DYNAMO THEORY: BASICS

Amitava Bhattacharjee

Princeton Plasma Physics Laboratory, Princeton University

Acknowledgments: Matthias Rempel, HAO/NCAR

Magnetic fields in the Universe

- Earth
  - Magnetic field present for $\sim 3.5 \cdot 10^9$ years, much longer than Ohmic decay time ($\sim 10^4$ years)
  - Strong variability on shorter time scales (10^3 years)
- Planets: Mercury, Jupiter, Saturn, Uranus, Neptune have large scale fields
- Sun
  - Magnetic fields from smallest observable scales to size of sun
  - 11 year cycle of large scale field
  - Ohmic decay time $\sim 10^9$ years (in absence of turbulence)
- Other stars
  - Stars with outer convection zone: similar to sun
  - Stars with outer radiation zone: most likely primordial fields
- Galaxies
  - Field structure coupled to observed matter distribution (e.g. spirals)
  - Is it primordial?

Why is the Universe magnetized?

D. Longcope lecture: Dynamo has 3 fundamental features:

- Electrically conducting fluid
- Fluid must have complex motions
- Motions must be vigorous enough (as measured by the Magnetic Reynolds number Rm = velocity x size/resistivity)

Toy example: Homopolar Dynamo

References for this lecture:

Heliophysics, Vol. 1, Chapter 3: M. Rempel

Heliophysics, Vol. 3, Chapter 6: P. Charbonneau

Heliophysics, Vol. 4, Chapter 6: S. Stanley

MHD equations

The full set of MHD equations combines the induction equation with the Navier-Stokes equations including the Lorentz-force:

$$
\begin{aligned}
\frac{\partial \varrho}{\partial t} &= -\nabla \cdot (\varrho \mathbf{v}) \\
\varrho \frac{\partial \mathbf{v}}{\partial t} &= -\varrho(\mathbf{v} \cdot \nabla)\mathbf{v} - \nabla p + \varrho \mathbf{g} + \frac{1}{\mu_0}(\nabla \times \mathbf{B}) \times \mathbf{B} + \nabla \cdot \tau \\
\varrho \frac{\partial e}{\partial t} &= -\varrho(\mathbf{v} \cdot \nabla)e - p\nabla \cdot \mathbf{v} + \nabla \cdot (\kappa \nabla T) + Q_\nu + Q_\eta \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})
\end{aligned}
$$

Assumptions:

- Validity of continuum approximation (enough particles to define averages)
- Non-relativistic motions, low frequencies

MHD equations

Viscous stress tensor τ

$$\begin{aligned}
\Lambda_{ik} &= \frac{1}{2}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right) \\
\tau_{ik} &= 2\varrho\nu\left(\Lambda_{ik} - \frac{1}{3}\delta_{ik}\nabla\cdot\mathbf{v}\right) \\
Q_\nu &= \tau_{ik}\Lambda_{ik}\ ,
\end{aligned}$$

Ohmic dissipation Q_η

$$Q_\eta = \frac{\eta}{\mu_0}(\nabla\times\mathbf{B})^2\ .$$

Equation of state

$$p = \frac{\varrho\, e}{\gamma - 1}\ .$$

ν, η and κ: viscosity, magnetic diffusivity and thermal conductivity
μ_0 denotes the permeability of vacuum

Induction equation

Using Ampere's law $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$ yields for the electric field in the laboratory frame

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla \times \mathbf{B}$$

leading to the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

with the magnetic diffusivity

$$\eta = \frac{1}{\mu_0 \sigma} .$$

Advection, diffusion, magnetic Reynolds number

L: typical length scale U: typical velocity scale L/U: time unit

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{v} \times \mathbf{B} - \frac{1}{R_m} \nabla \times \mathbf{B} \right)$$

with the magnetic Reynolds number

$$R_m = \frac{U L}{\eta} \; .$$

$R_m \ll 1$: diffusion dominated regime

$$\frac{\partial \mathbf{B}}{\partial t} = \eta \Delta \mathbf{B} \; .$$

Only decaying solutions with decay (diffusion) time scale

$$\tau_{\mathrm{d}} \sim \frac{L^2}{\eta}$$

Advection, diffusion, magnetic Reynolds number

$R_m \gg 1$ advection dominated regime (ideal MHD)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Equivalent expression

$$\frac{\partial \mathbf{B}}{\partial t} = -(\mathbf{v} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{v} - \mathbf{B}\,\nabla \cdot \mathbf{v}$$

- advection of magnetic field
- amplification by shear (stretching of field lines)
- amplification through compression

Dynamos: Motivation

- For $\mathbf{v} = 0$ magnetic field decays on timescale $\tau_d \sim L^2/\eta$
- Earth and other planets:
  - Evidence for magnetic field on earth for $3.5 \cdot 10^9$ years while $\tau_d \sim 10^4$ years
  - Permanent rock magnetism not possible since $T > T_{\text{Curie}}$ and field highly variable $\longrightarrow$ field must be maintained by active process
- Sun and other stars:
  - Evidence for solar magnetic field for $\sim 300\,000$ years (^{10}Be)
  - Most solar-like stars show magnetic activity independent of age
  - Indirect evidence for stellar magnetic fields over life time of stars
  - But $\tau_d \sim 10^9$ years!
  - Primordial field could have survived in radiative interior of sun, but convection zone has much shorter diffusion time scale ~ 10 years (turbulent diffusivity)

Advection, diffusion, magnetic Reynolds number

Object	$\eta[\mathrm{m}^2/\mathrm{s}]$	$L[\mathrm{m}]$	$U[\mathrm{m/s}]$	R_m	τ_d
earth (outer core)	2	10^6	10^{-3}	300	10^4 years
sun (plasma conductivity)	1	10^8	100	10^{10}	10^9 years
sun (turbulent conductivity)	10^8	10^8	100	100	3 years
liquid sodium lab experiment	0.1	1	10	100	10 s

Mathematical definition of dynamo

S bounded volume with the surface ∂S, $\mathbf{B}$ maintained by currents contained within S, $B \sim r^{-3}$ asymptotically,

$$\begin{aligned}
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) && \text{in } S \\
\nabla \times \mathbf{B} &= 0 && \text{outside } S \\
[\mathbf{B}] &= 0 && \text{across } \partial S \\
\nabla \cdot \mathbf{B} &= 0
\end{aligned}$$

$\mathbf{v} = 0$ outside S, $\mathbf{n} \cdot \mathbf{v} = 0$ on ∂S and

$$E_{\rm kin} = \int_S \frac{1}{2} \varrho \mathbf{v}^2 \, dV \leq E_{\rm max} \quad \forall\, t$$

