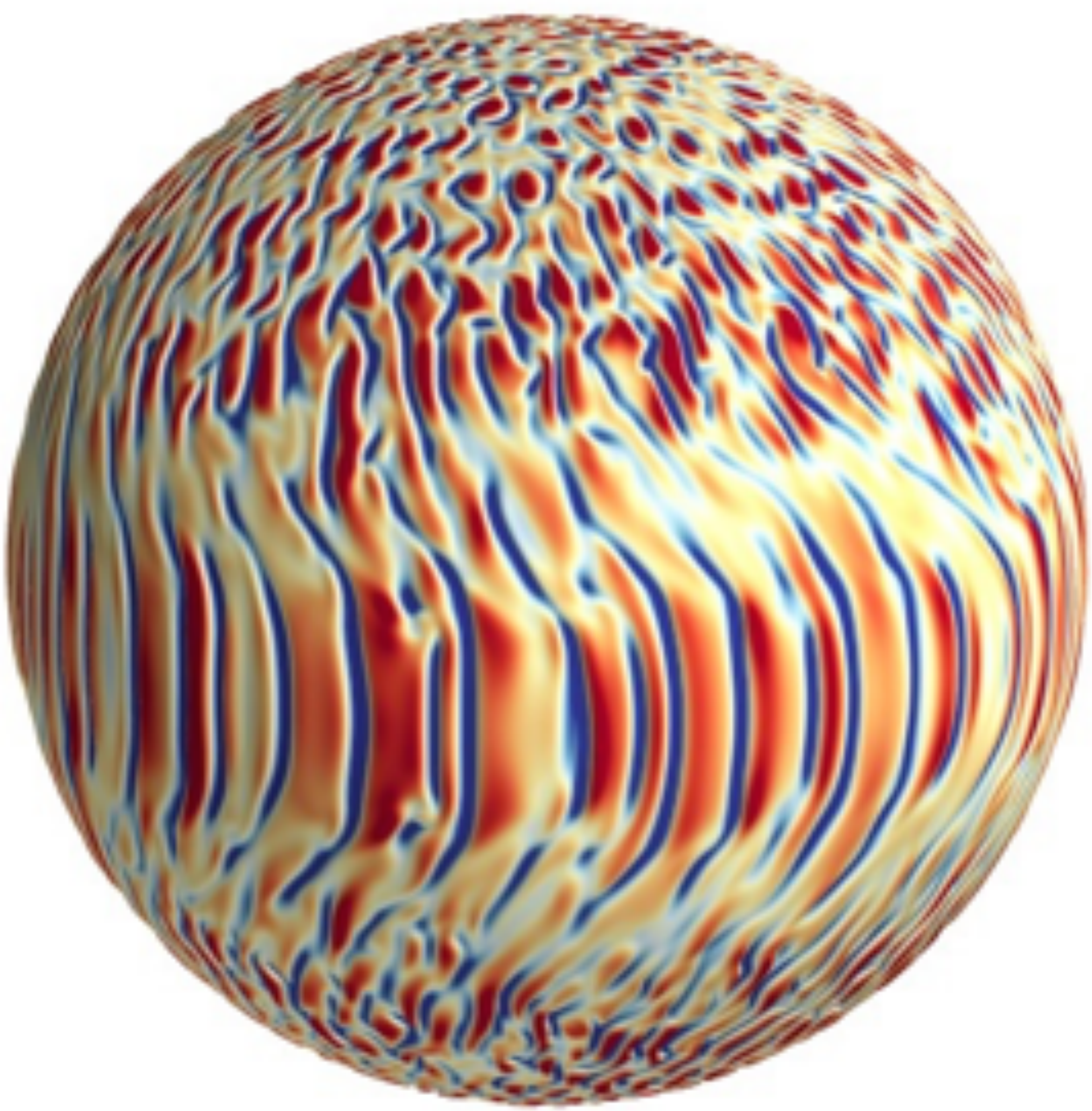
$$\frac{\partial}{\partial t}\left(\rho \mathcal{L}\right) = -\boldsymbol{\nabla}\cdot\left(\mathcal{F}_{mc} + \mathcal{F}_{rs}\right)$$

Reynolds stress

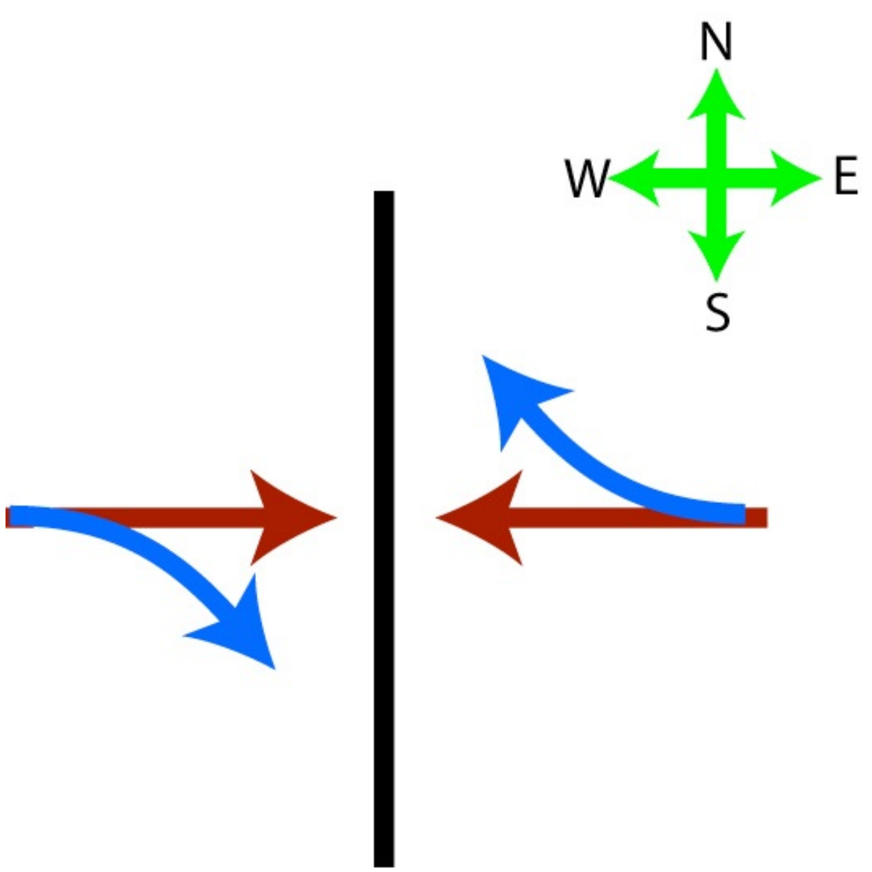
$$\mathcal{F}_{rs} = \langle \rho \lambda \mathbf{v}'_m v'_\phi \rangle$$

Conservation of angular momentum

Angular Momentum Transport



Coriolis-induced tilting of convective structures



$$\langle v'_\theta v'_\phi \rangle > 0$$

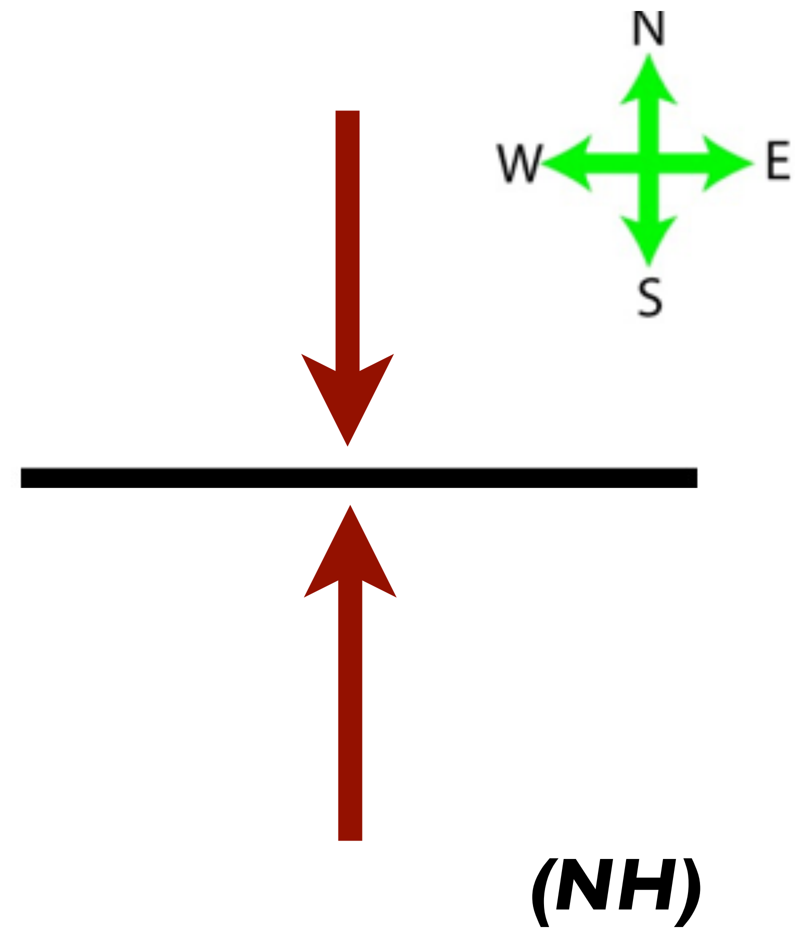
(NH)

angular momentum transport toward the equator

Question

Consider a downflow lane oriented East-West in the northern hemisphere

Which direction would you expect the convective angular momentum transport (Reynold stress) to be?



Reynolds stress

$$\mathcal{F}_{rs} = \langle \rho \lambda \mathbf{v}'_m v'_\phi \rangle$$

Angular Momentum Transport

Question

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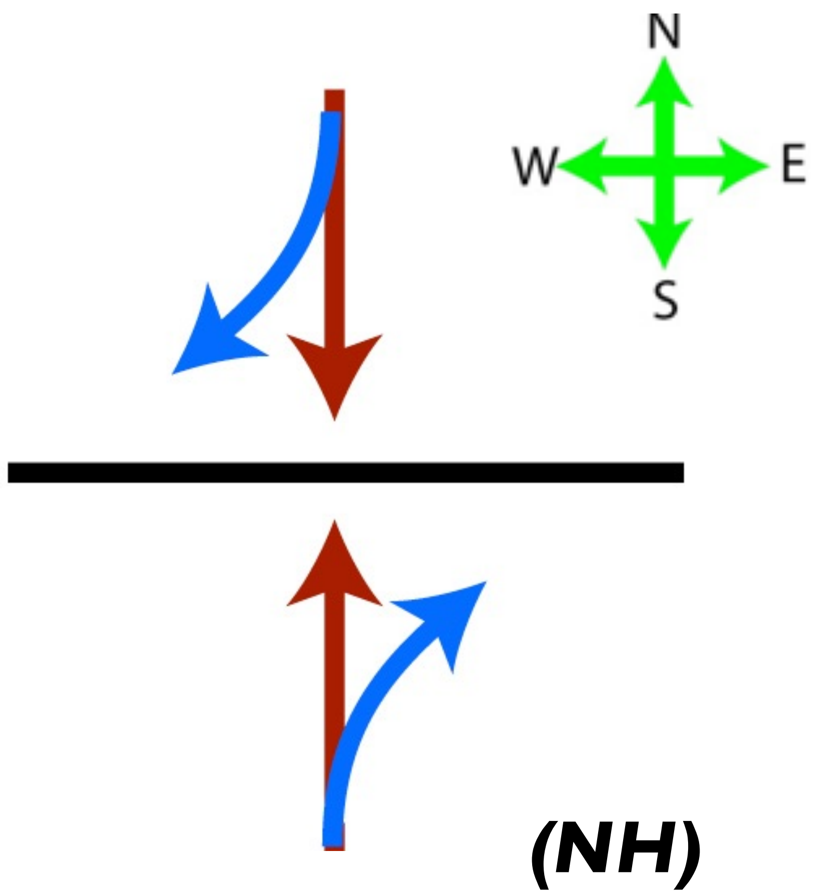
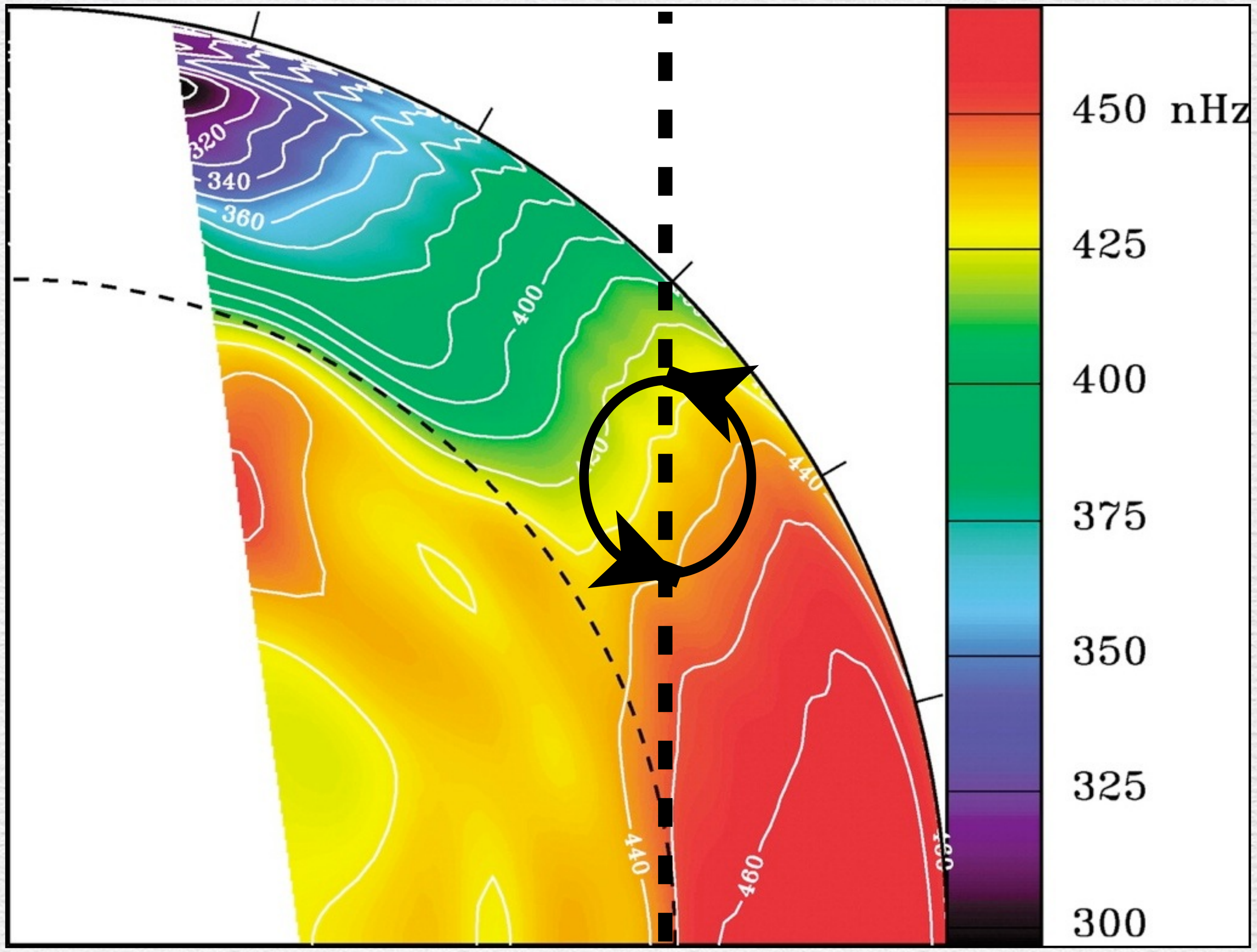


$$\langle v'_\theta v'_\phi \rangle < 0$$

Toward the poles!

Differential Rotation

ΔΩ Established by convective angular momentum transport (Reynolds Stress)



Conical orientation of Ω surfaces attributed to thermal gradients

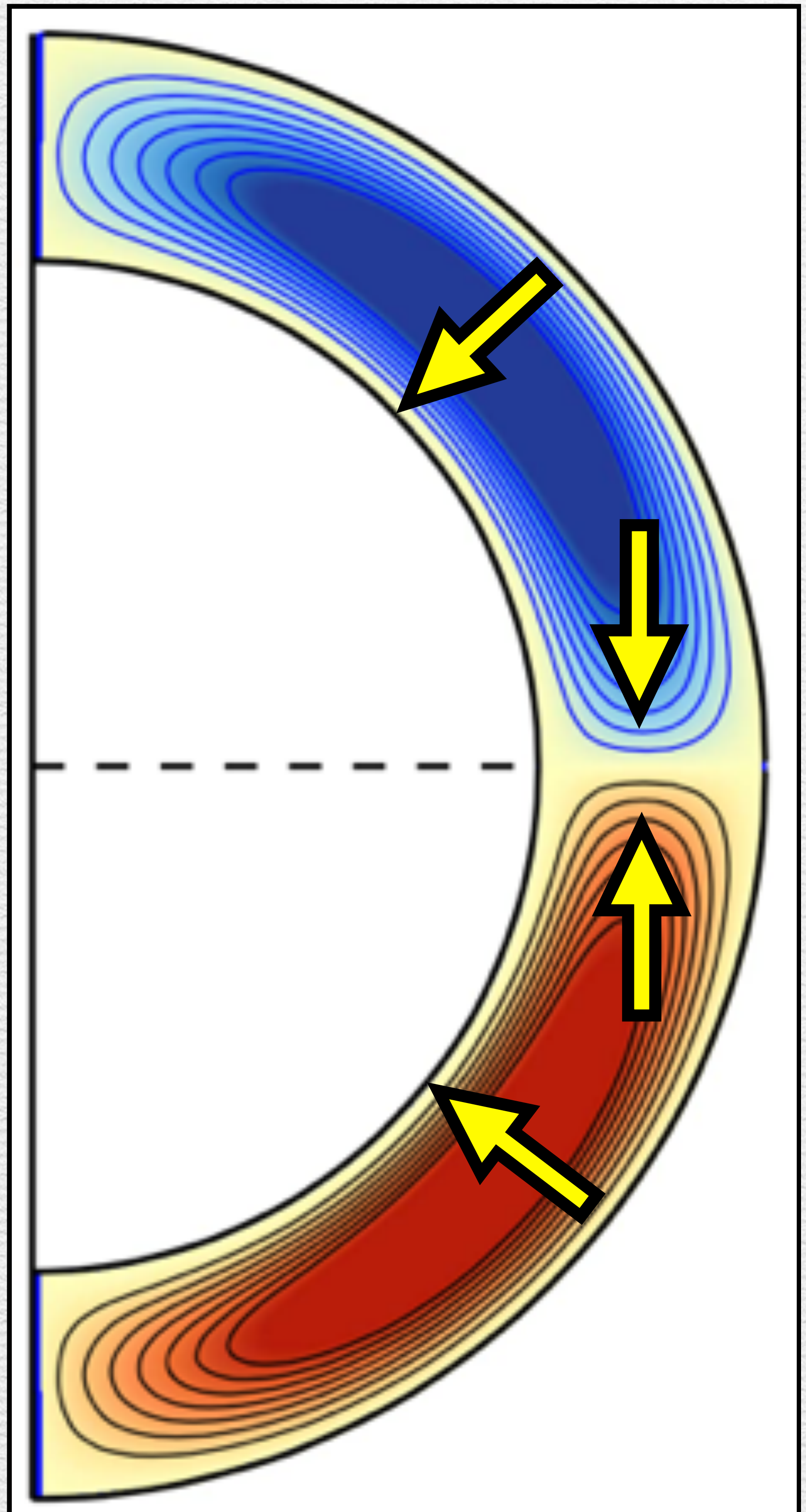
$$\frac{\partial \Omega^2}{\partial z} = \frac{g}{r \lambda C_P} \frac{\partial \langle S \rangle}{\partial \theta}$$