$\mathbf{v}$ is a dynamo if an initial condition $\mathbf{B} = \mathbf{B}_0$ exists so that

$$E_{\rm mag} = \int_{-\infty}^{\infty} \frac{1}{2\mu_0} \mathbf{B}^2 \, dV \geq E_{\rm min} \quad \forall\, t$$

Large scale/small scale dynamos

Decompose the magnetic field into large scale part and small scale part (energy carrying scale of turbulence) $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{B}'$:

$$E_{\mathrm{mag}} = \int \frac{1}{2\mu_0} \overline{\mathbf{B}}^2 \, dV + \int \frac{1}{2\mu_0} \overline{\mathbf{B}'^2} \, dV .$$

- Small scale dynamo: $\overline{\mathbf{B}}^2 \ll \overline{\mathbf{B}'^2}$
- Large scale dynamo: $\overline{\mathbf{B}}^2 \geq \overline{\mathbf{B}'^2}$

Almost all turbulent (chaotic) velocity fields are small scale dynamos for sufficiently large R_m, large scale dynamos require additional large scale symmetries (see second half of this lecture)

What means large/small in practice (Sun)?

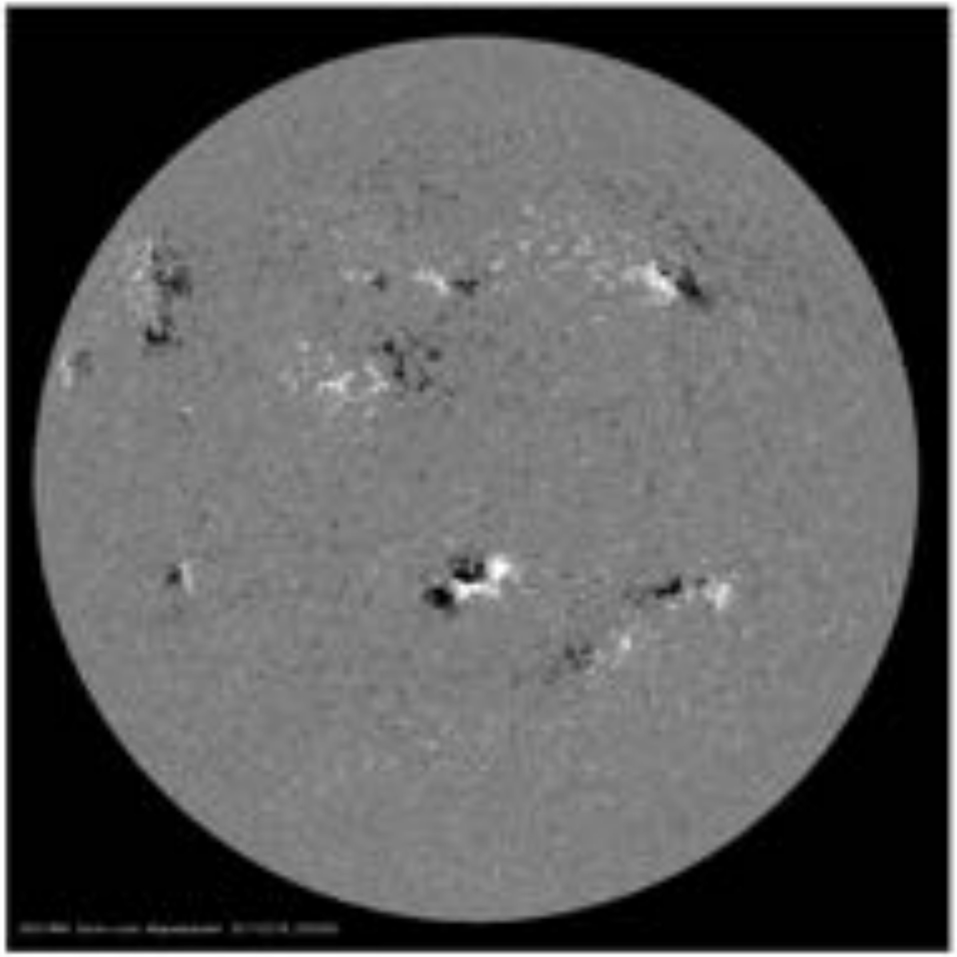
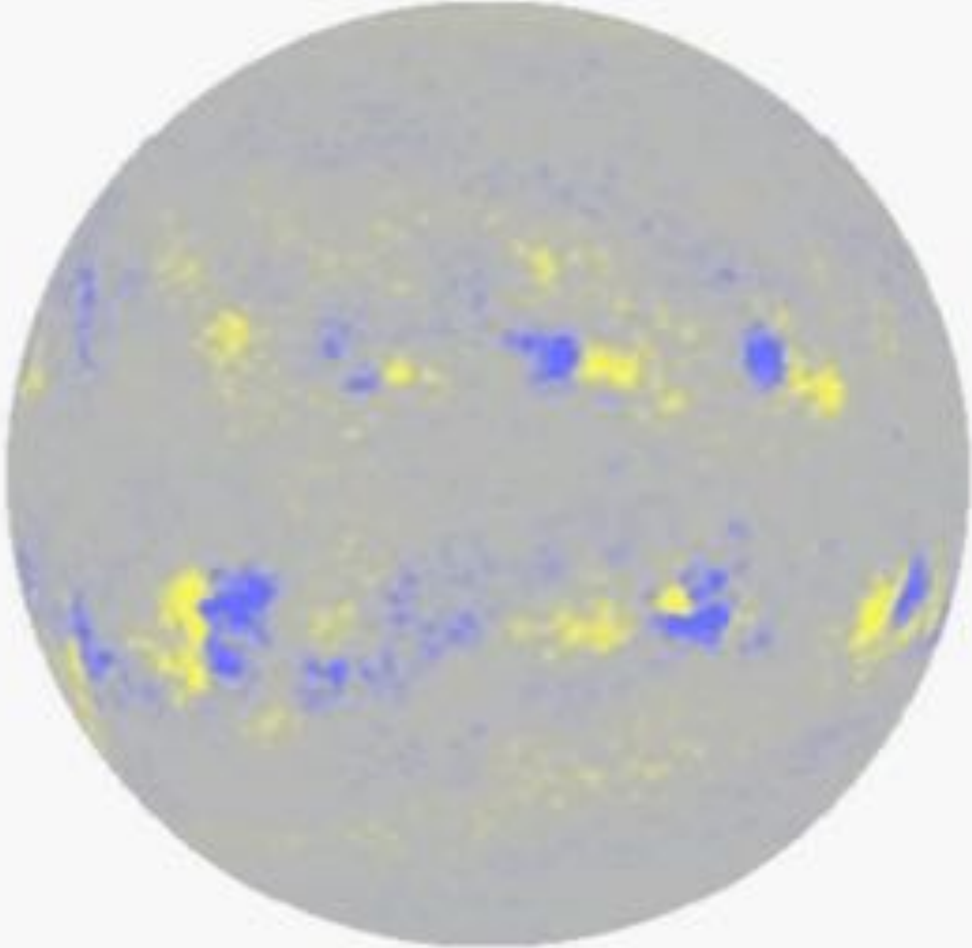
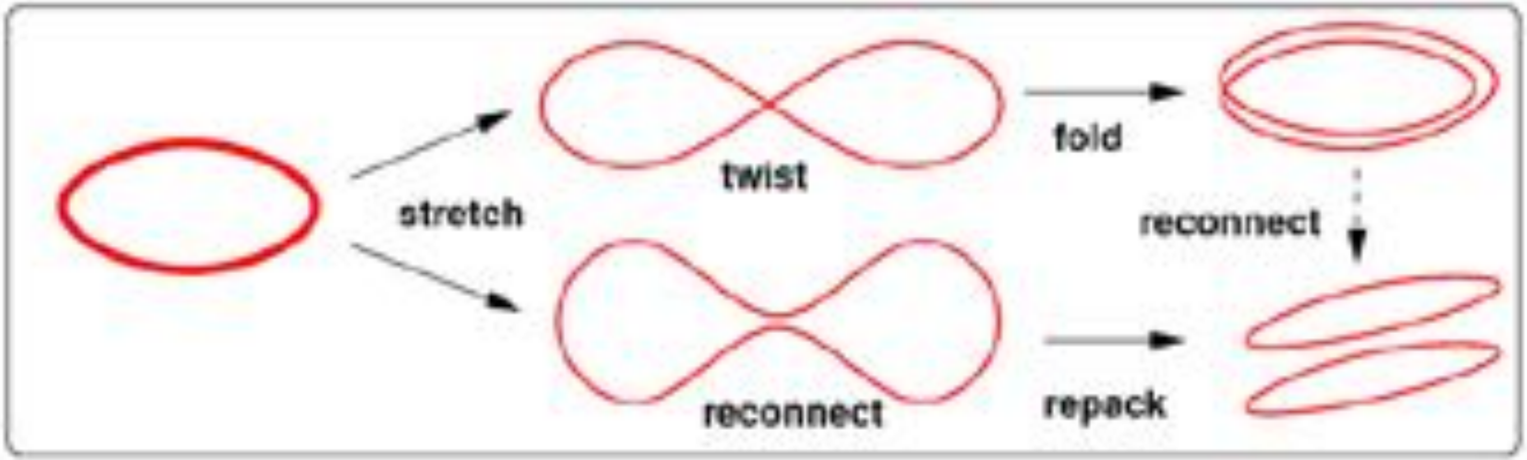