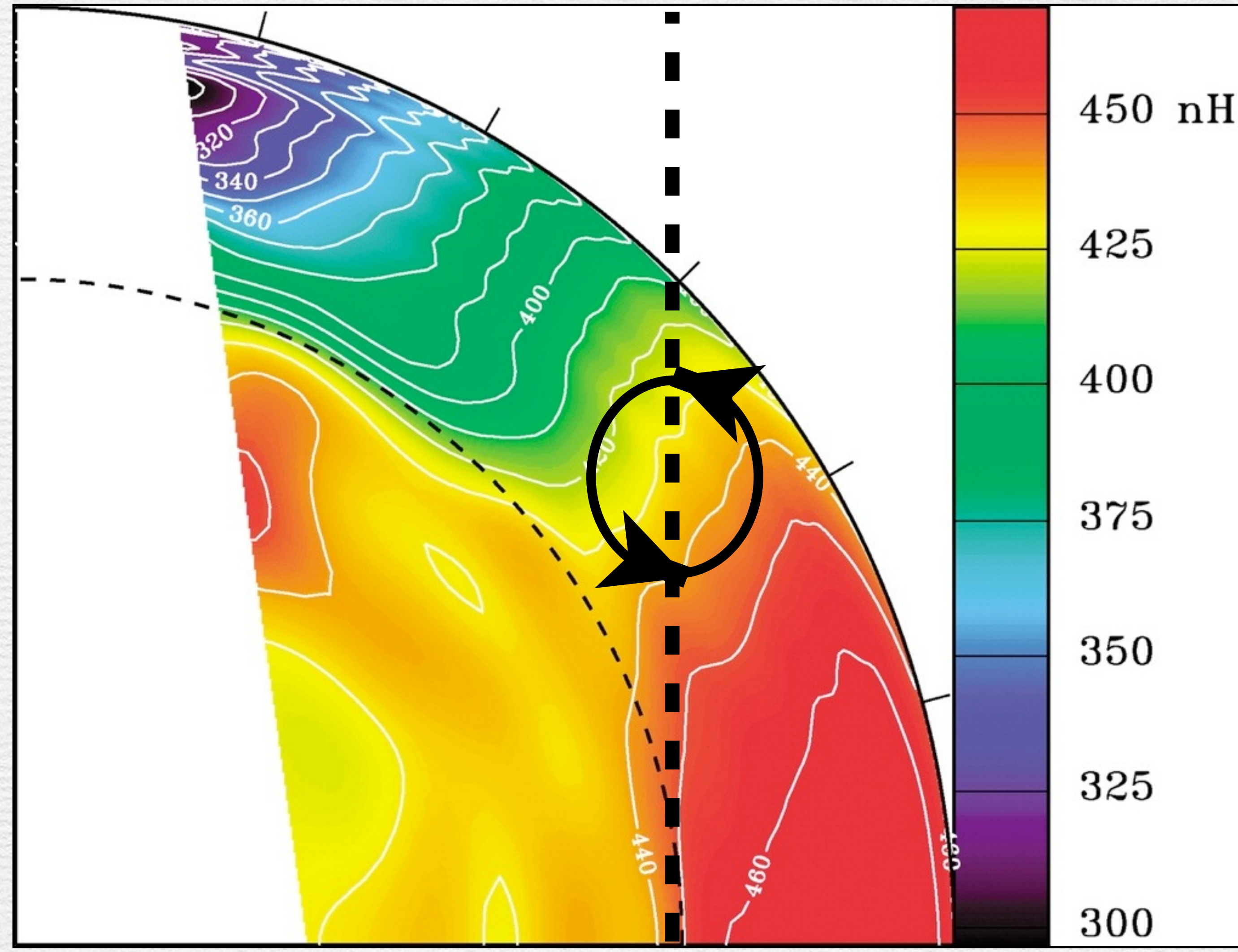
Thermal Wind Balance

Warm poles ($\partial\langle S\rangle/\partial\theta < 0$ in NH) needed to offset inertia of differential rotation

Required amplitudes of thermal variations tiny: one part in 10^5 (δT ~ 10K relative to 2.2million K background)

Meridional Circulation



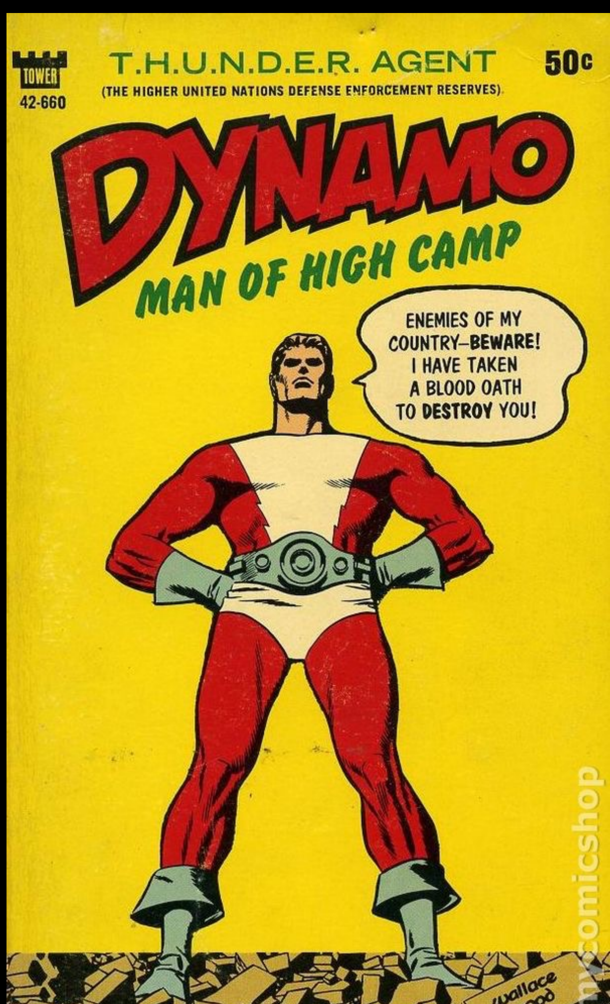


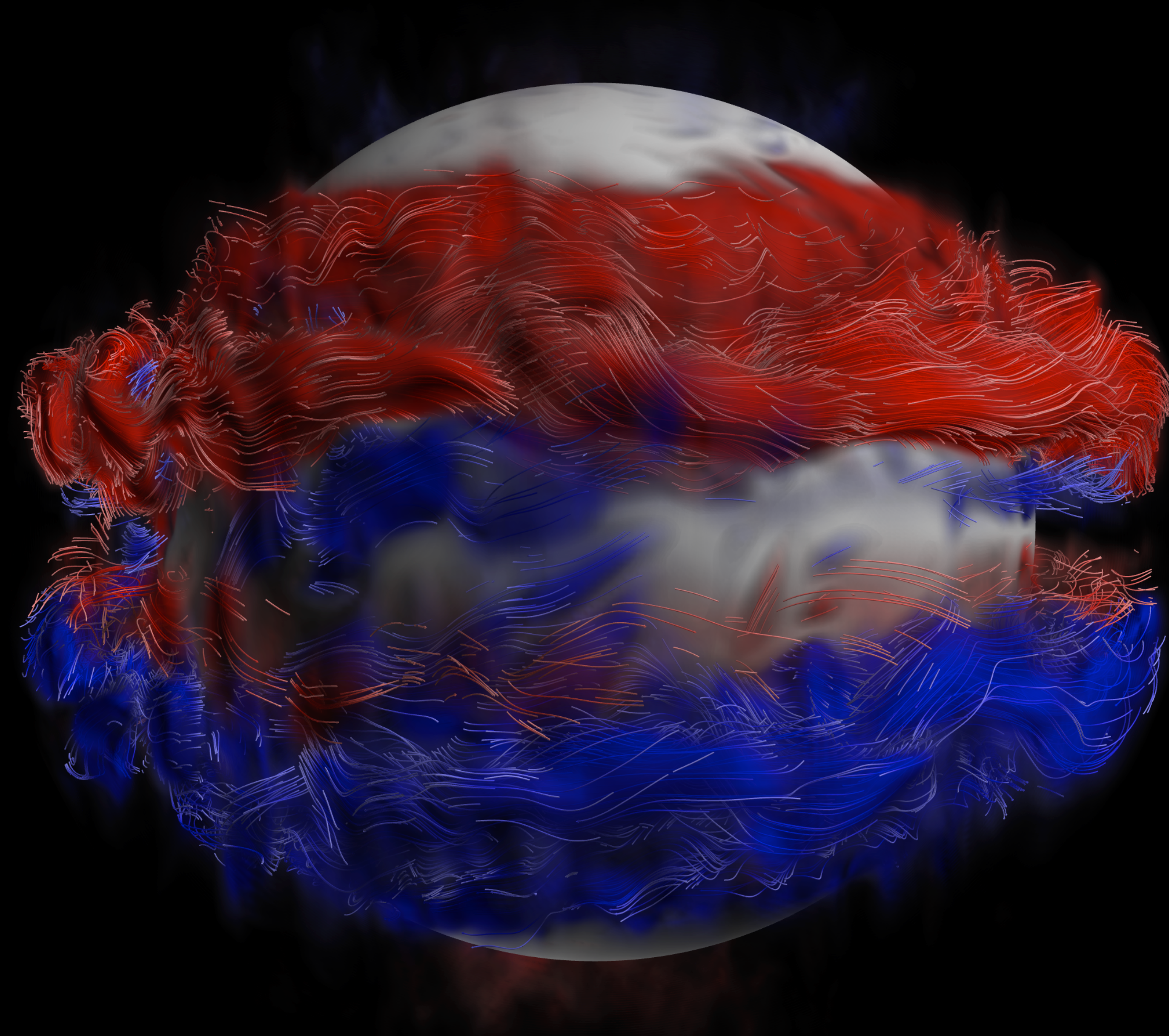
Convective angular momentum transport also thought to be responsible for MC

Featherstone & Miesch (2015)

Part 3 (of 3)

- ★ Solar Magnetism
- ★ Solar Convection and Mean Flows
- ★ Solar Dynamo Models





Lesson #1 in Solar Dynamo Theory:
If the velocity is specified (kinematic), the induction equation is Linear

Note: this is the definition of "kinematic"

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

Profound implications (immensely useful for theory)

Asking whether or not a given (steady) velocity field will or will not be a dynamo then reduces to a linear instability problem

Solutions are a linear superposition of different modes, each with its own (complex) eigenvalue and eigenfunction

Real part of eigenvalue indicates whether the solution exponentially grows or exponentially decays

Imaginary part determines whether or not the solution is oscillatory (cyclic)

Lesson #2 in Solar Dynamo Theory:
No real dynamo in nature is kinematic

Profound pain in the neck (..or opportunity, depending on your perspective)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

This suggests two classes of dynamos:

Essentially Kinematic:

Make it stop!

Small seed field that is initially kinematic (too weak to induce a significant Lorentz force) grows exponentially until it becomes big enough to modify the velocity field

This brings up the crucial issue of: Dynamo Saturation

Essentially Nonlinear:

Make it go!

The velocity field that gives rise to the dynamo mechanism depends on the existence of the field

The focus then shifts toward: Dynamo Excitation

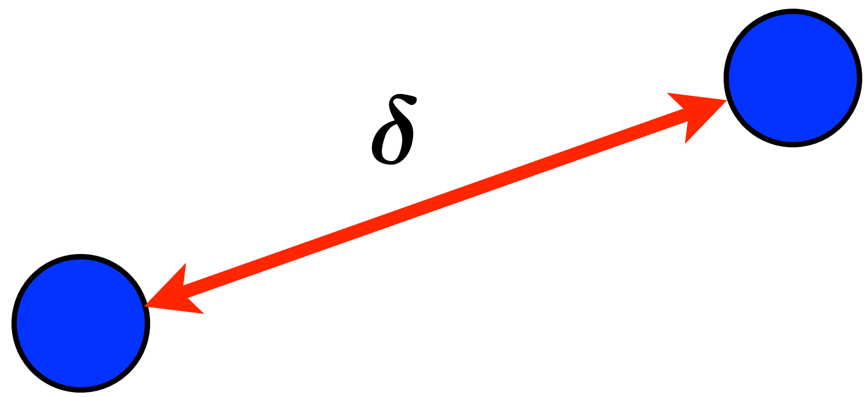
Chaotic stretching

Chaotic fluid trajectories amplify magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B} - \eta \boldsymbol{\nabla} \times \mathbf{B})$$

$$\frac{D\mathbf{B}}{Dt} = \frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\,\mathbf{B} = (\mathbf{B} \cdot \boldsymbol{\nabla})\,\mathbf{v} - \mathbf{B}\,(\boldsymbol{\nabla} \cdot \mathbf{v}) - \boldsymbol{\nabla} \times (\eta \boldsymbol{\nabla} \times \mathbf{B})$$

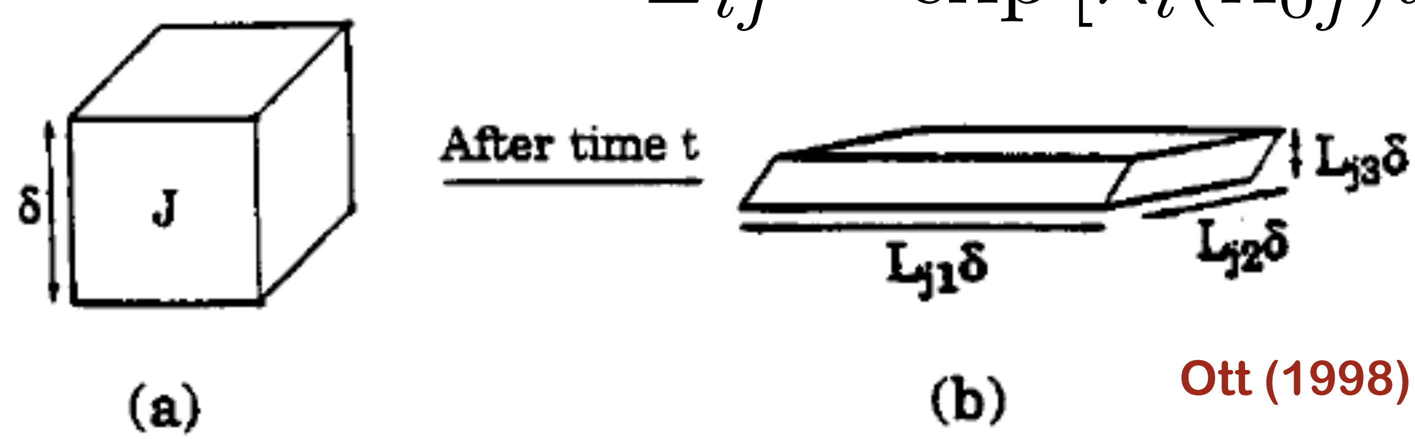
If $\boldsymbol{\nabla} \cdot \mathbf{v} = \eta = 0$ **then**



λ= Local Lyapunov exponents

$$\frac{D\mathbf{B}}{Dt} = (\mathbf{B} \cdot \boldsymbol{\nabla})\,\mathbf{v}$$

$$L_{ij} = \exp\left[\lambda_i(\mathbf{x}_{0j})t\right]$$



Ott (1998)

$$\frac{d\boldsymbol{\delta}}{dt} = (\boldsymbol{\delta} \cdot \boldsymbol{\nabla})\,\mathbf{v}$$

$$\frac{d\delta_i(\mathbf{x}_0, t)}{dt} = \mathcal{J}_{ij}(\mathbf{x}_0, t)\ \delta_j\,(\mathbf{x}_0, t)$$

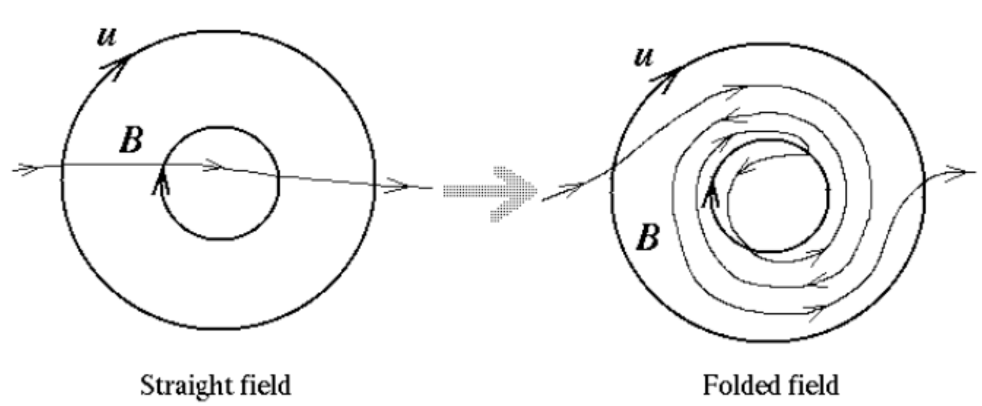
$$\lambda_1 + \lambda_2 + \lambda_3 = 0$$

Local Dynamo Action in the Sun and Stars

Granulation: $\tau_c \sim$ 10-15 min
Giant Cells: $\tau_c \sim$ days - months

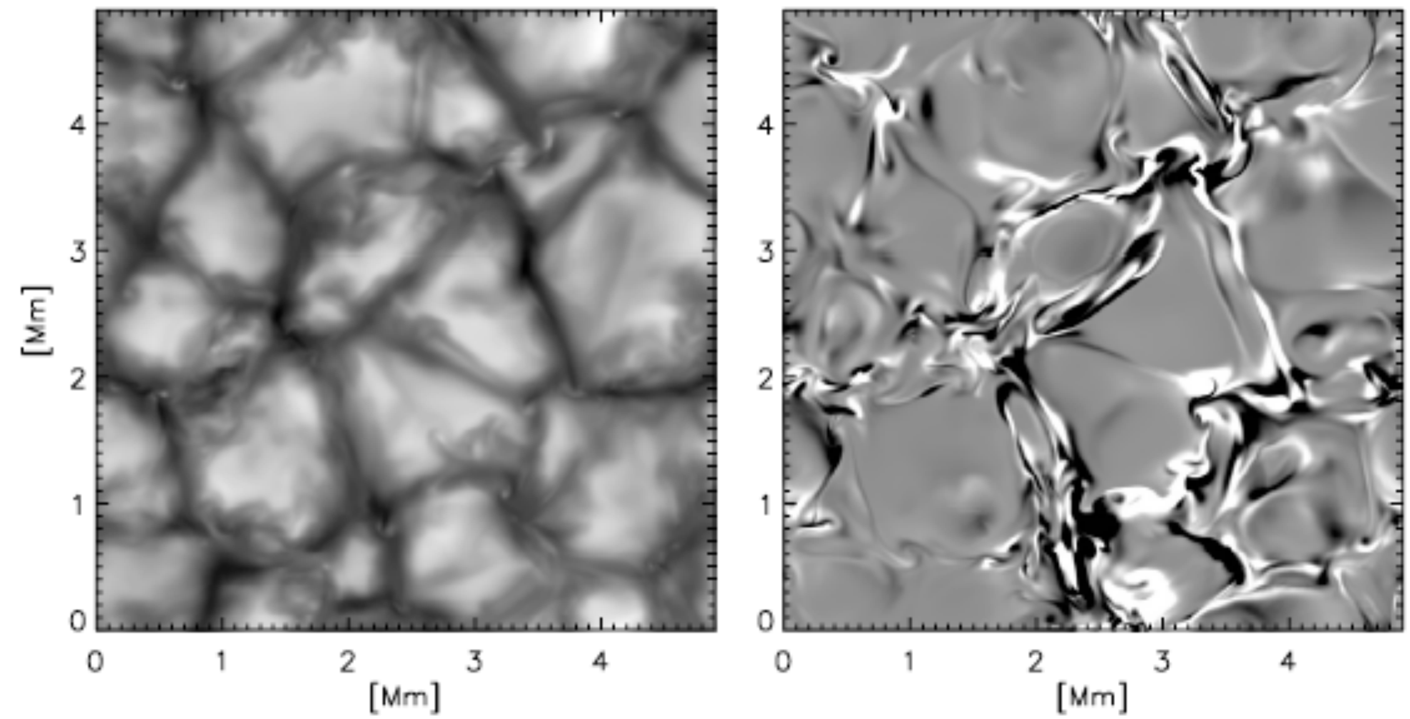
Granulation may generate field locally by chaotic stretching with little regard for the deeper convection zone

Folding of B field generates scales smaller than the v field itself

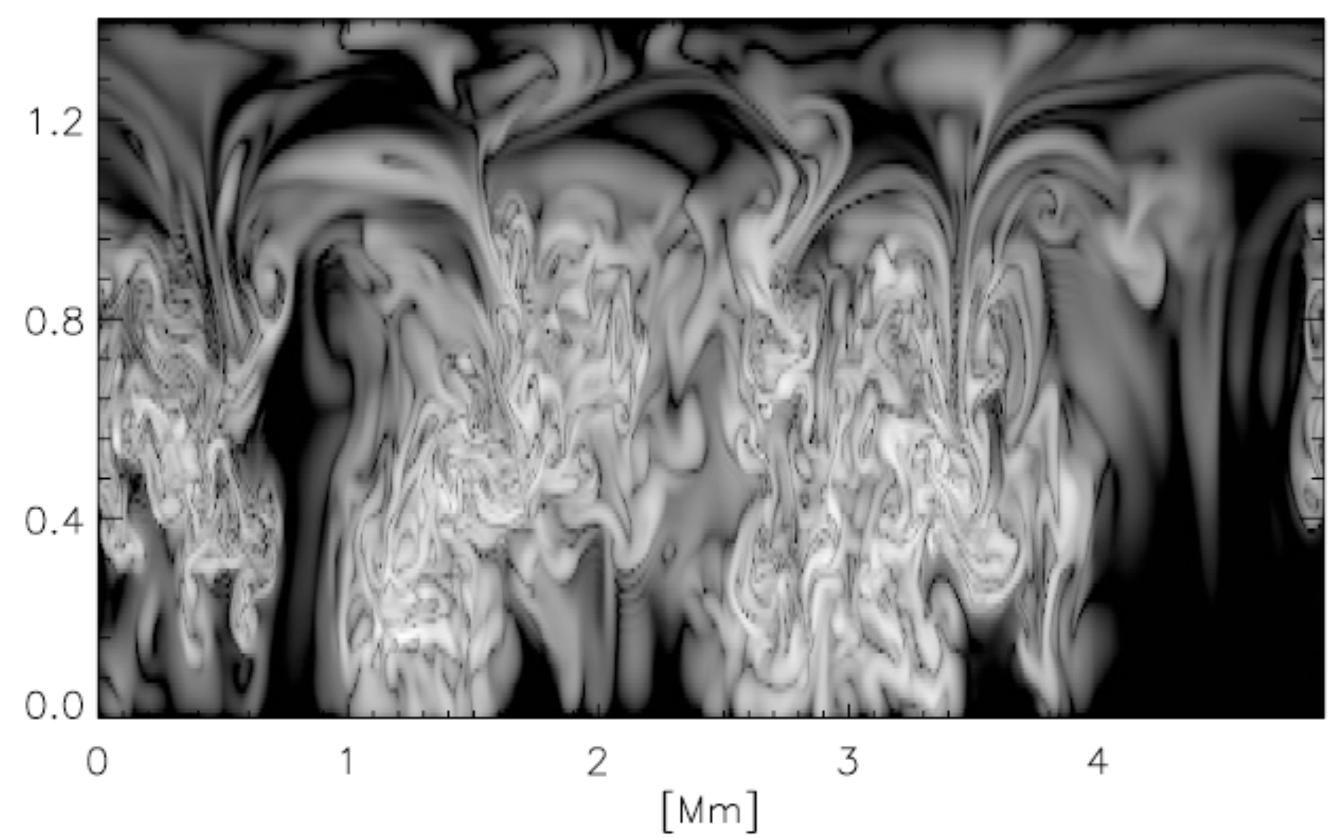


Schekochihin et al (2004)

Flux expulsion and reconnection produce strong horizontal fields near photosphere



Schussler & Vogler (2008)



Turbulent flows beget turbulent fields!

Types of Dynamos

define
Small-scale dynamo

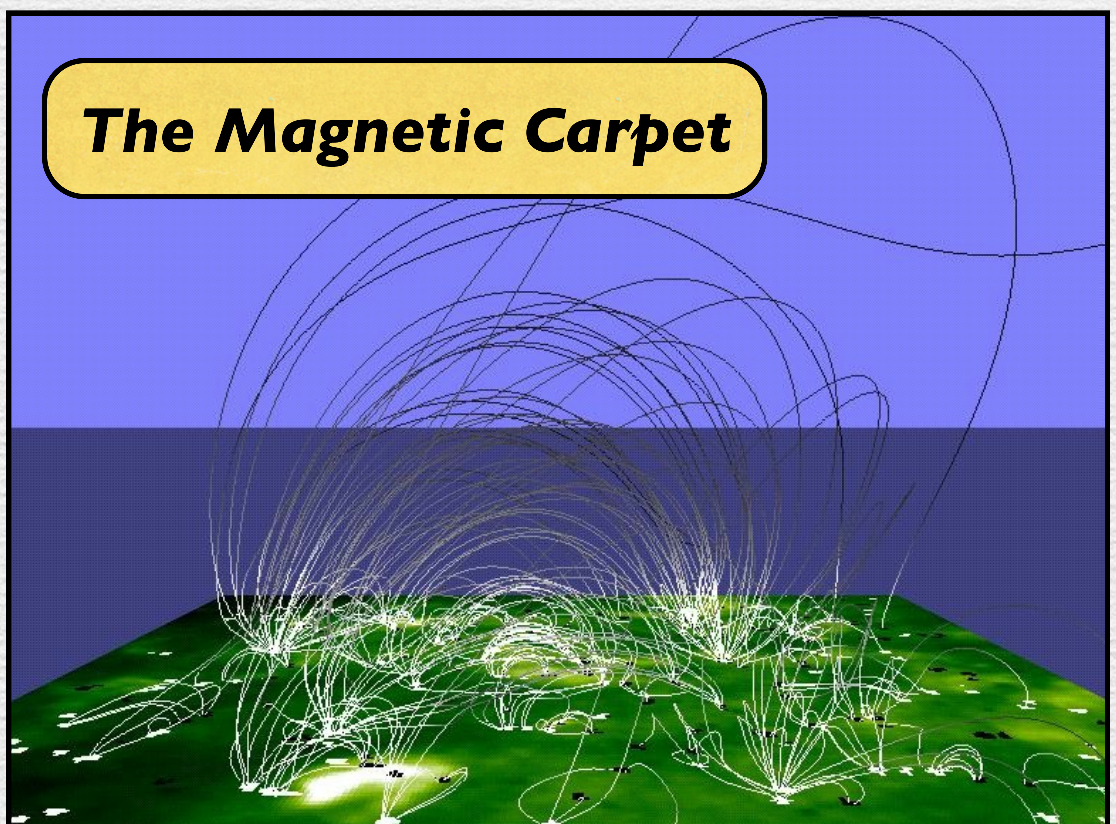
Generates magnetic fields on scales smaller than the velocity field

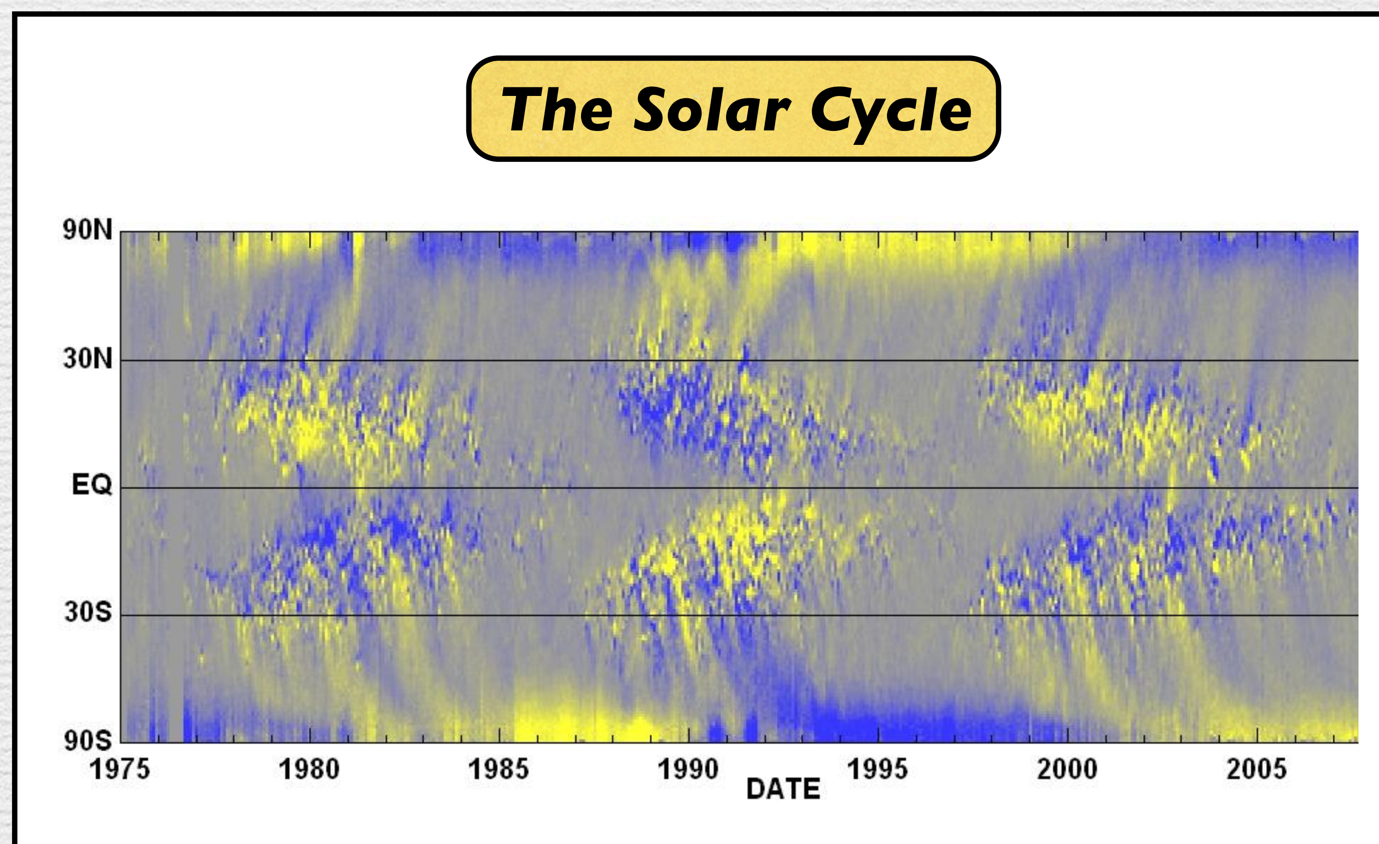
$$\ell_B \sim \mathrm{Rm}^{-1/2}\ \ell_v \ll \ell_v$$

define
Large-scale dynamo

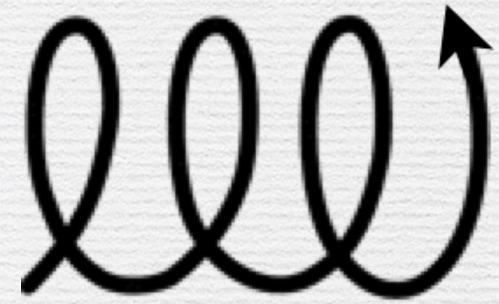
Generates magnetic fields on scales larger than the velocity field

$$\ell_B >> \ell_v$$





Recipe for Building Large-Scale Fields



Lagrangian Chaos

- Builds magnetic energy

Rotational Shear

- Builds large-scale toroidal flux (W-effect)
- Enhances dissipation of small-scale fields
- Promotes magnetic helicity flux

Helicity

- Rotation and stratification generate kinetic helicity
- Kinetic helicity generates magnetic helicity
- Upscale spectral transfer of magnetic helicity generates large-scale fields
  - Local transfer: inverse cascade of magnetic helicity
  - Nonlocal transfer: α-effect

Specific manifestations of a more general (and more profound) phenomenon

$$H_k = \langle \boldsymbol{\omega} \cdot \boldsymbol{v} \rangle$$

$$H_m = \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle$$

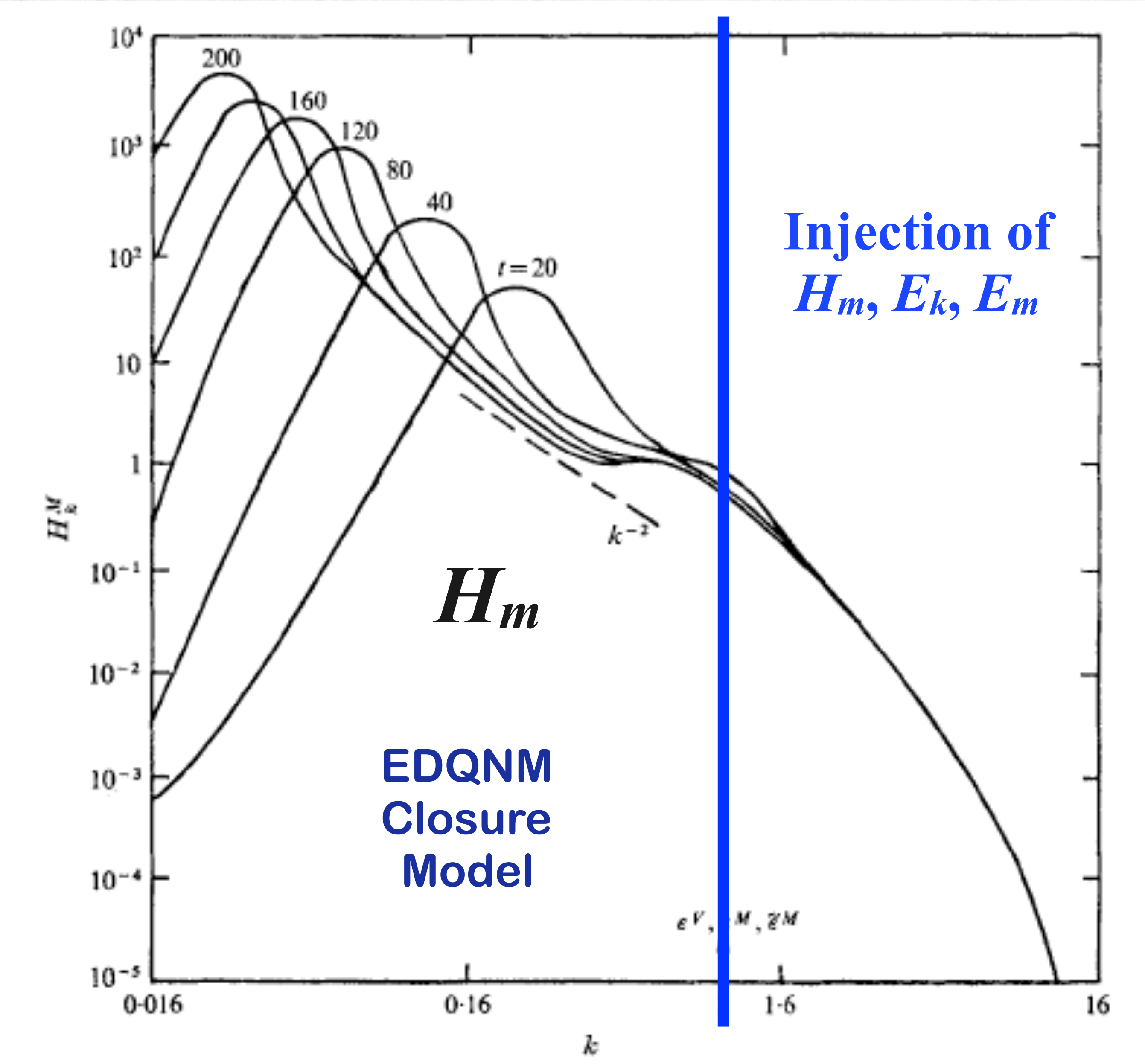
$$H_c = \langle \boldsymbol{J} \cdot \boldsymbol{B} \rangle$$

$$\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{v}$$

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$$

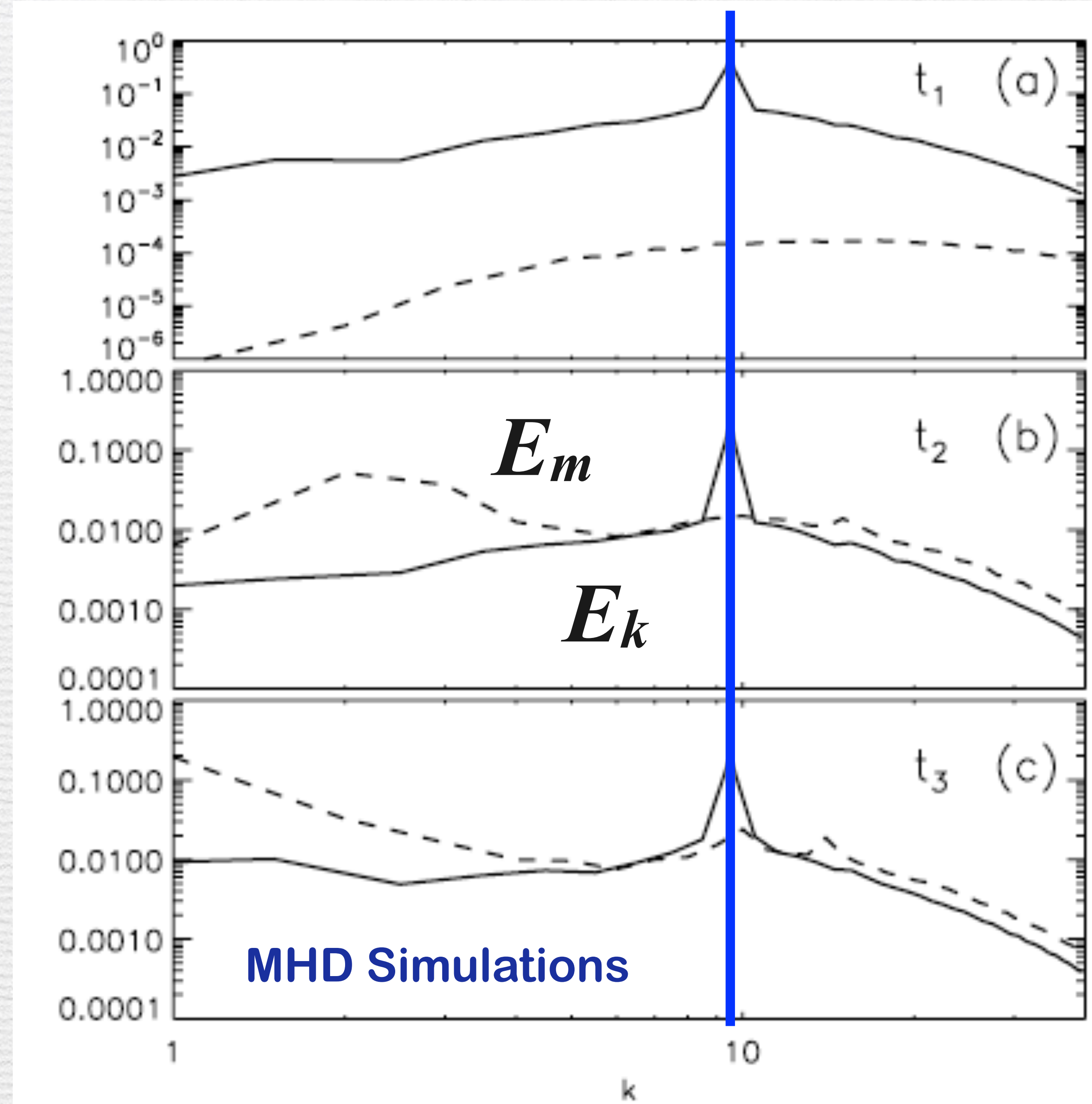
$$\boldsymbol{J} = \frac{c}{4\pi} \boldsymbol{\nabla} \times \boldsymbol{B}$$

Inverse Cascade of Magnetic Helicity



Pouquet, Frisch & Leorat (1976)

Injection of E_k, H_k



Alexakis, Mininni & Pouquet (2006)

Magnetic Helicity is conserved in the limit $\eta \longrightarrow 0$

Provides an essential link between large and small scales

If you twist the field on small scales, large scales will respond

Large Scale Dynamos: The Mean Induction Equation

Ω-effect | **Meridional circulation** | **Diffusion (plasma)**

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \lambda \overline{\mathbf{B}}_p \cdot \boldsymbol{\nabla} \Omega \ \hat{\phi} + \boldsymbol{\nabla} \times \left(\overline{\mathbf{v}}_m \times \overline{\mathbf{B}} \right) + \eta \nabla^2 \overline{\mathbf{B}} + \boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}}$$

Kinematic, mean-field models

- **Specify Ω, V_m, $\mathcal{E}$ as a function of r, θ, t, ⟨B⟩**

Non-kinematic mean-field models:

- **Solve mean momentum, continuity, and energy equations to obtain Ω, V_m, as a function of r, θ, t, ⟨B⟩**
- **Still have to specify $\mathcal{E}$ as a function of r, θ, t, ⟨B⟩**
- **Also have to specify convective momentum, heat transport as a function of mean fields (hydro analogues of $\mathcal{E}$)**

3D MHD convection simulations:

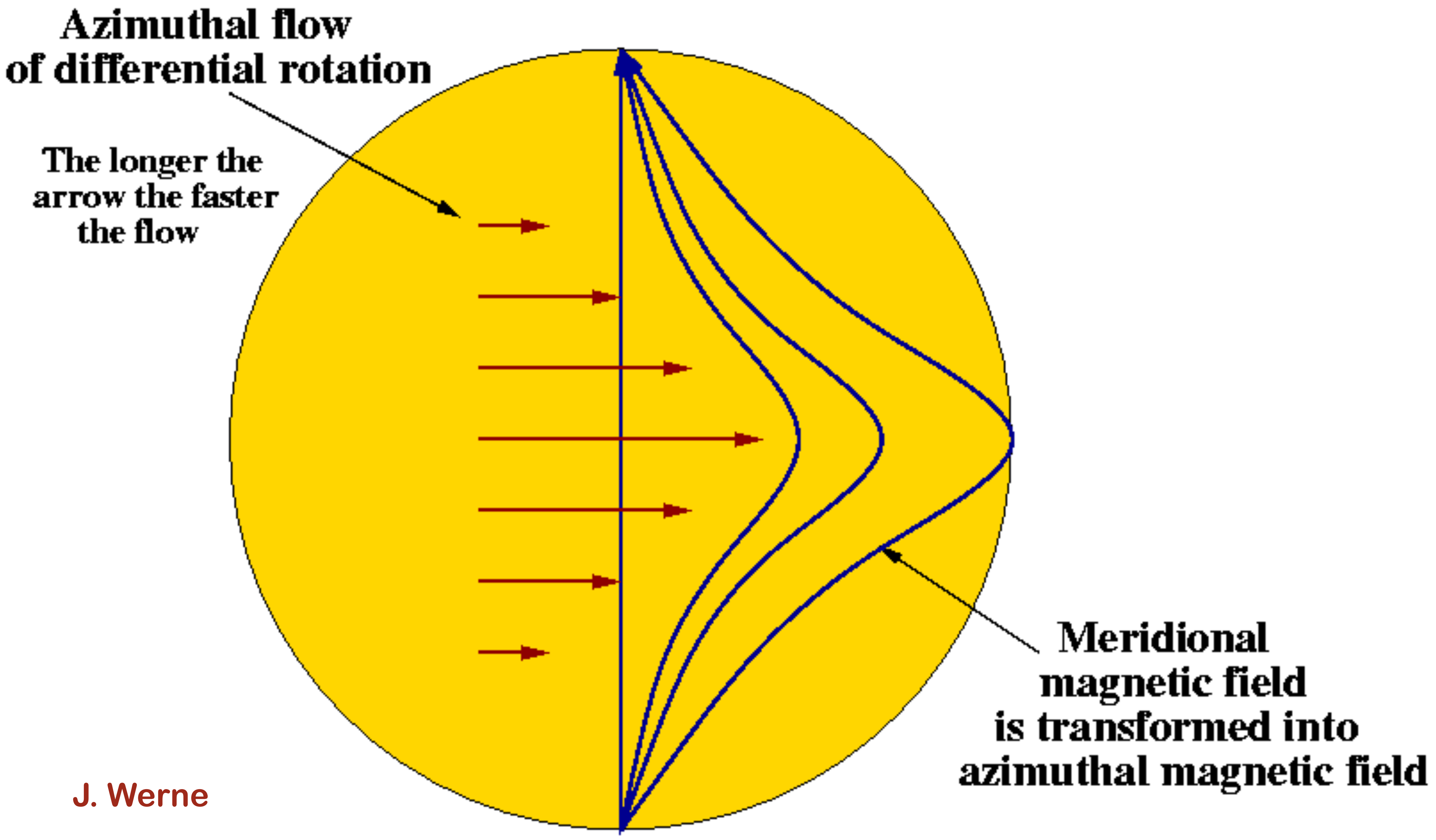
- **Solve 3D momentum, continuity, energy, and induction equations to obtain Ω, V_m, $\mathcal{E}$ as a function of r, θ, t, ⟨B⟩**

Plasma diffusion is typically neglected ($\eta = 0$)

The Ω-effect

Converts poloidal to toroidal field and amplifies it

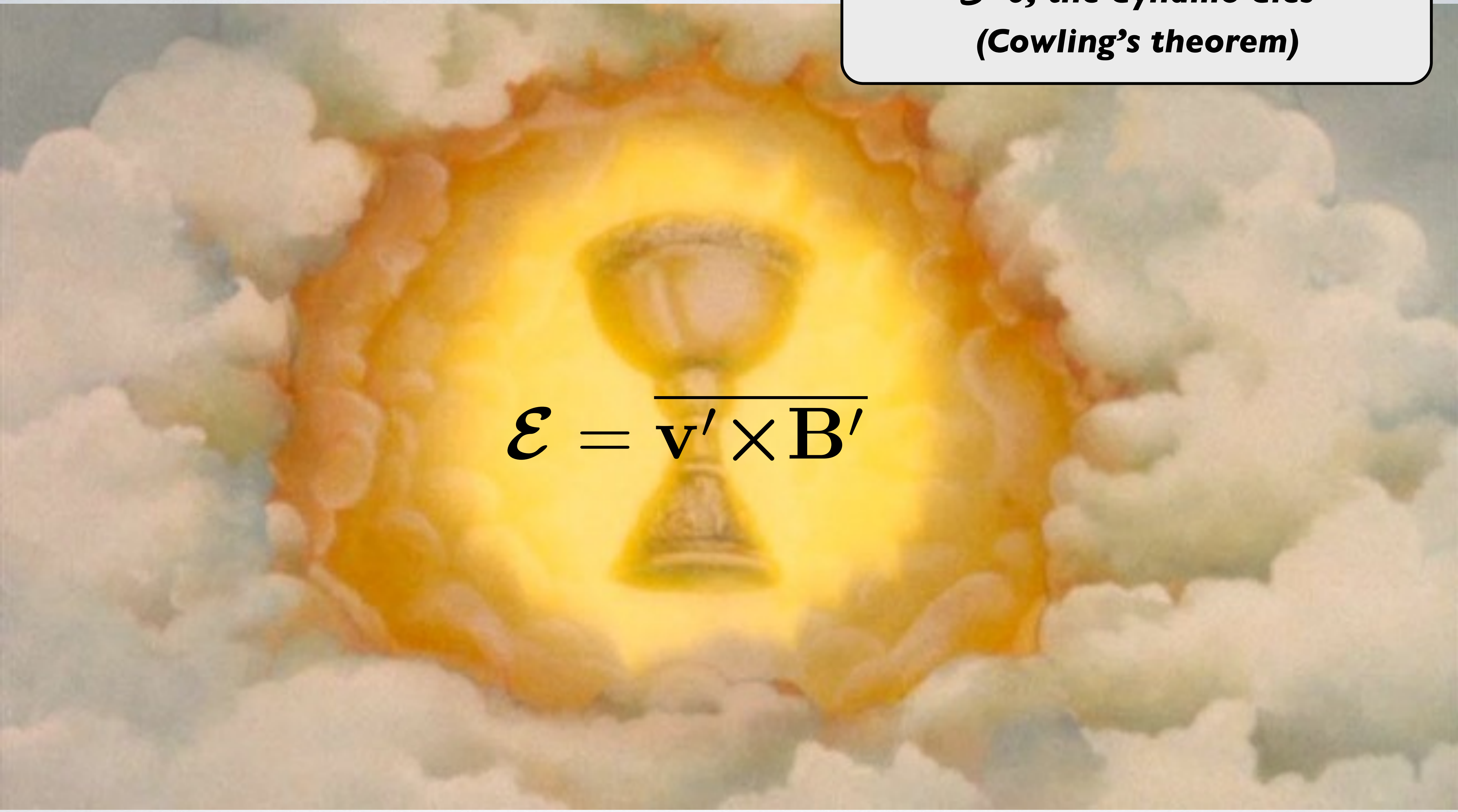
...by tapping the kinetic energy of the differential rotation



J. Werne

The Fluctuating emf

Straightforward to show that if $\mathcal{E}$=0, the dynamo dies (Cowling's theorem)



$$\mathcal{E} = \overline{\mathbf{v}' \times \mathbf{B}'}$$

How can a non-axisymmetric flow across magnetic field lines produce an axisymmetric field?

$$\mathbf{v}' = \mathbf{v} - \overline{\mathbf{v}}$$

$$\mathbf{B}' = \mathbf{B} - \overline{\mathbf{B}}$$

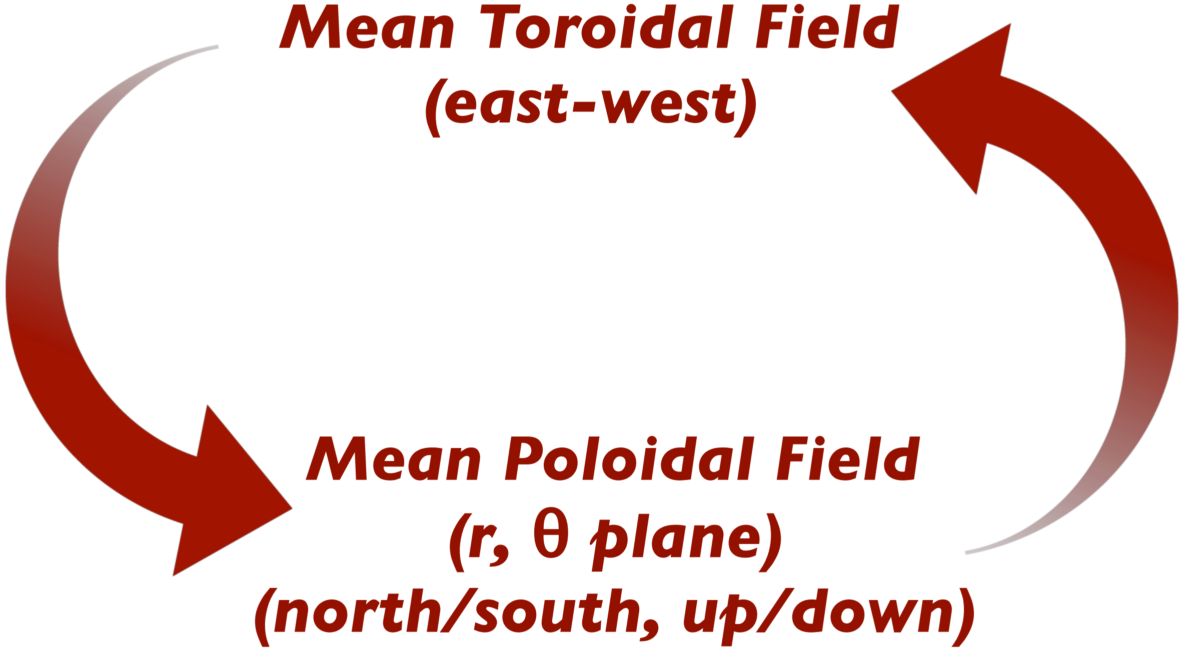
One way: The turbulent α-effect

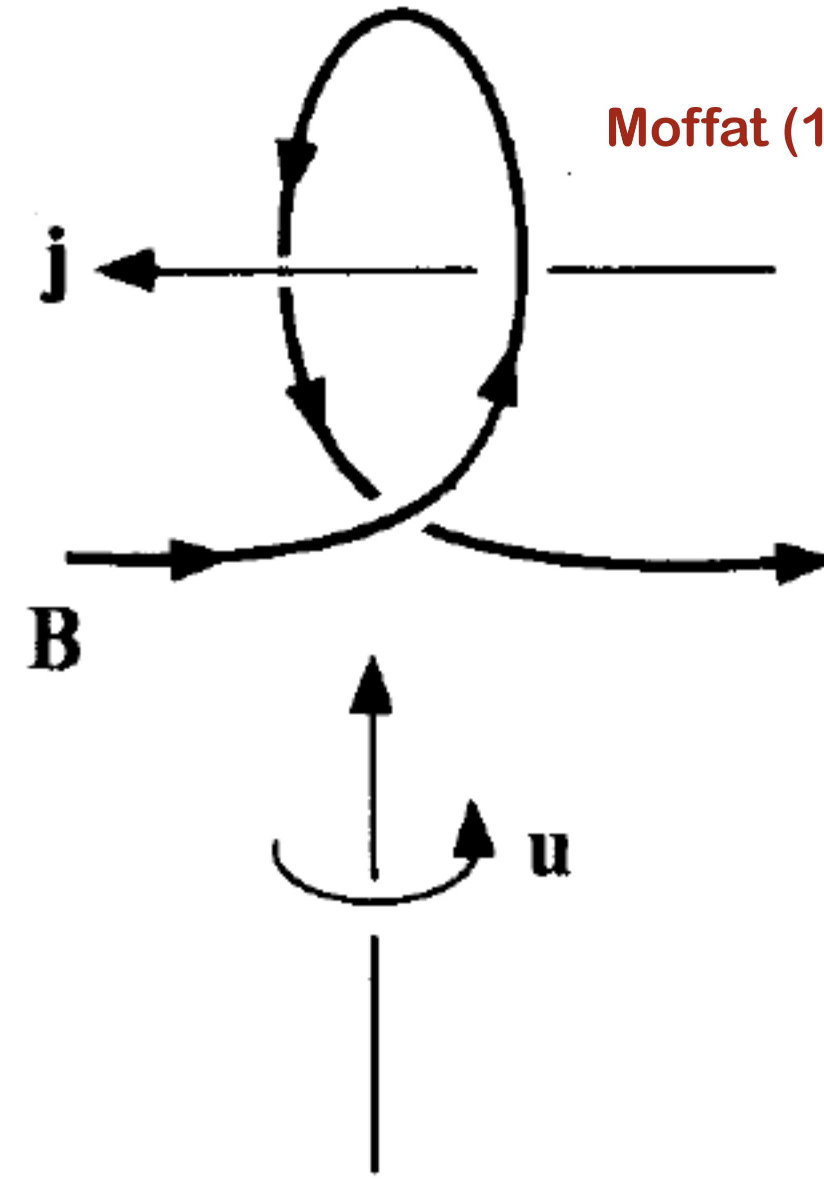
Helical motions (lift, twist) can induce an emf that is parallel to the mean field

$$\mathcal{E} = \overline{\mathbf{v}' \times \mathbf{B}'} = \alpha \overline{\mathbf{B}}$$

This creates mean poloidal (r, θ) field from toroidal (ϕ) field

which closes the Dynamo Loop





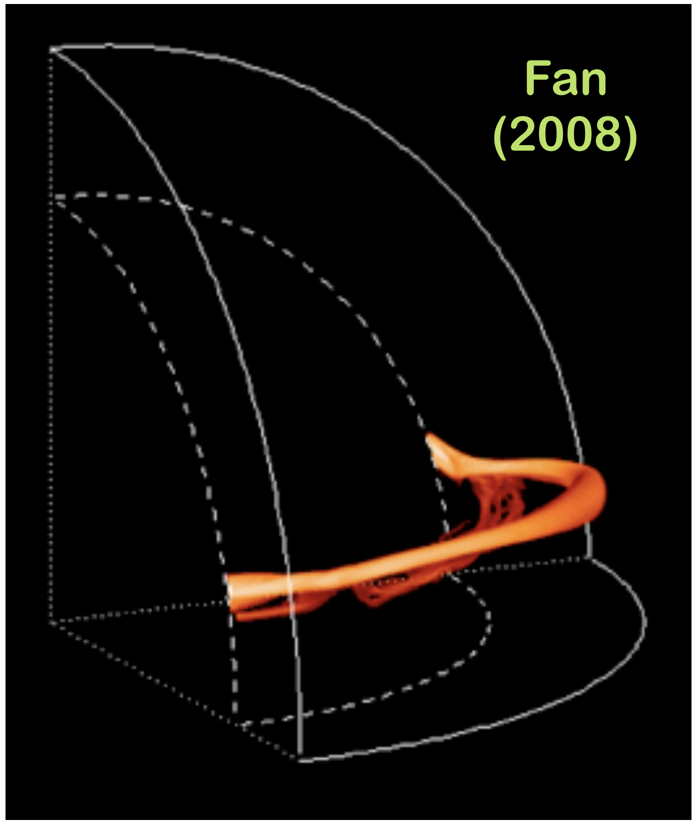
Moffat (1978)