Figure: Full disk magnetogram SDO/HMI



Large scale/small scale dynamos



- Amplification through field line stretching
- Twist-fold required to repack field into original volume
- Magnetic diffusivity allows for change of topology

What is Magnetic Reconnection?

If a plasma is perfectly conducting, that is, it obeys the ideal Ohm's law,

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

B-lines are frozen in the plasma, and no reconnection occurs.

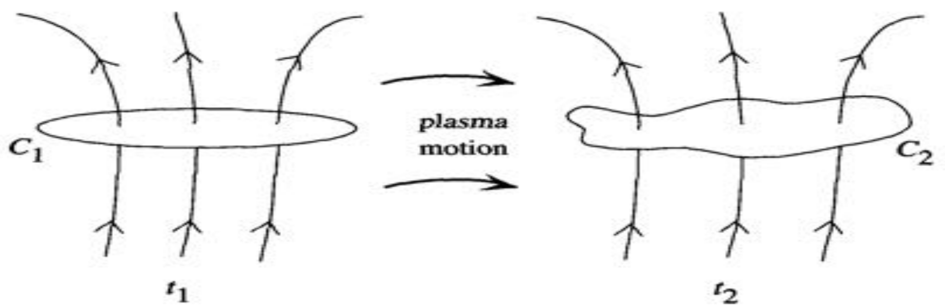


Fig. 1.6. Magnetic flux conservation: if a curve C_1 is distorted into C_2 by plasma motion, the flux through C_1 at t_1 equals the flux through C_2 at t_2.

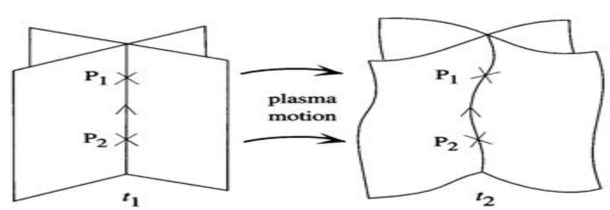


Fig. 1.7. Magnetic field-line conservation: if plasma elements P_1 and P_2 lie on a field line at time t_1, then they will lie on the same line at a later time t_2.

What is magnetic reconnection? (continued)