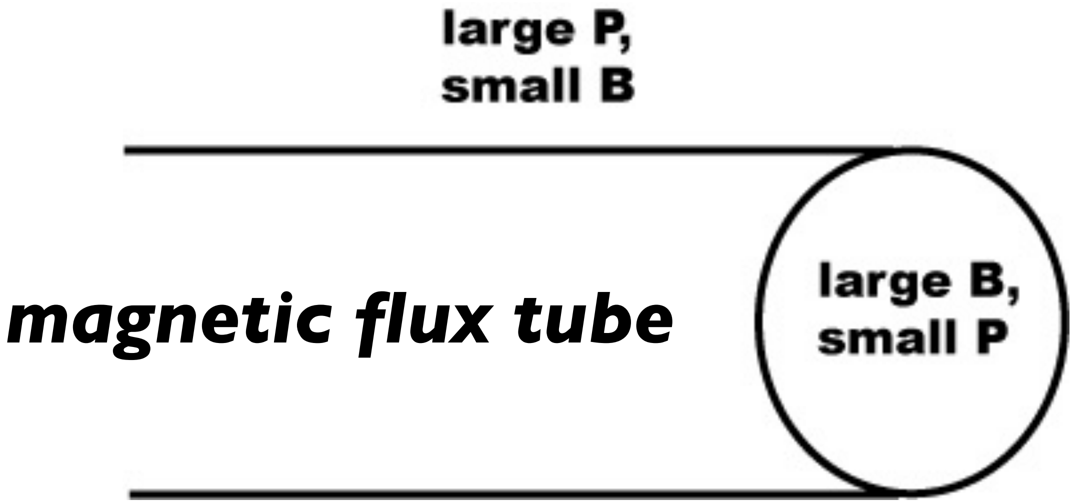
Linked to kinetic, magnetic helicity

Linked to large-scale dynamo action

Illustrates the 3D nature of dynamos

Another Way: Starts with Magnetic Buoyancy





$$P = \mathcal{R}\rho T$$

$$P^{(tube)} + P_m^{(tube)} \approx P^{(ext)}$$

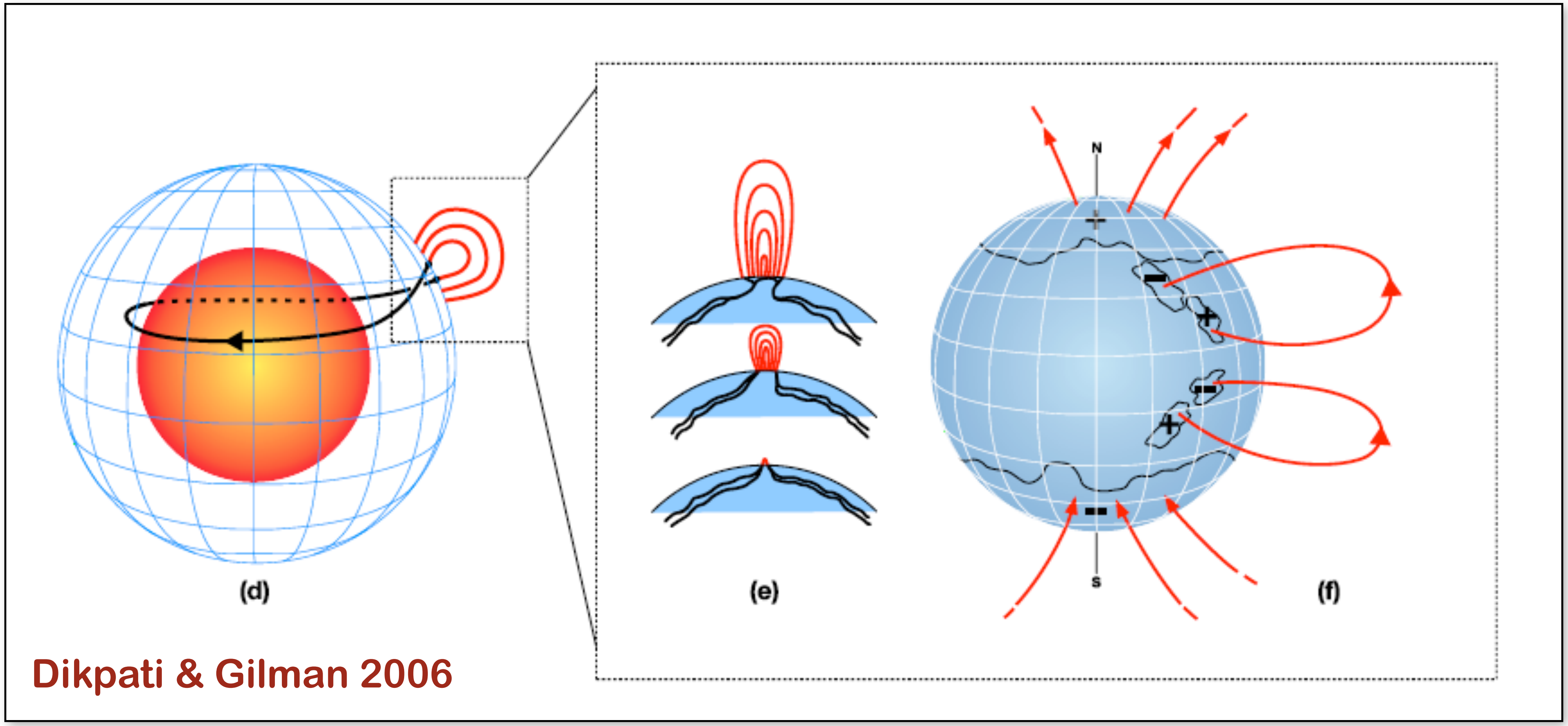
$$P_m = \frac{B^2}{8\pi}$$

$$P^{(tube)} \approx P^{(ext)} - P_m^{(tube)} < P^{(ext)}$$

If $T^{(tube)} \approx T^{(ext)}$

$$\rho^{(tube)} < \rho^{(ext)}$$

The Babcock-Leighton Mechanism

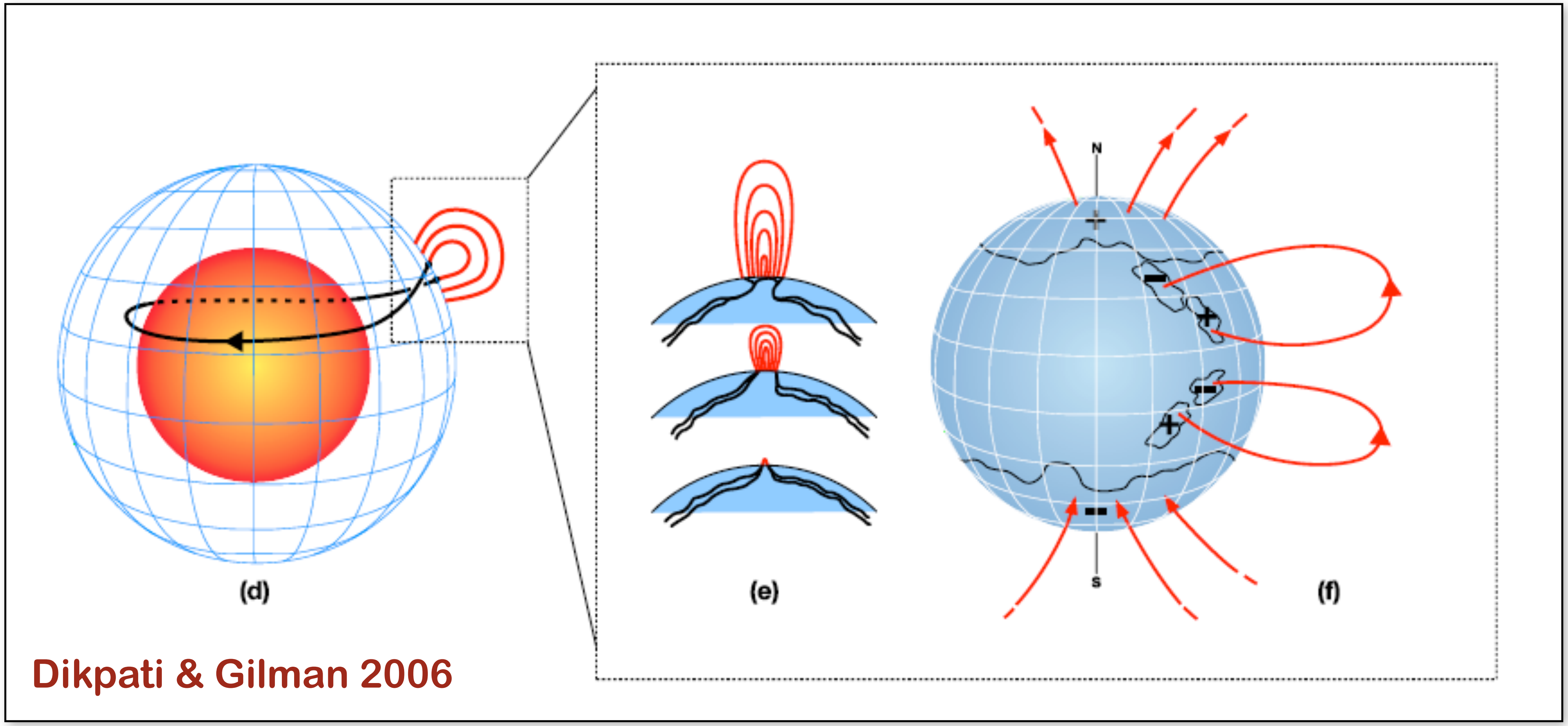


Trailing member of the spot pair is displaced poleward relative to leading edge by the Coriolis force (Joy's law: the higher the latitude, the more the tilt)

Polarity of trailing spot is opposite to pre-existing polar field

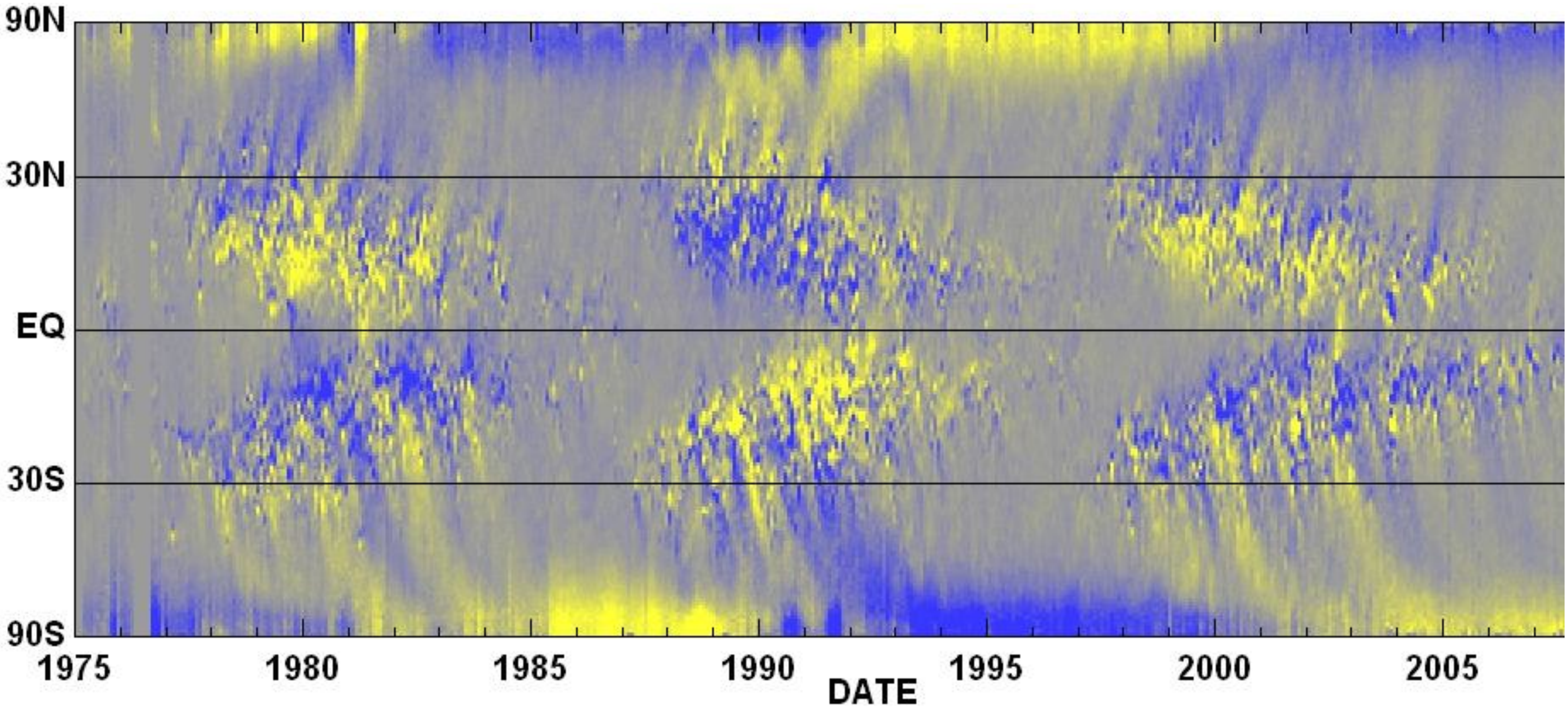
Dispersal of many spots by convection and meridional flow acts to reverse the pre-existing poloidal field

Question



What if all the spots emerged at high latitudes (> 50°) instead of low latitudes (< 50°)

Would you expect this to help or to hinder a Babcock-Leighton dynamo? Why?



Strongest evidence in favor of BL Mechanism

We see it happening!

The flux emerging in sunspots/active regions is about 2 orders of magnitude larger than the flux needed to reverse the (surface) dipole moment

Commonly used components of $\mathcal{E}$

$$\mathcal{E} = \overline{\mathbf{v}' \times \mathbf{B}'}$$

(in Mean-Field Dynamo Models)

Turbulent α-effect $\mathcal{E} = \alpha \overline{\mathbf{B}} + \ldots$

Amplificaton

Babcock-Leighton Mechanism

Often parameterized as a non-local α-effect in which a poloidal source near the surface depends on the toroidal field near the CZ base

Mathematically and functionally very similar to turbulent α-effect but physical justification is very different *(essentially nonlinear vs essentially kinematic)*

Turbulent Diffusion $\mathcal{E} = \eta_t \nabla \times \overline{\mathbf{B}} + \ldots$

Magnetic Pumping $\mathcal{E} = \gamma \times \overline{\mathbf{B}} + \ldots$

Transport

Babcock-Leighton Dynamo Models

Dikpati & Gilman 2006

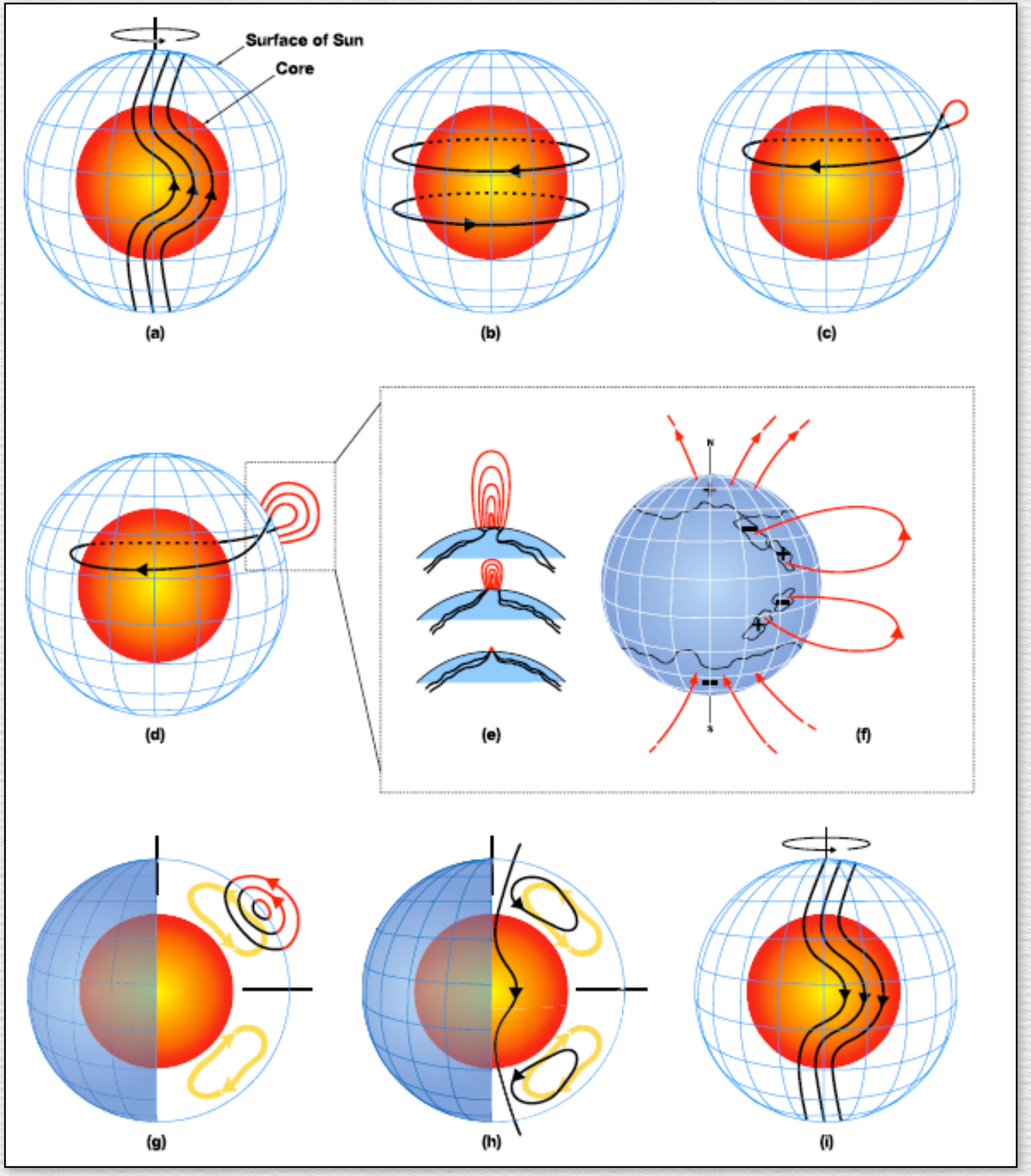
Poloidal field is generated by the Babcock-Leighton Mechanism

Cycle period is regulated by the equatorward advection of toroidal field by the meridional circulation at the base of the CZ