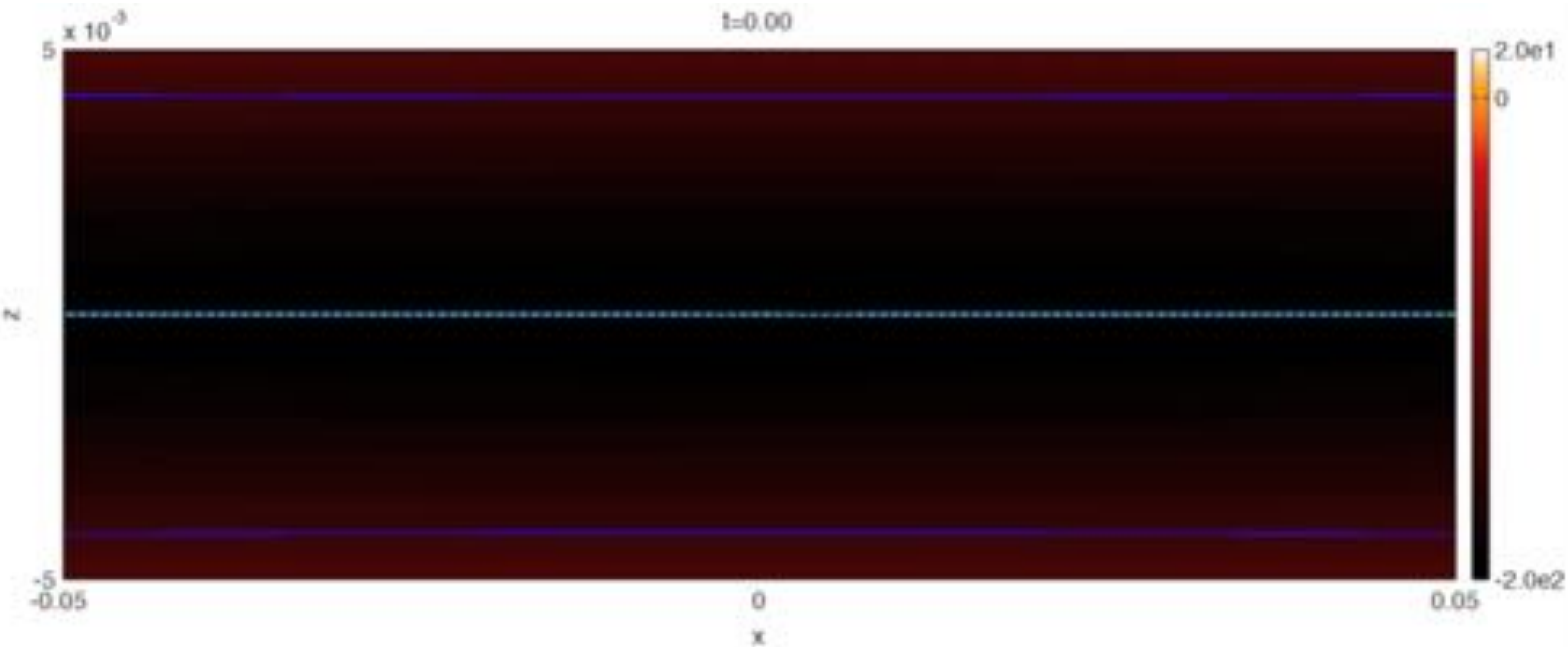
Departures from ideal behavior, represented by

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} / c = \mathbf{R}, \quad \nabla \times \mathbf{R} \neq \mathbf{0}$$

break ideal topological invariants, allowing field lines to reconnect.

In the generalized Ohm's law for weakly collisional or collisionless plasmas, **R** contains resistivity, Hall current, electron inertia and pressure.

The thin current sheet is explosively stable over a critical Lundquist number, forming, ejecting, and coalescing a hierarchy of plasmoids.



Courtesy: Y.-M. Huang

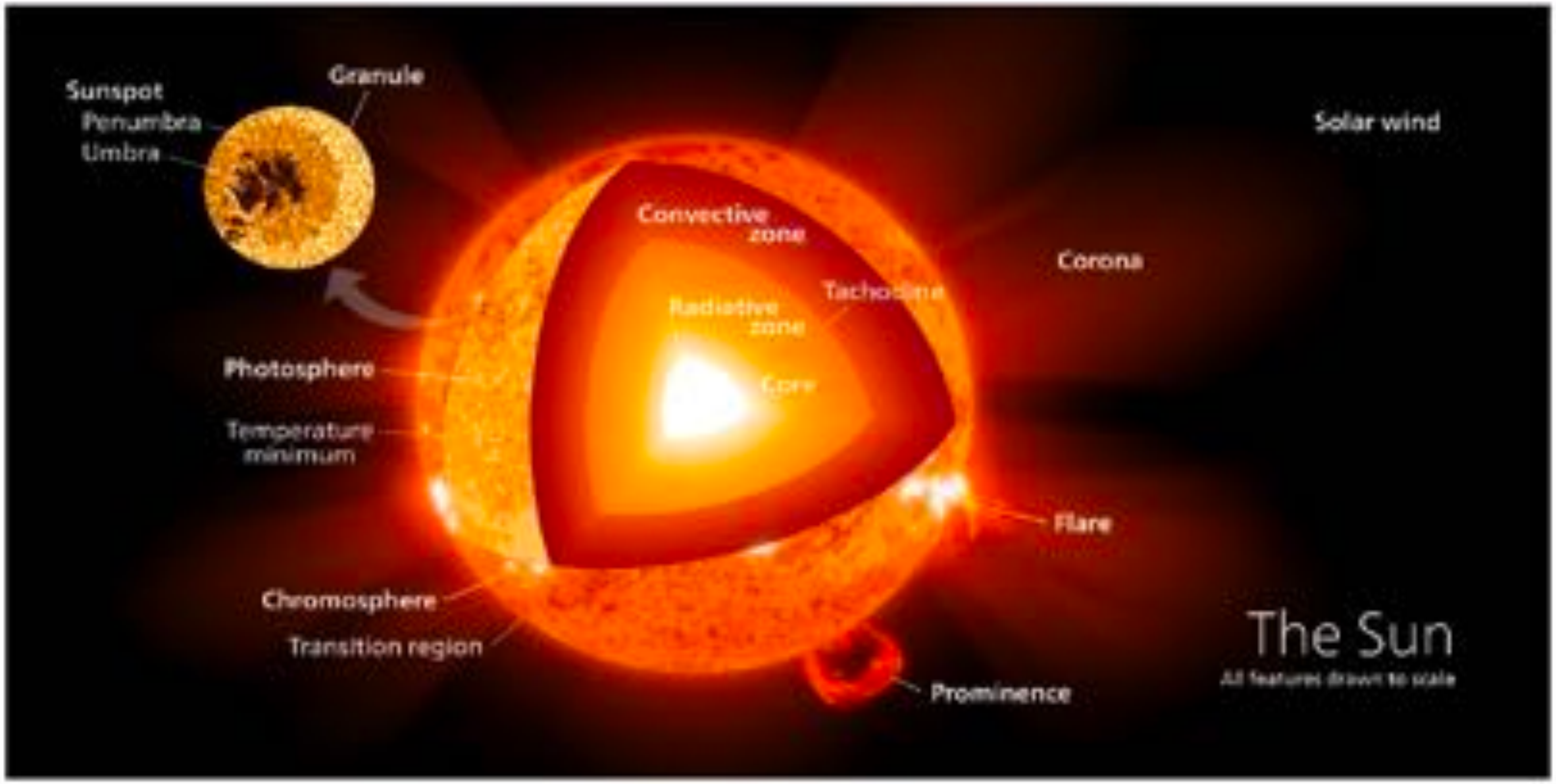
Slow/fast dynamos

Influence of magnetic diffusivity on growth rate

- Fast dynamo: growth rate independent of R_m (stretch-twist-fold mechanism)
- Slow dynamo: growth rate limited by resistivity (stretch-reconnect-repack)

- Fast dynamos relevant for most astrophysical objects since $R_m \gg 1$
- Dynamos including (resistive) reconnection steps can be fast provided the reconnection is fast

Solar structure ($\beta \gg 1$)



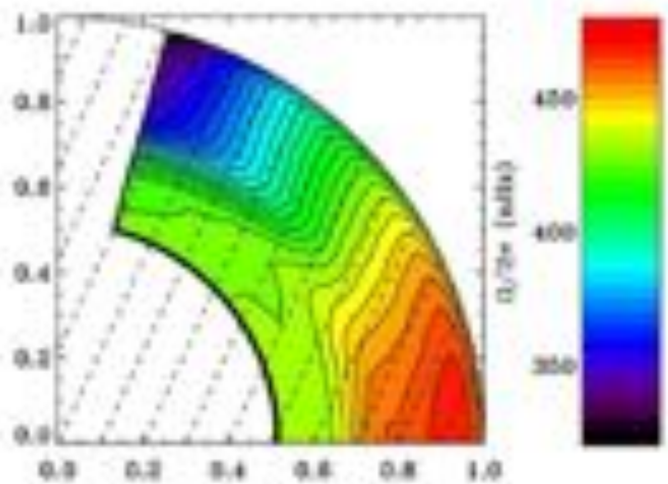
- Fusion dominated by proton-proton chain in core
- Inner radiative zone
- Tachocline: thin shear layer boundary
- Outer convective zone: vital for solar dynamo

Differential rotation and meridional flow

Induction effects of axisymmetric flows on axisymmetric field:

$$\mathbf{B} = B\mathbf{e}_\Phi + \nabla \times (A\mathbf{e}_\Phi)$$
$$\mathbf{v} = v_r\mathbf{e}_r + v_\theta\mathbf{e}_\theta + \Omega\, r \sin\theta\mathbf{e}_\Phi$$

Differential rotation most dominant shear flow in stellar convection zones:



Meridional flow by-product of DR, observed as poleward surface flow

Differential rotation and meridional flow

Spherical geometry:

$$\frac{\partial B}{\partial t} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rv_r B) + \frac{\partial}{\partial \theta}(v_\theta B)\right) =$$

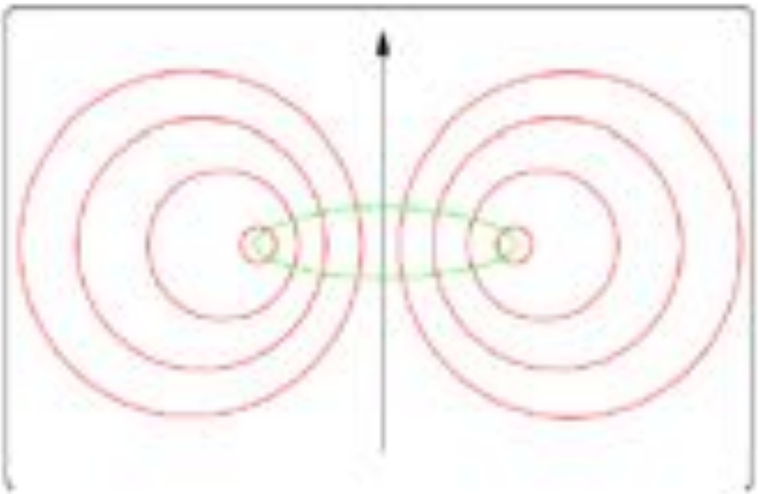
$$r \sin \mathbf{B}_p \cdot \nabla\Omega + \eta\left(\Delta - \frac{1}{(r\sin\theta)^2}\right) B$$

$$\frac{\partial A}{\partial t} + \frac{1}{r\sin\theta}\mathbf{v}_p \cdot \nabla(r\sin\theta A) = \eta\left(\Delta - \frac{1}{(r\sin\theta)^2}\right) A$$

- Meridional flow: Independent advection of poloidal and toroidal field
- Differential rotation: Source for toroidal field (if poloidal field not zero)
- Diffusion: Sink for poloidal and toroidal field
- No term capable of maintaining poloidal field against Ohmic decay!

Cowling's anti-dynamo theorem

A stationary axisymmetric magnetic field with currents limited to a finite volume in space cannot be maintained by a velocity field with finite amplitude.



Ohm's law of the form $\mathbf{j} = \sigma \mathbf{E}$ only decaying solutions, focus here on $\mathbf{j} = \sigma(\mathbf{v} \times \mathbf{B})$.
On O-type neutral line $\mathbf{B}_p$ is zero, but $\mu_0 \mathbf{j}_t = \nabla \times \mathbf{B}_p$ has finite value, but cannot be maintained by $(\mathbf{v} \times \mathbf{B})_t = (\mathbf{v}_p \times \mathbf{B}_p)$.

Large scale dynamo theory

Some history:

- 1919 Sir Joeseph Larmor: Solar magnetic field maintained by motions of conducting fluid?
- 1937 Cowling's anti-dynamo theorem and many others
- 1955 Parker: decomposition of field in axisymmetric and non-axisymmetric parts, average over induction effects of non-axisymmetric field
- 1964 Braginskii, Steenbeck, Krause: Mathematical frame work of mean field theory developed
- last 3 decades 3D dynamo simulations

Meanfield induction equation

Average of induction equation:

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(\overline{\mathbf{v}' \times \mathbf{B}'} + \overline{\mathbf{v}} \times \overline{\mathbf{B}} - \eta \nabla \times \overline{\mathbf{B}} \right)$$

New term resulting from small scale effects:

$$\overline{\mathcal{E}} = \overline{\mathbf{v}' \times \mathbf{B}'}$$

Fluctuating part of induction equation:

$$\left(\frac{\partial}{\partial t} - \eta \Delta \right) \mathbf{B}' - \nabla \times (\overline{\mathbf{v}} \times \mathbf{B}') = \nabla \times \left(\mathbf{v}' \times \overline{\mathbf{B}} + \mathbf{v}' \times \mathbf{B}' - \overline{\mathbf{v}' \times \mathbf{B}'} \right)$$

Kinematic approach: $\mathbf{v}'$ assumed to be given

- Solve for $\mathbf{B}'$, compute $\overline{\mathbf{v}' \times \mathbf{B}'}$ and solve for $\overline{\mathbf{B}}$
- Term $\mathbf{v}' \times \mathbf{B}' - \overline{\mathbf{v}' \times \mathbf{B}'}$ leading to higher order correlations (closure problem)

Mean field expansion of turbulent induction effects

Exact expressions for $\overline{\mathcal{E}}$ exist only under strong simplifying assumptions (see homework assignment).