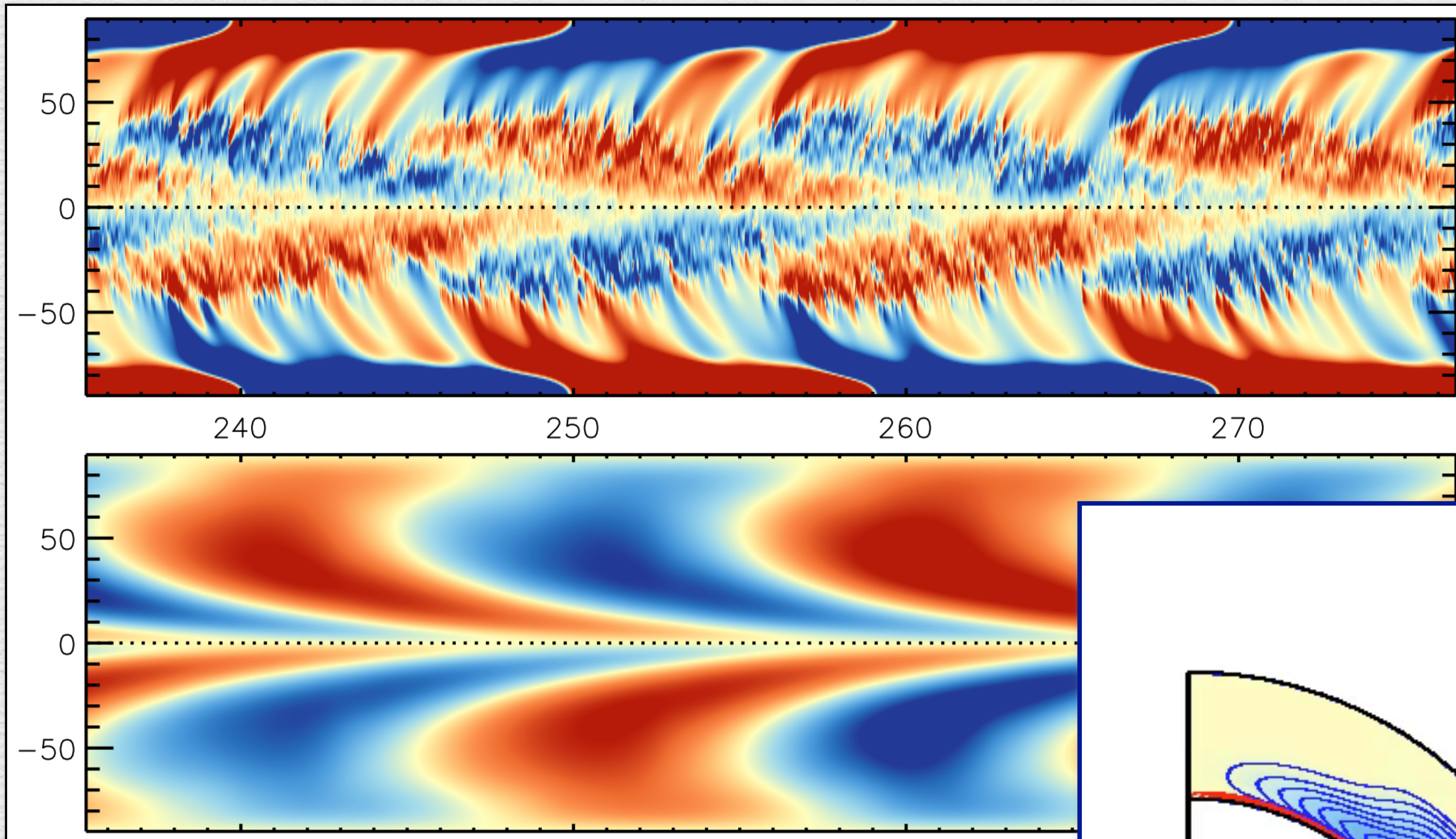
2-3 m/s gives you about 11 years

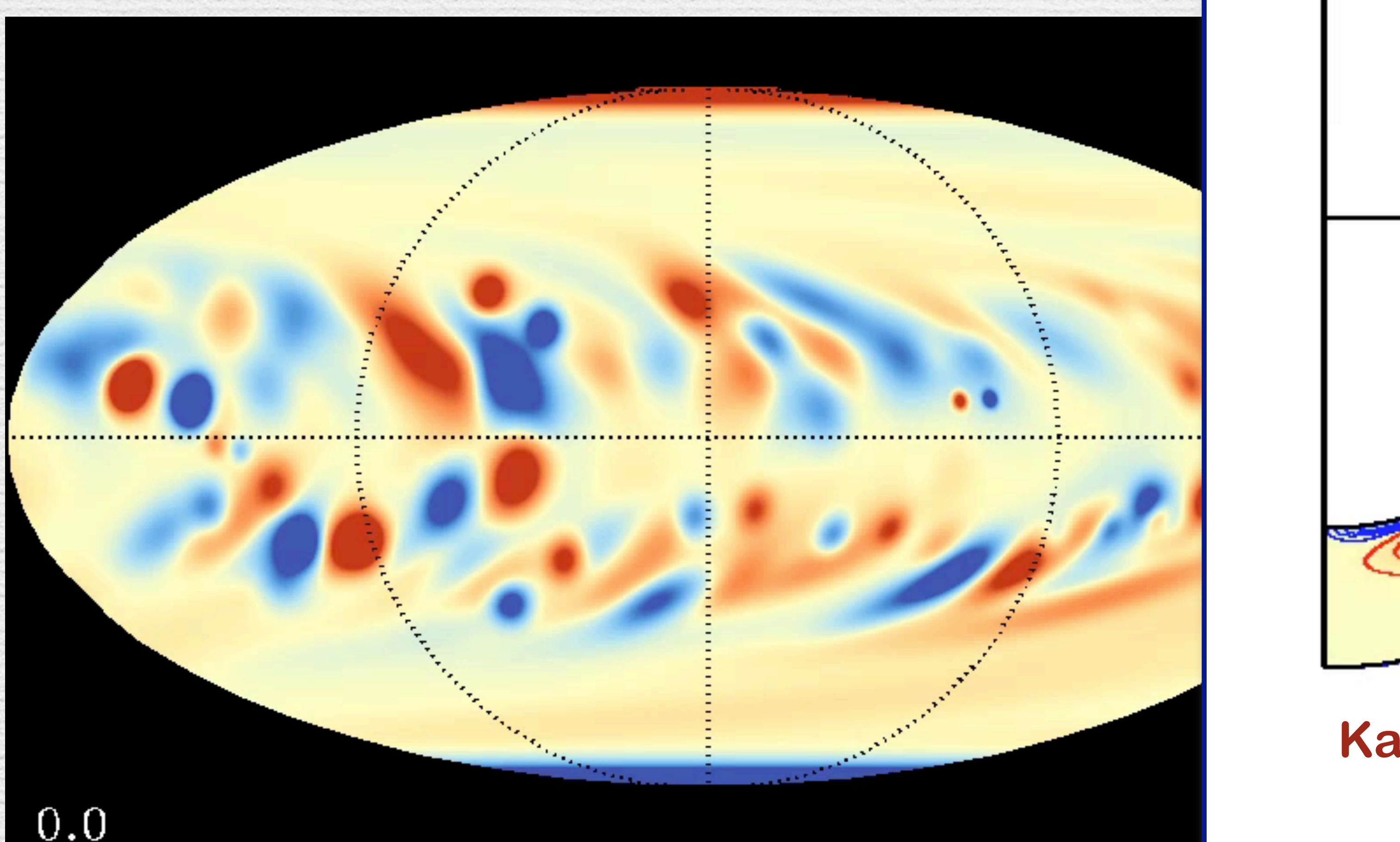
For this reason, they are also called

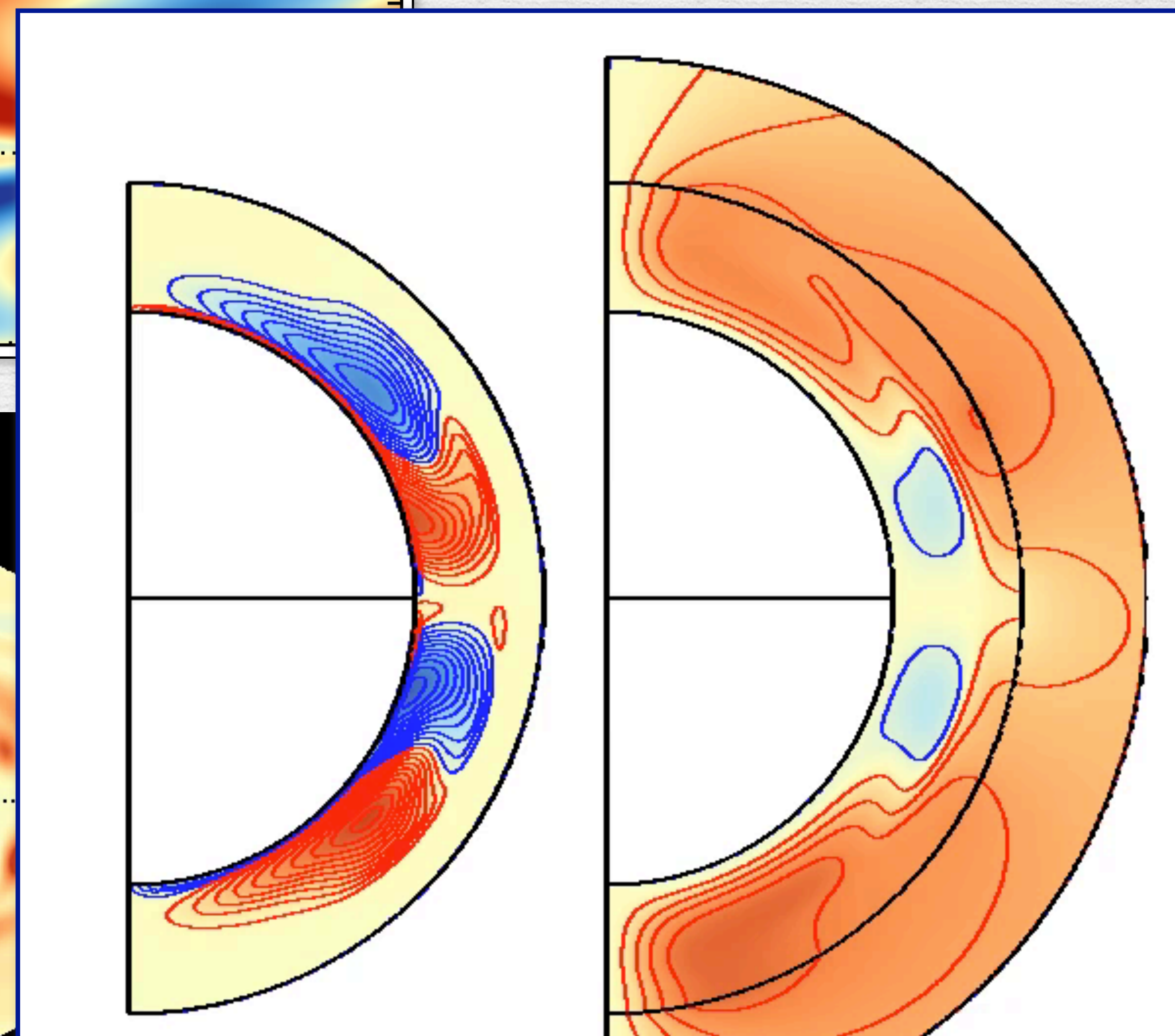
Flux-Transport Dynamo Models



Reversal of dipole moment is a cumulative effect arising from the cross-equatorial cancellation of many active region (as in the Sun)







Karak & Miesch 2017

Convective Dynamo Models

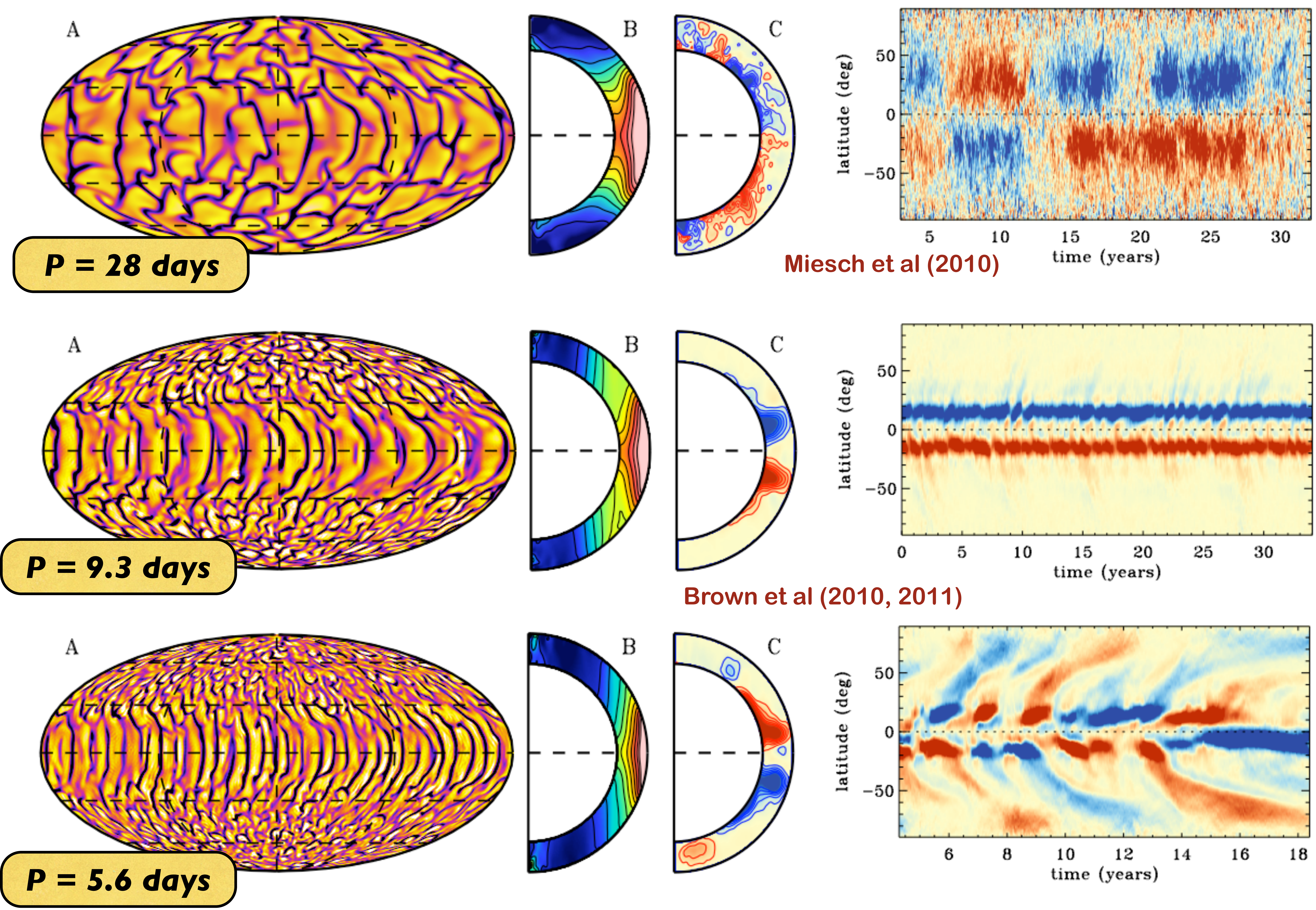
Early Work Based on Mean-Field Theory

- Beginning with Parker (1955); dominated through the early 1990's
- Differential rotation key in getting magnetic cycles
- α-Ω dynamo models
- Usually kinematic and (2D); An advantage for early insights but now regarded as a liability

Recent Focus has shifted to 3D MHD simulations

- Similar to the planetary dynamo models we discussed earlier
- $\mathcal{E} = \overline{\mathbf{v}' \times \mathbf{B}'}$ calculated explicitly - No parameterizations!
- Not kinematic: DR, MC set up self-consistently by the convective motions and nonlinear feedbacks fully included
- Dramatic progress in recent years but parameter regimes still far from the Sun and stars (Ra, Ek, Rm, Pm…)

Helicity & Shear promote magnetic self-organization

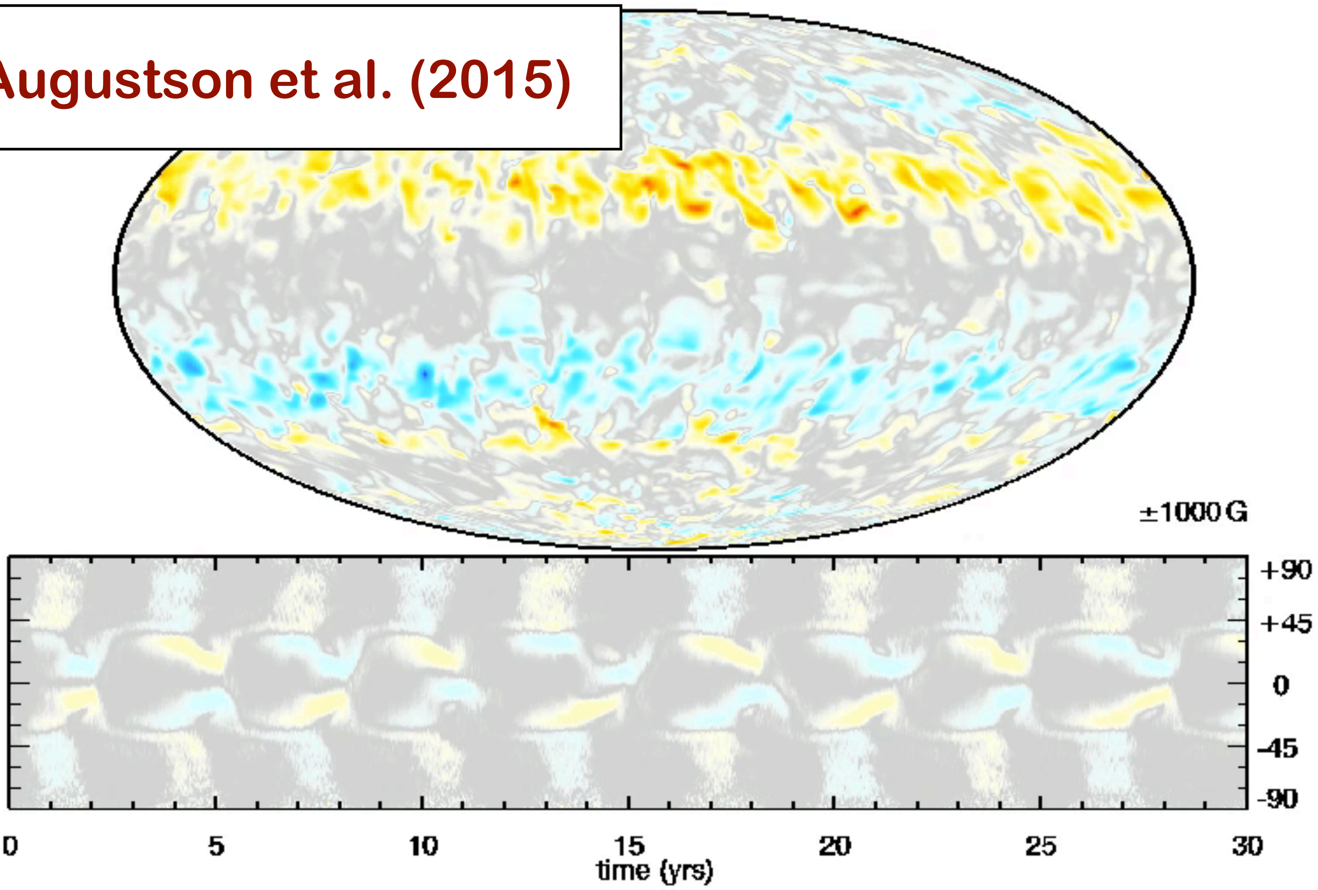


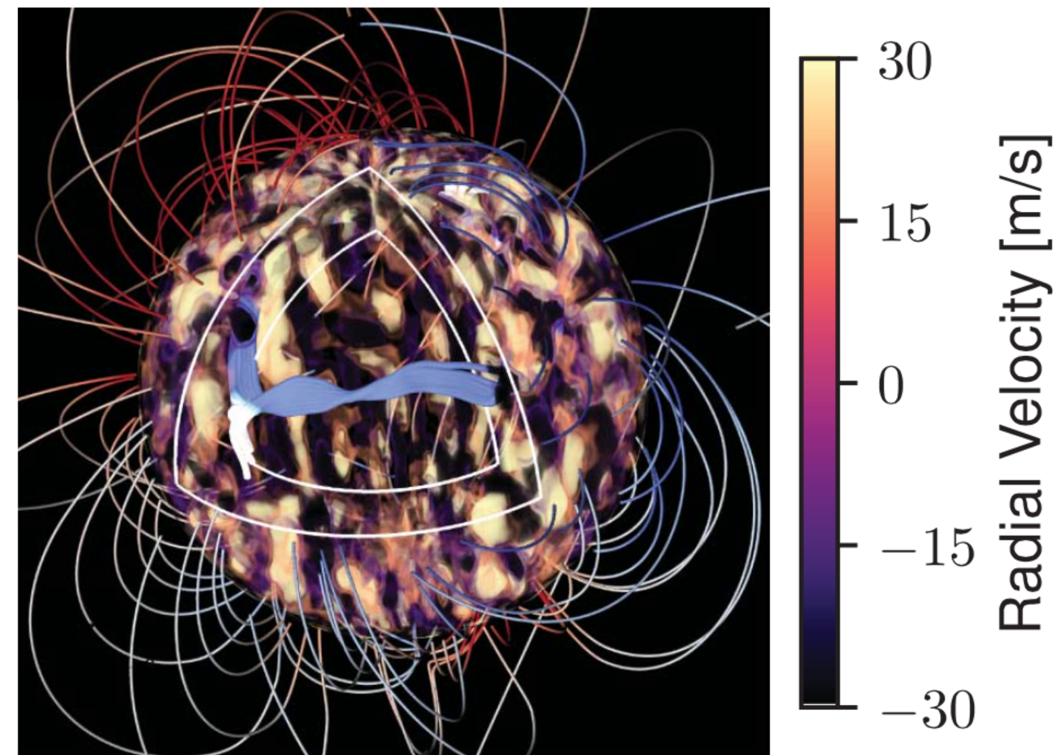
Magnetic Cycles in Convective Dynamos

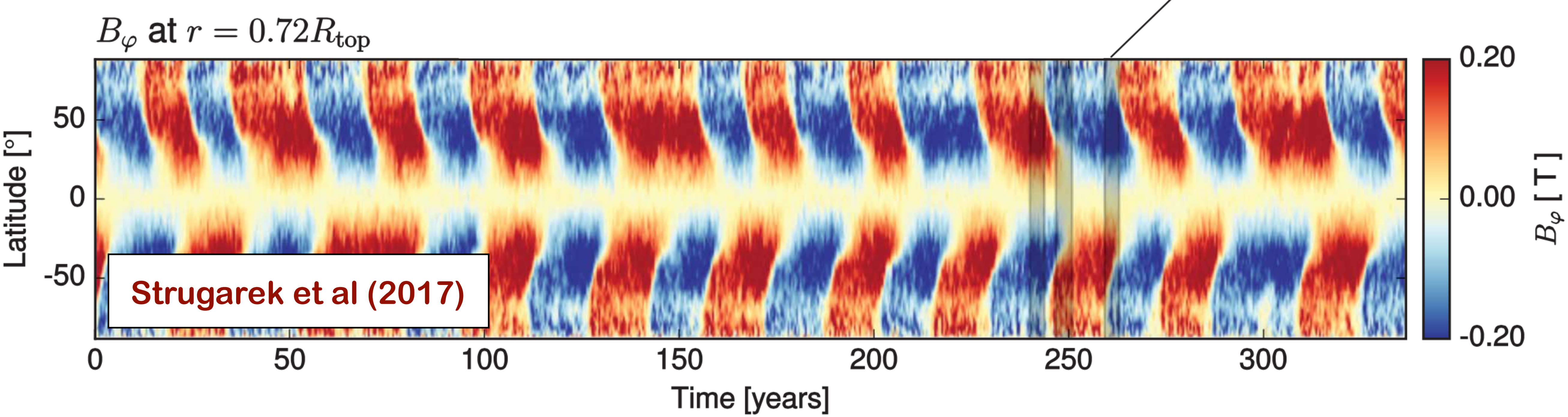
$$\frac{P_{cyc}}{P_{rot}} \sim 10^2$$

(Sun ~ 140)

Augustson et al. (2015)



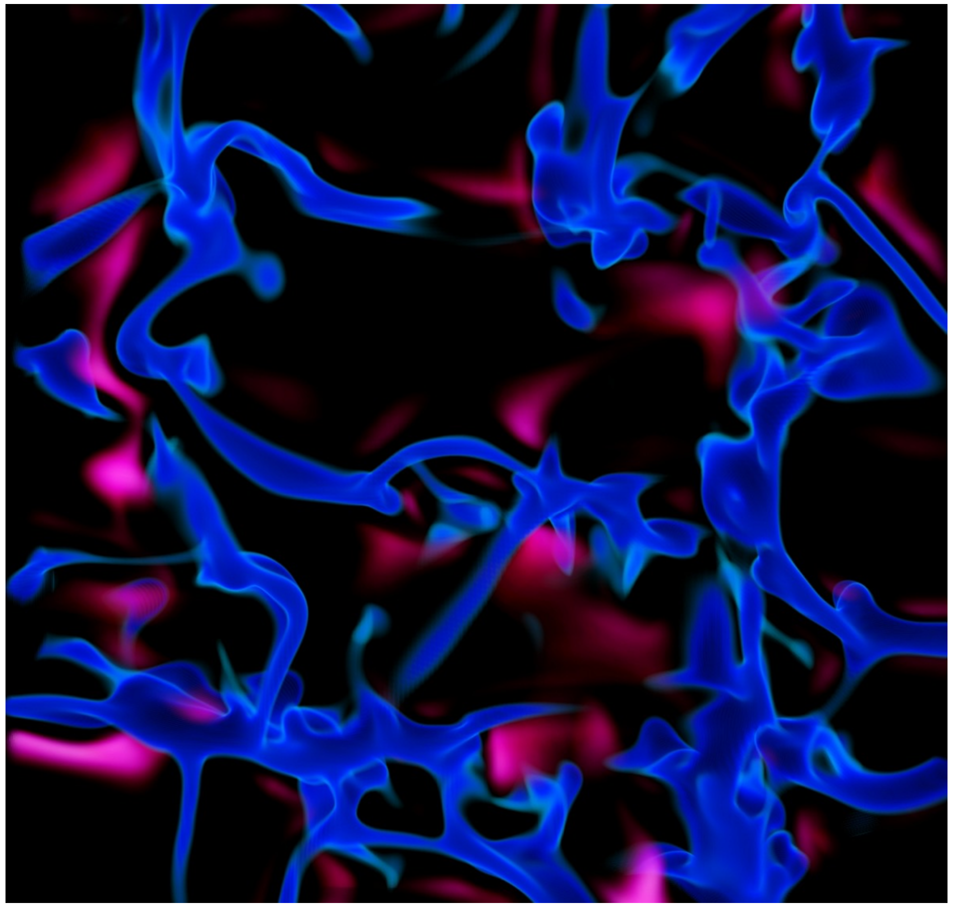




So which one is right?