In general $\overline{\mathcal{E}}$ is a linear functional of $\overline{\mathbf{B}}$:

$$\overline{\mathcal{E}}_i(\mathbf{x},t) = \int_{-\infty}^{\infty} d^3x' \int_{-\infty}^{t} dt' \, \mathcal{K}_{ij}(\mathbf{x},t,\mathbf{x}',t') \, \overline{B}_j(\mathbf{x}',t') \, .$$

Can be simplified if a sufficient scale separation is present:

- $l_c \ll L$
- $\tau_c \ll \tau_L$

Leading terms of expansion:

$$\overline{\mathcal{E}}_i = a_{ij}\overline{B}_j + b_{ijk}\frac{\partial \overline{B}_j}{\partial x_k}$$

Symmetry constraints

Decomposing a_{ij} and $\partial \overline{B}_j / \partial x_k$ into symmetric and antisymmetric components:

$$
\begin{aligned}
a_{ij} &= \underbrace{\frac{1}{2}(a_{ij} + a_{ji})}_{\alpha_{ij}} + \underbrace{\frac{1}{2}(a_{ij} - a_{ji})}_{-\varepsilon_{ijk}\gamma_k} \\
\frac{\partial \overline{B}_j}{\partial x_k} &= \frac{1}{2}\left(\frac{\partial \overline{B}_j}{\partial x_k} + \frac{\partial \overline{B}_k}{\partial x_j}\right) + \underbrace{\frac{1}{2}\left(\frac{\partial \overline{B}_j}{\partial x_k} - \frac{\partial \overline{B}_k}{\partial x_j}\right)}_{-\frac{1}{2}\varepsilon_{jkl}(\nabla \times \overline{\mathbf{B}})_l}
\end{aligned}
$$

Leads to:

$$
\overline{\mathcal{E}}_i = \alpha_{ik}\overline{B}_k + \varepsilon_{ijk}\gamma_j\overline{B}_k - \underbrace{\frac{1}{2}b_{ijk}\varepsilon_{jkl}}_{\beta_{il} - \varepsilon_{ilm}\delta_m}(\nabla \times \overline{\mathbf{B}})_l + \ldots
$$

Symmetry constraints

Overall result:

$$\overline{\mathcal{E}} = \alpha\overline{\mathbf{B}} + \gamma \times \overline{\mathbf{B}} - \beta \nabla \times \overline{\mathbf{B}} - \delta \times (\nabla \times \overline{\mathbf{B}}) + \dots$$

With:

$$\alpha_{ij} = \frac{1}{2}(a_{ij} + a_{ji}) , \qquad \gamma_i = -\frac{1}{2}\varepsilon_{ijk} a_{jk}$$

$$\beta_{ij} = \frac{1}{4}(\varepsilon_{ikl} b_{jkl} + \varepsilon_{jkl} b_{ikl}) , \qquad \delta_i = \frac{1}{4}(b_{jji} - b_{jij})$$

Simplified expressions

Assuming $|\mathbf{B}'| \ll |\overline{\mathbf{B}}|$ in derivation + additional simplification for (quasi) isotropic, non-mirror symmetric, (weakly) inhomogeneous turbulence:

$$\overline{v_i' v_j'} \sim \delta_{ij},\ \alpha_{ij} = \alpha\delta_{ij},\ \beta_{ij} = \eta_t\delta_{ij}$$

Leads to:

$$\frac{\partial\overline{\mathbf{B}}}{\partial t} = \nabla \times \left[\alpha\overline{\mathbf{B}} + (\overline{\mathbf{v}} + \gamma) \times \overline{\mathbf{B}} - (\eta + \eta_t)\nabla \times \overline{\mathbf{B}}\right]$$

with the scalar quantities

$$\alpha = -\frac{1}{3}\tau_c\overline{\mathbf{v}' \cdot (\nabla \times \mathbf{v}')}, \quad \eta_t = \frac{1}{3}\tau_c\overline{\mathbf{v}'^2}$$

and vector

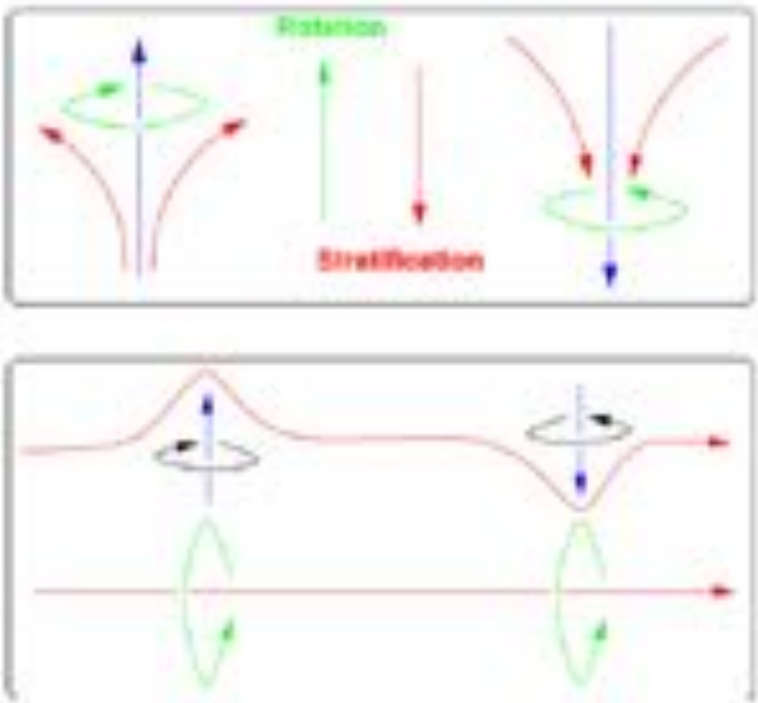
$$\gamma = -\frac{1}{6}\tau_c\nabla\overline{\mathbf{v}'^2} = -\frac{1}{2}\nabla\eta_t$$

Expressions are independent of η (in this approximation): fast dynamo

Kinematic α-effect

$$\alpha = -\frac{1}{3}\tau_c \overline{\mathbf{v}' \cdot (\nabla \times \mathbf{v}')} \quad H_k = \overline{\mathbf{v}' \cdot (\nabla \times \mathbf{v}')} \text{ kinetic helicity}$$

Requires rotation + additional preferred direction (stratification)



$\alpha\Omega$-dynamo

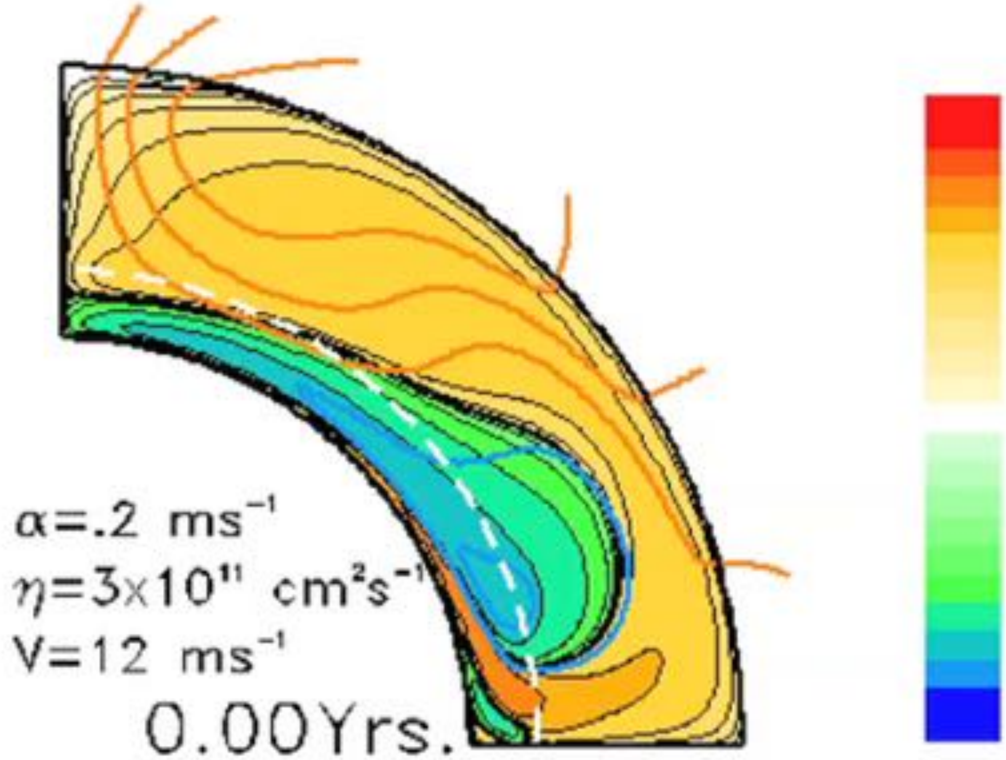
$$
\begin{aligned}
\frac{\partial B}{\partial t} &+ \frac{1}{r}\left(\frac{\partial}{\partial r}(r v_r B) + \frac{\partial}{\partial \theta}(v_\theta B)\right) = r \sin \mathbf{B}_p \cdot \nabla \Omega \\
&+ \eta\left(\Delta - \frac{1}{(r\sin\theta)^2}\right) B \\
\frac{\partial A}{\partial t} &+ \frac{1}{r\sin\theta}\mathbf{v}_p \cdot \nabla (r\sin\theta A) = \alpha B + \eta\left(\Delta - \frac{1}{(r\sin\theta)^2}\right) A
\end{aligned}
$$

- Dimensionless measure for strength of Ω- and α-effect

$$D_\Omega = \frac{R^2 \Delta\Omega}{\eta_t} \qquad D_\alpha = \frac{R\alpha}{\eta_t}$$

- Dynamo excited if dynamo number

$$D = D_\Omega D_\alpha > D_{crit}$$



α=.2 ms⁻¹
η=3x10¹¹ cm²s⁻¹
V=12 ms⁻¹
0.00Yrs.

Non-kinematic effects

Proper way to treat them: 3D simulations

- Still very challenging
- Has been successful for geodynamo, but not for solar dynamo

Semi-analytical treatment of Lorentz-force feedback in mean field models:

- Macroscopic feedback: Change of the mean flow (differential rotation, meridional flow) through the mean Lorentz-force

$$\overline{\mathbf{f}} = \overline{\mathbf{j}} \times \overline{\mathbf{B}} + \overline{\mathbf{j}' \times \mathbf{B}'}$$

- Mean field model including mean field representation of full MHD equations
- Microscopic feedback: Change of turbulent induction effects (e.g. α-quenching)

Microscopic feedback

Symmetry of momentum and induction equation $\mathbf{v}' \leftrightarrow \mathbf{B}'/\sqrt{\mu_0\varrho}$:

$$
\begin{aligned}
\frac{d\mathbf{v}'}{dt} &= \frac{1}{\mu_0\varrho}(\overline{\mathbf{B}}\cdot\nabla)\mathbf{B}' + \ldots \\
\frac{d\mathbf{B}'}{dt} &= (\overline{\mathbf{B}}\cdot\nabla)\mathbf{v}' + \ldots \\
\overline{\boldsymbol{\mathcal{E}}} &= \overline{\mathbf{v}'\times\mathbf{B}'}
\end{aligned}
$$