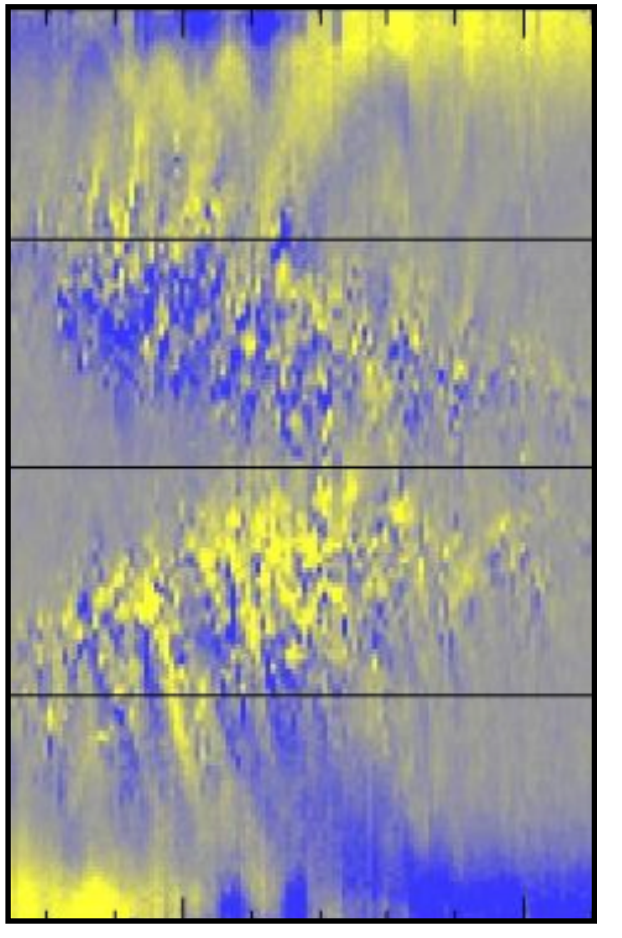
Is the Sun running a convective dynamo or a Babcock-Leighton dynamo?

In other words: what generates the large-scale poloidal field?



Helical convection

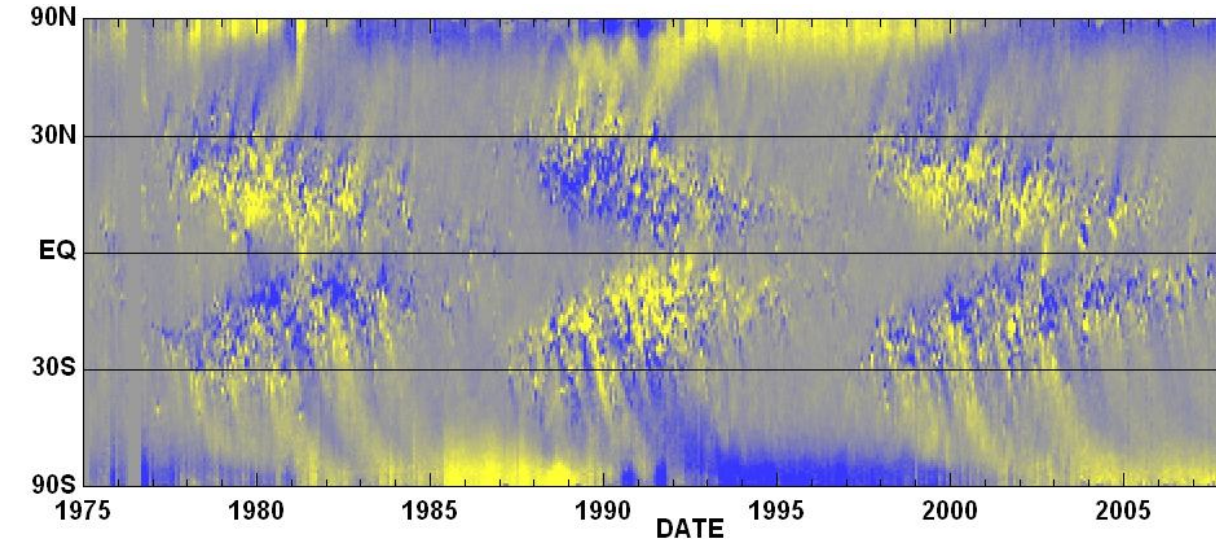
Emergence and dispersal of active regions



We don't know yet!
It's probably a combination of both!

Why 11 years?

Cycle linked to propagation of toroidal flux (Butterfly diagram)



Three ways to get propagation

❧ Meridional circulation

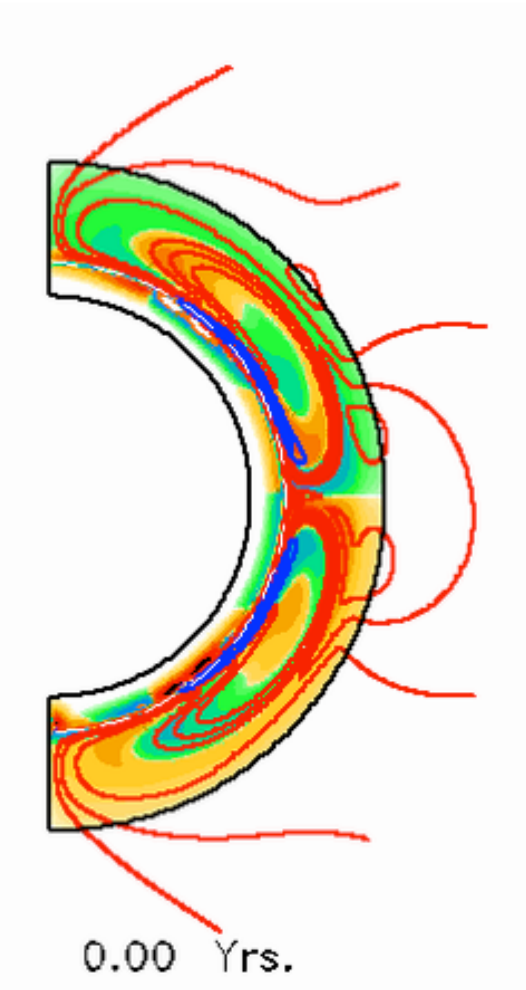
- Flux-Transport Dynamo models
- 2-3 m/s at CZ base

❧ Turbulent transport

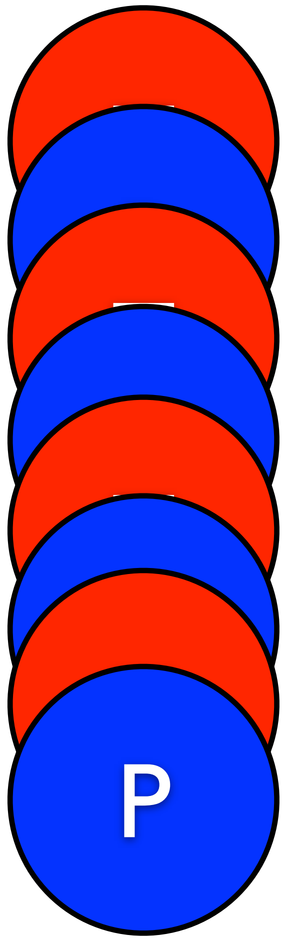
- magnetic pumping
- Mean-Field and convective dynamos

❧ Dynamo wave

- Early α-Ω dynamo models
- some modern convective dynamos



Dikpati & Gilman 2006



Rotation-Activity Correlation

Stellar observations indicate

$$B \propto \Omega^n$$

with n ≈ 2

Out until a

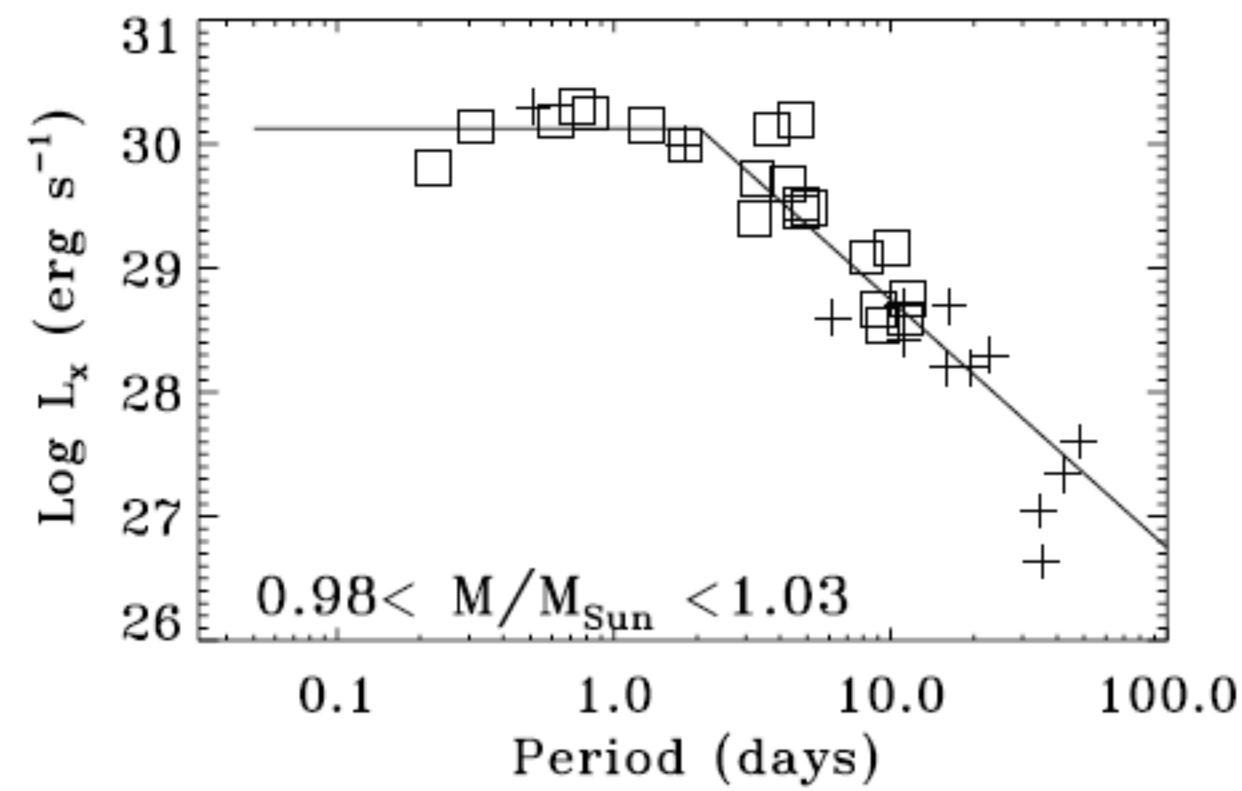
Saturation Regime

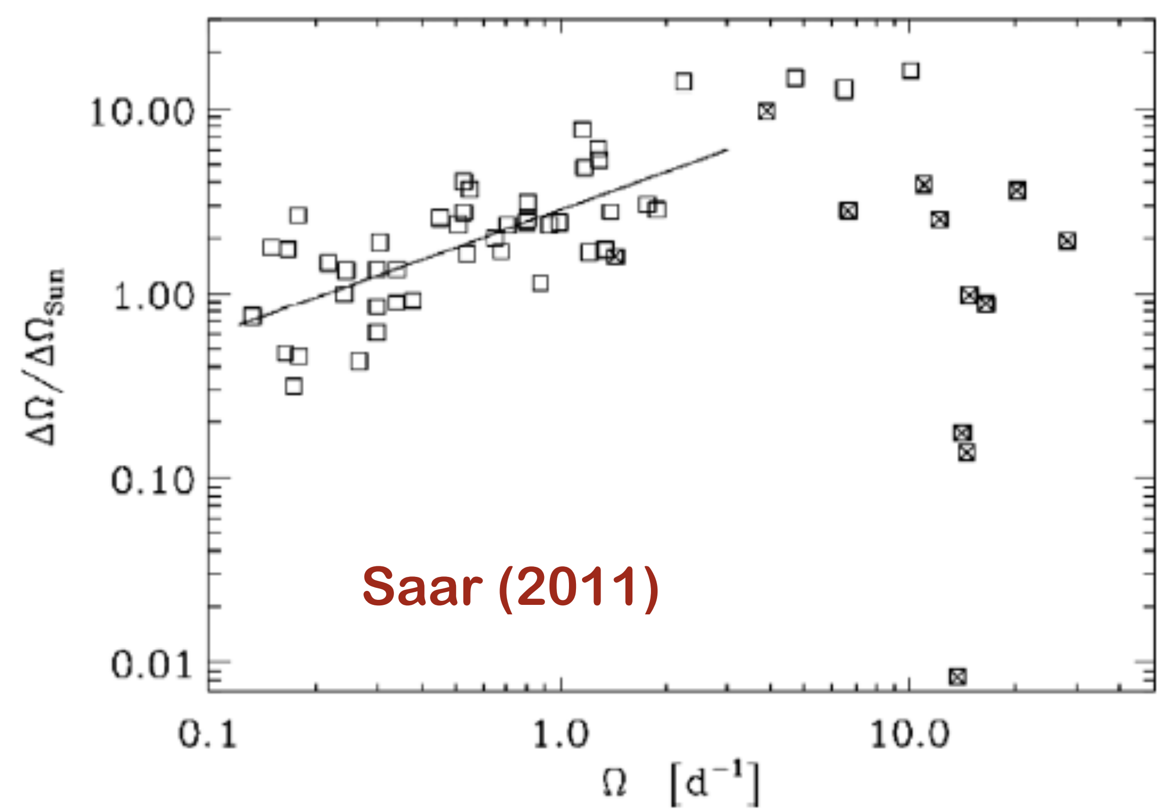
(≳10 $\Omega_\odot$ for solar-like stars) where B becomes independent of Ω and DR is suppressed

Convection simulations show similar behavior

This saturation regime may proved a promising link between planets and stars

Pizzolato et al (2003)

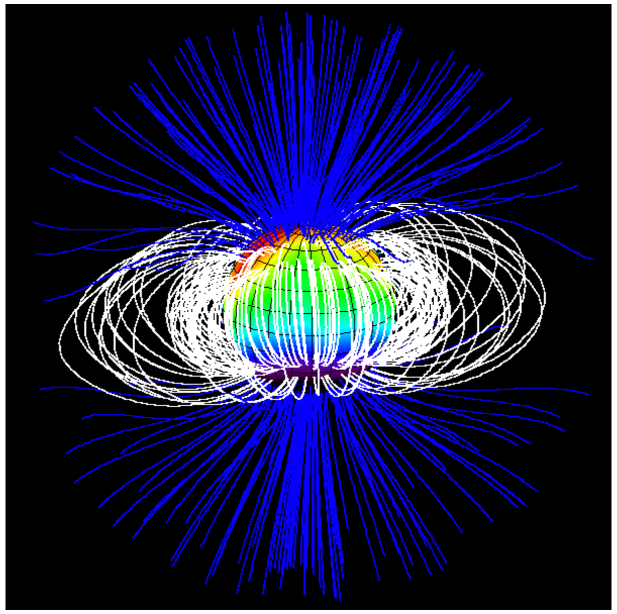




Puzzles

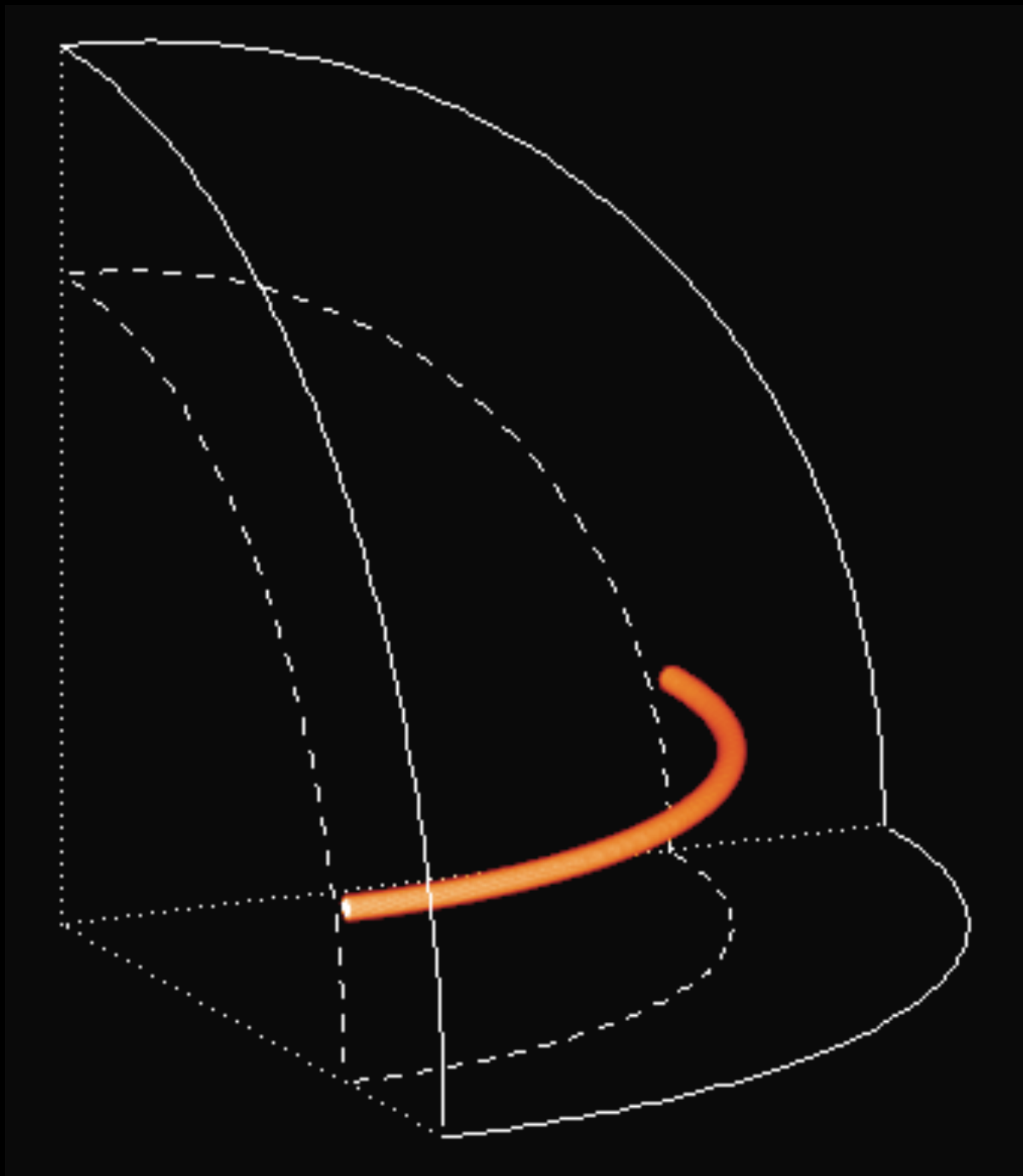
- **Amplitude and Structure of Deep Solar Convection**
  - **What is the Rossby number in the deep CZ?**
- **Mean Flows**
  - **How are the thermal gradients needed for conical Ω surfaces established?**
  - **What is the subsurface structure of the MC?**
- **The Solar Dynamo**
  - **How and where is mean poloidal field being generated?**
  - **How do convective dynamos produce sunspots/active regions and what role do they play in the dynamo?**
  - **How do small-scale and large-scale dynamo action couple?**
  - **What sets the 11-year period?**
  - **What sets the amplitude of solar cycles?**

Looking to the stars may help!



Donati et al (2006)

Supplemental Slides



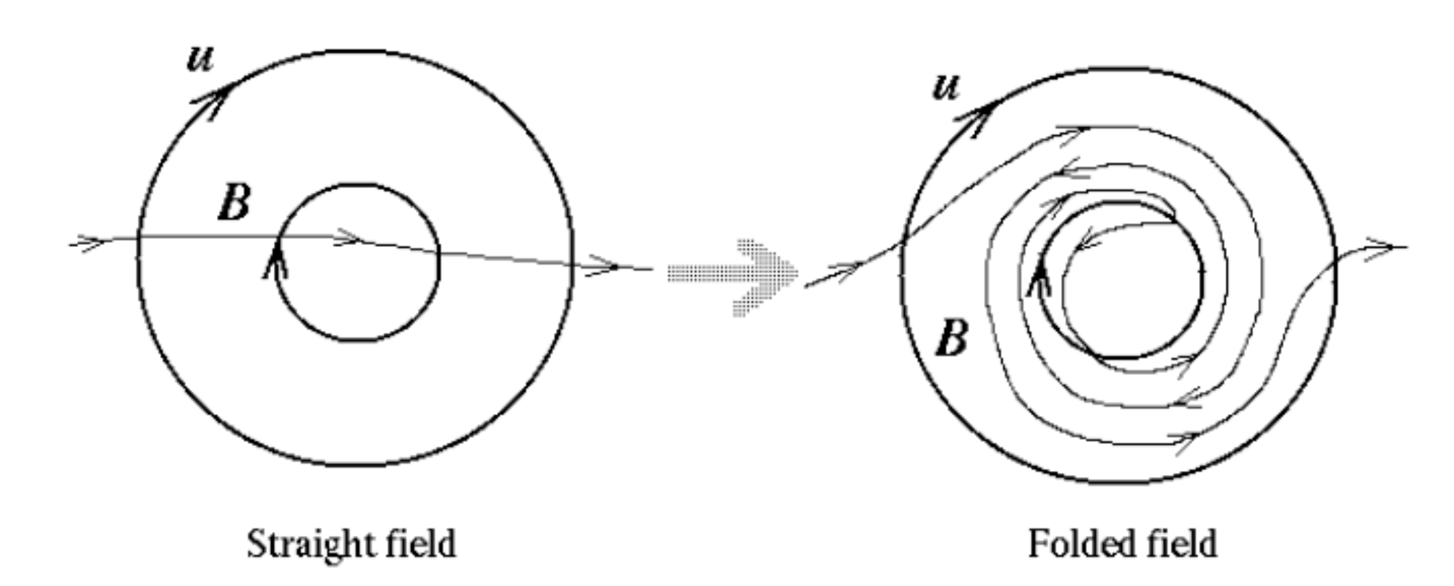
What forces might this flux tube be experiencing?

$$\frac{\partial \mathbf{v}}{\partial t} = -\left(\rho \mathbf{v} \cdot \boldsymbol{\nabla}\right) \mathbf{v} - 2\rho \boldsymbol{\Omega} \times \mathbf{v} - \boldsymbol{\nabla}\left(P + P_m\right) + (4\pi)^{-1}\left(\mathbf{B} \cdot \boldsymbol{\nabla}\right) \mathbf{B} + \rho \mathbf{g} - \boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{D}}$$

Spatially smooth, temporally chaotic flows work best

$$R_m = \frac{UL}{\eta}$$

$$P_m = \frac{\nu}{\eta}$$

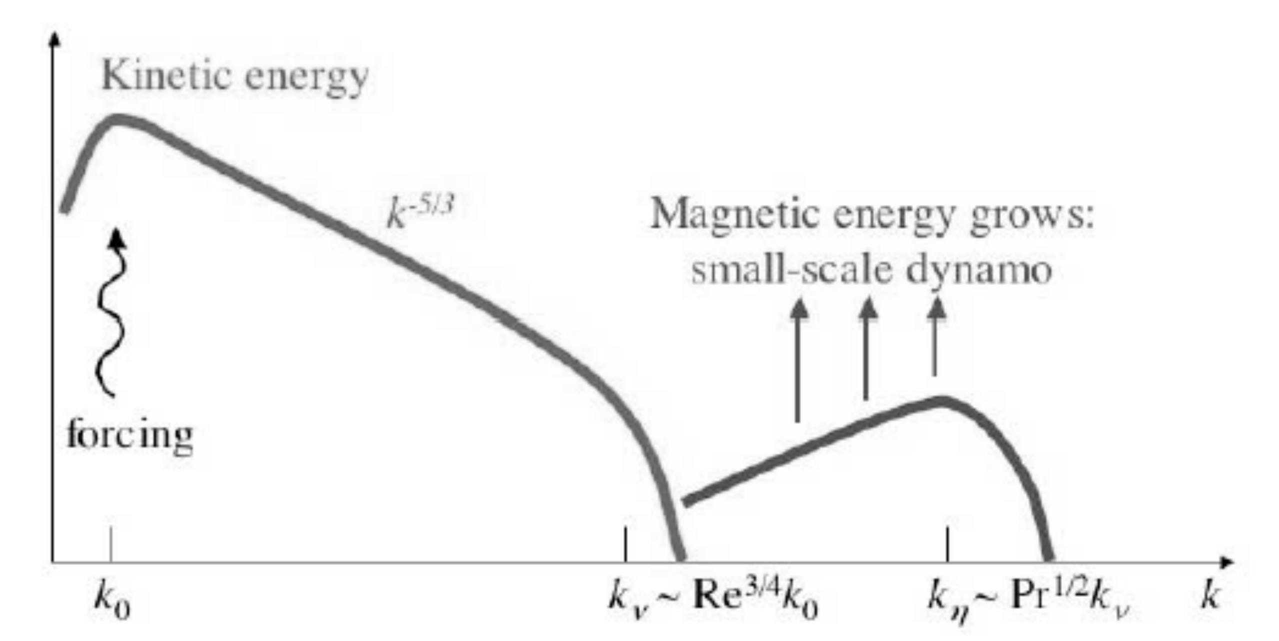


Schekochihin et al (2004)

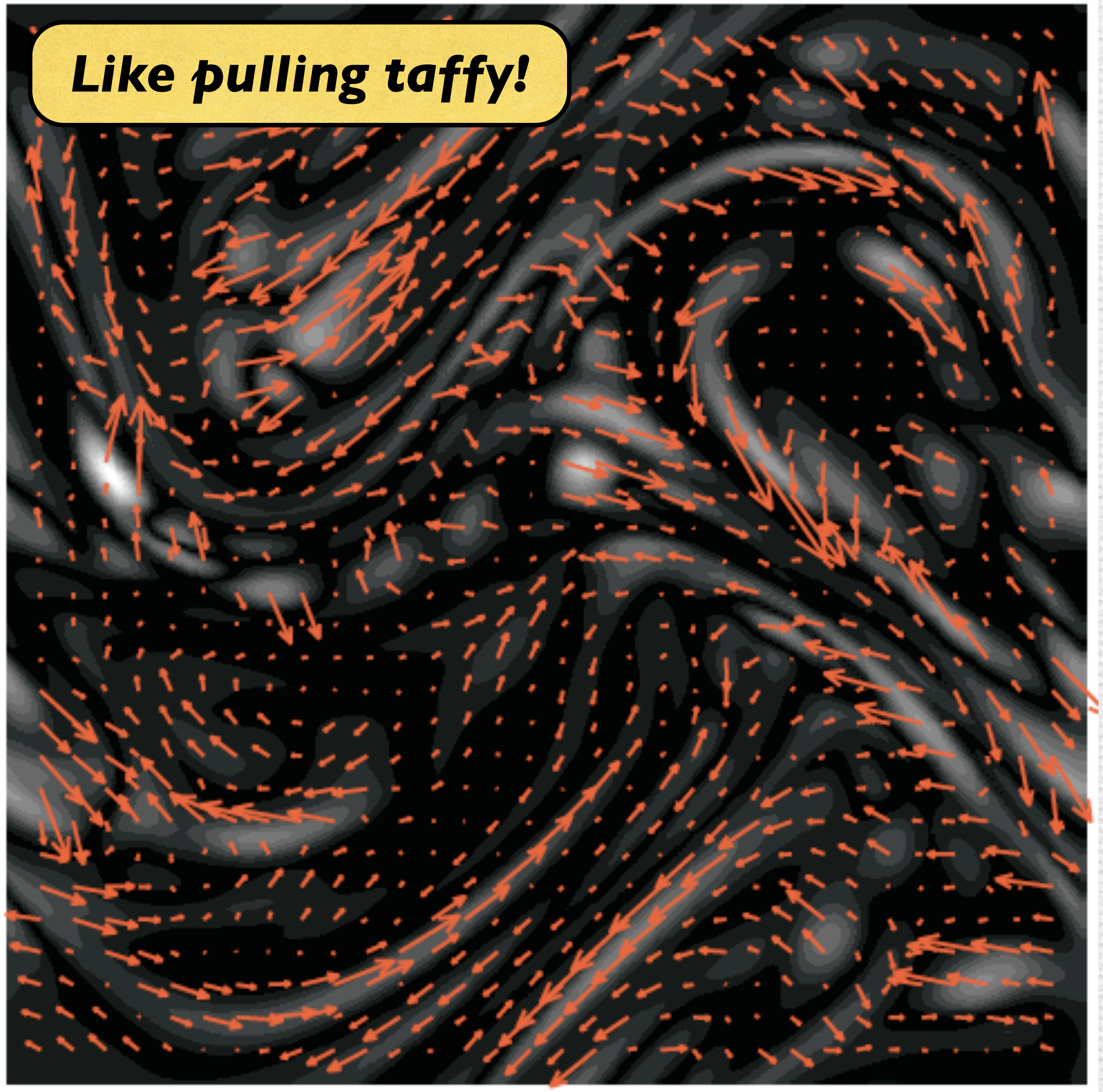
If P_m > 1 then turbulent dynamos build fields on sub-viscous scales

Magnetic energy peaks near resistive scale

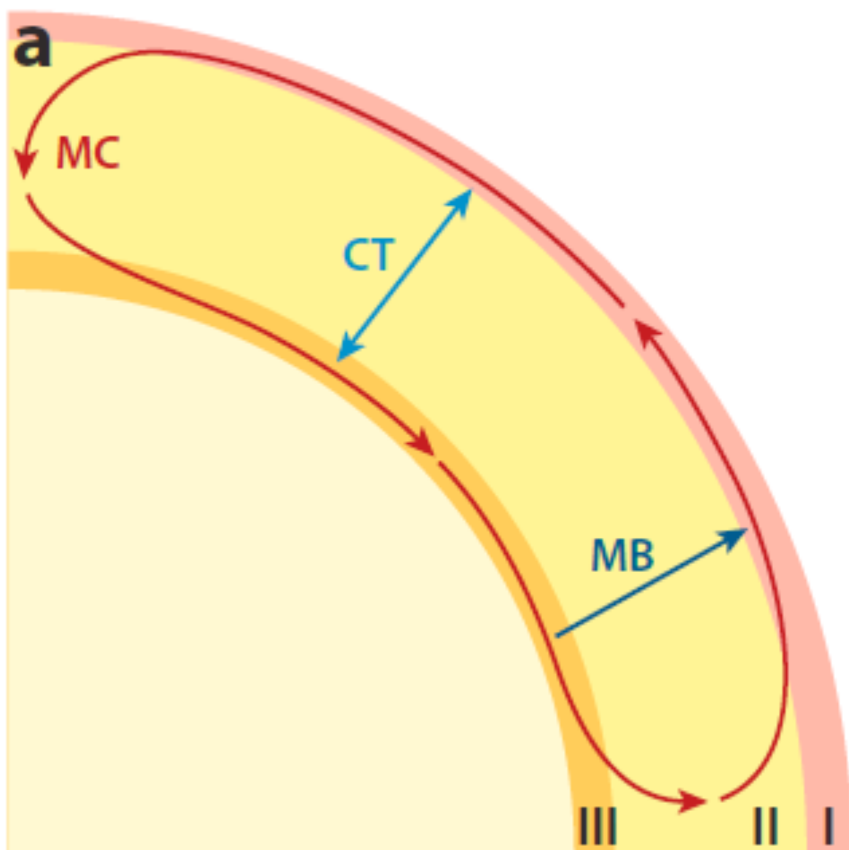
Turbulent flows beget turbulent fields!

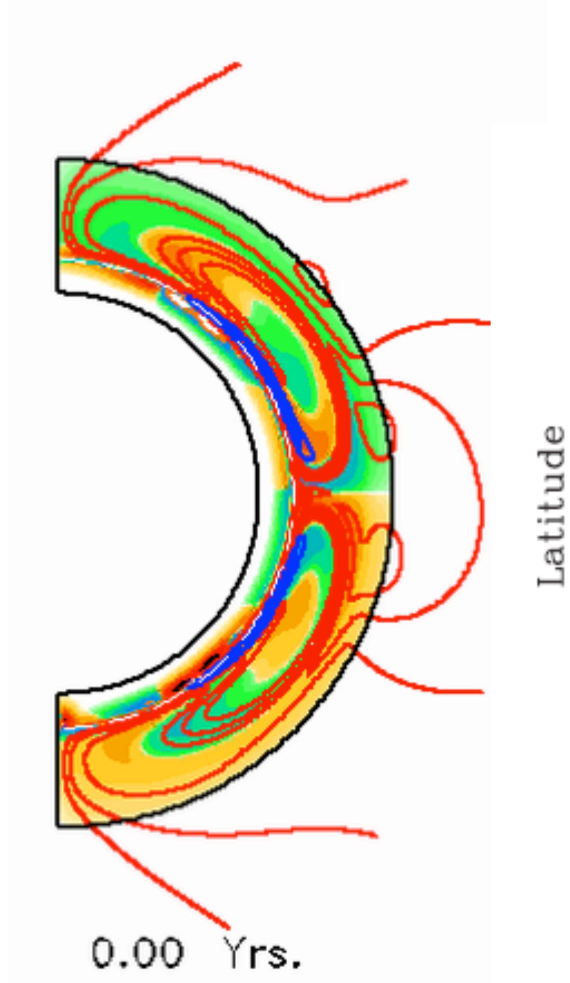


Like pulling taffy!

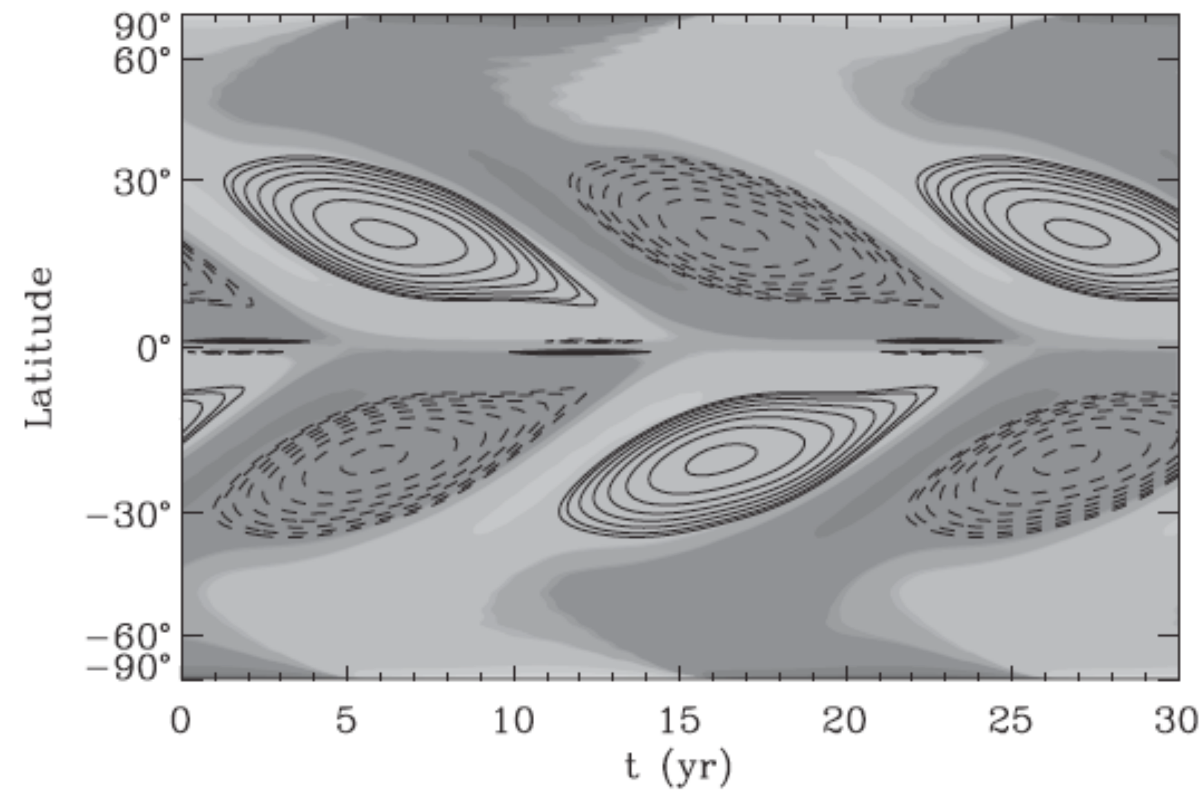


Solar Dynamo Models





Dikpati & Gilman



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	Toroidal field generation	Poloidal field generation	Principal coupling mechanisms	Cycle period determined by
BLFT models	Region III	Region I	MC, MB	Meridional flow
Interface models	Region III	Region II	CT	Dynamo waves[a]

a. Dispersion relation involving α, $\Delta\,\Omega$, and η_t.

Large Scale Dynamos: The Mean Induction Equation

Go back to our basic induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B}) + \eta \boldsymbol{\nabla}^2 \mathbf{B}$$

Note:
The B field in the Sun is clearly not axisymmetric. Still, the solar cycle does have an axisymmetric component so that's a good place to start

Now just average over longitude and rearrange a bit
(other averages are possible but we'll stick to this for simplicity)

The equation for the mean field comes out to be

$$\lambda = r \sin \theta$$

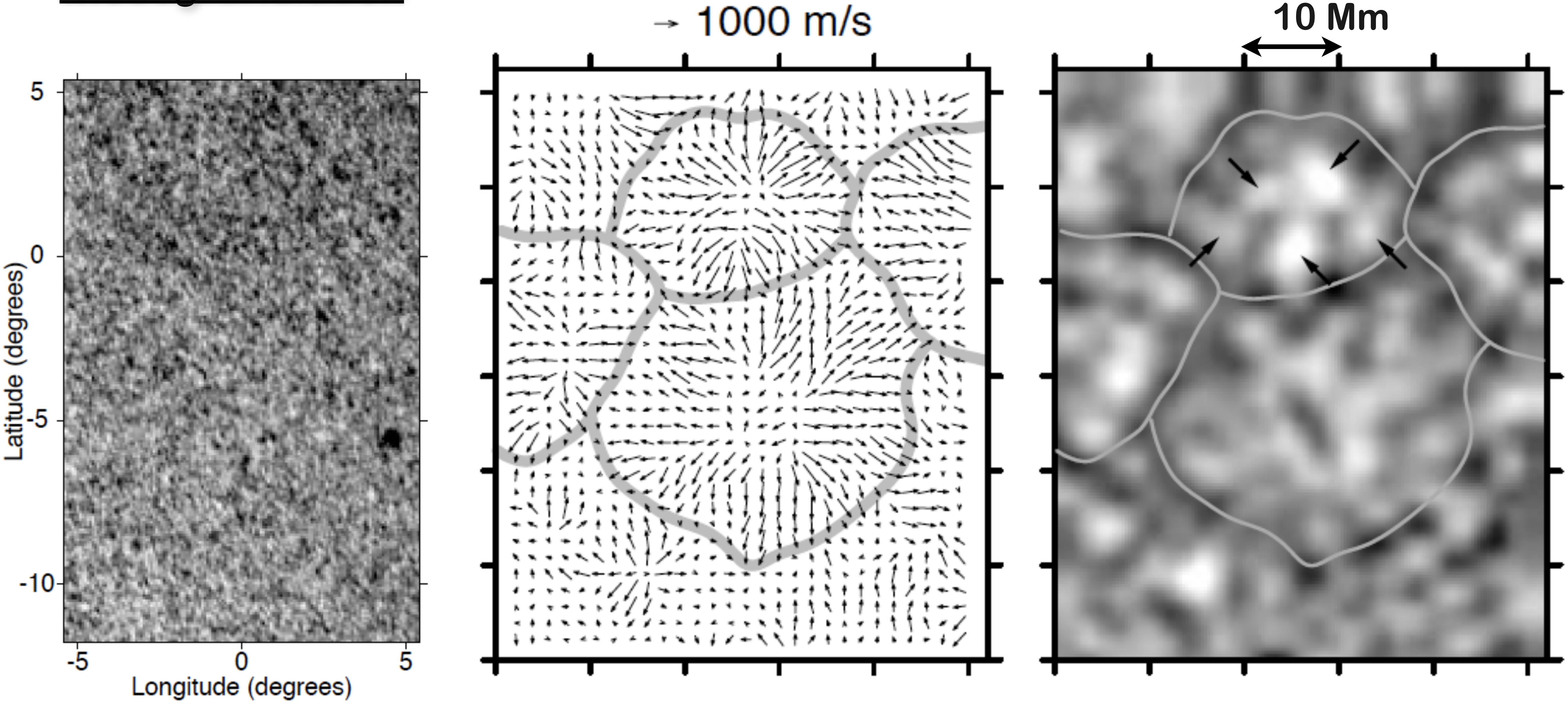
$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \lambda \overline{\mathbf{B}}_p \cdot \boldsymbol{\nabla} \Omega \ \hat{\phi} + \boldsymbol{\nabla} \times \left(\overline{\mathbf{v}}_m \times \overline{\mathbf{B}} \right) + \eta \nabla^2 \overline{\mathbf{B}} + \boldsymbol{\nabla} \times \mathcal{E}$$

Ω-effect — **Meridional circulation** — **Diffusion (molecular)** — **Fluctuating emf**

No assumptions made up to this point beyond the basic MHD induction equation

Straightforward to show that if $\mathcal{E}$=0, the dynamo dies (Cowling's theorem)

Mesogranulation



Shine, Simon & Hurlburt (2000)

Most readily seen in horizontal velocity divergence maps obtained from local correlation tracking (LCT)

Vertical velocity and temperature signatures of mesogranulation and supergranulation are still elusive
hard to verify that they are convection per se

$L \sim 5$ Mm
$t \sim 3\text{-}4$ hr