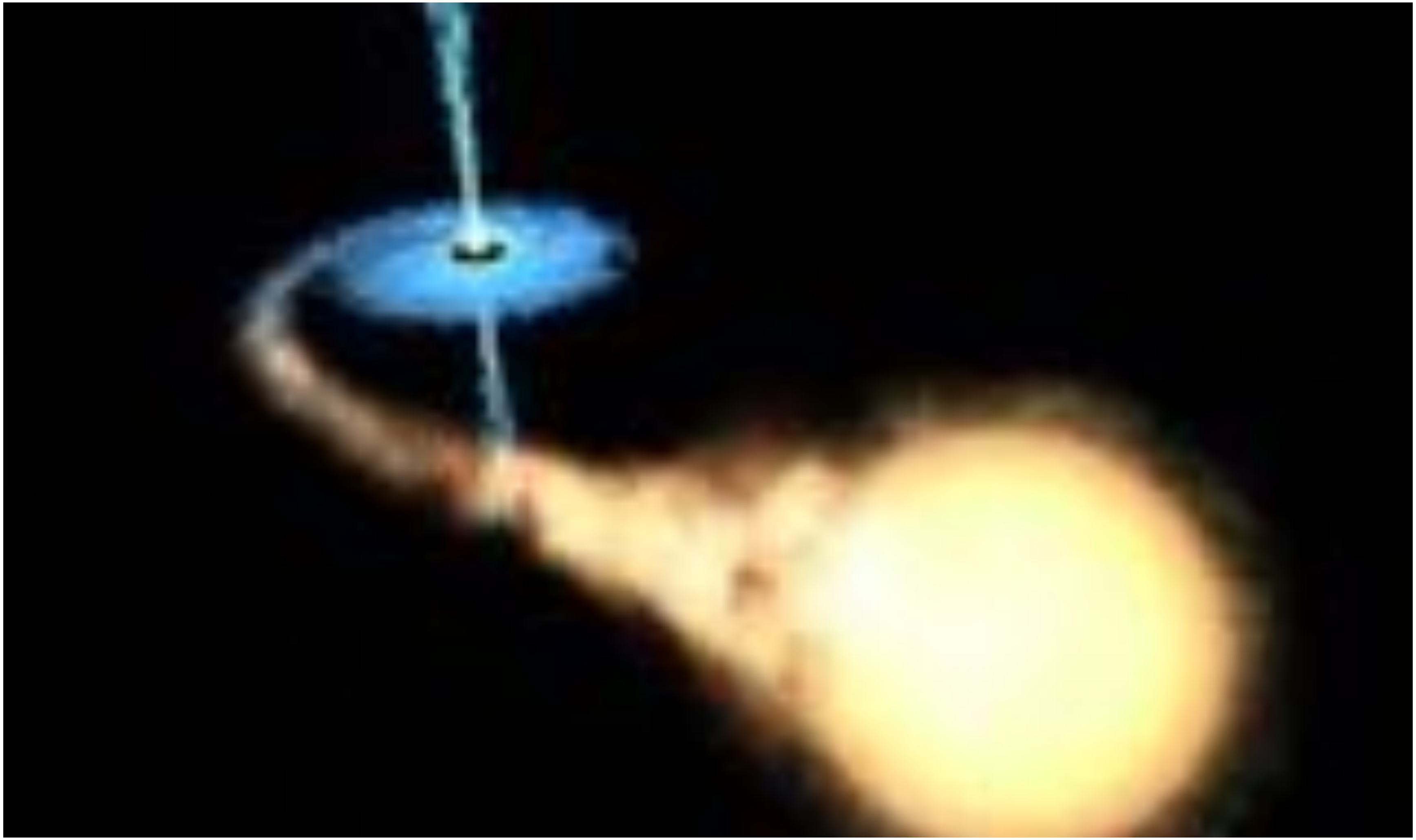
Strongly motivates magnetic term for α-effect (Pouquet et al. 1976):

$$
\alpha = \frac{1}{3}\tau_c\left(\frac{1}{\varrho}\overline{\mathbf{j}'\cdot\mathbf{B}'} - \overline{\boldsymbol{\omega}'\cdot\mathbf{v}'}\right)
$$

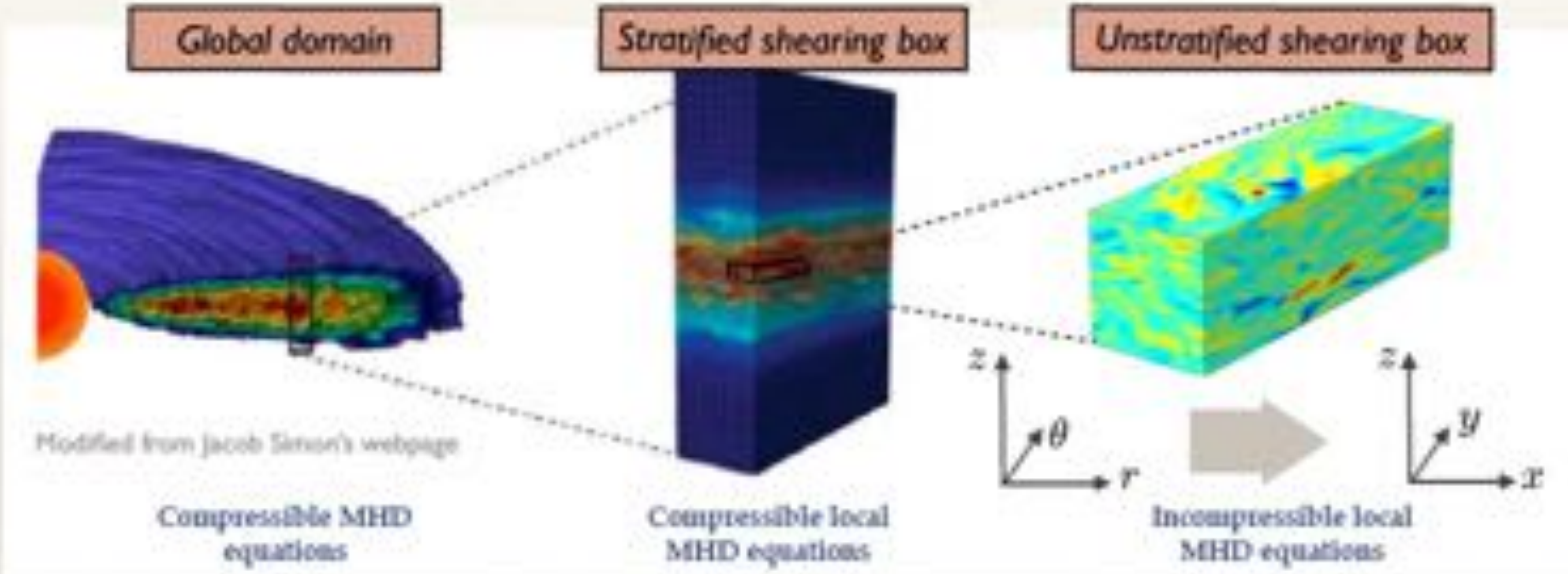
Challenges to Kinematic Mean-Field Dynamo Theory

- Smallest scales grows most rapidly (*Kulsrud and Anderson 1972, Boldyrev et al. 2005*)
- Due to constraints of magnetic helicity conservation, small-scale fields act back to decrease the large-scale field growth drastically---the problem of "catastrophic quenching" (*Gruzinov and Diamond 1994, Cattaneo and Hughes 2009*). But this challenge could be addressed by transporting helicity (*Blackman and Field 2002, Subramanian and Brandenburg 2004, Ebrahimi and B. 2014, Tobias and Cattaneo 2014*)
- At even moderate Rm, the fast-growing small-scale dynamo implies that velocity fluctuations should always be accompanied by magnetic field fluctuations of a similar magnitude (*Schekochihin et al. 2004*), questioning the relevance of the classical kinematic theory.

Accretion Disks near a Black Hole

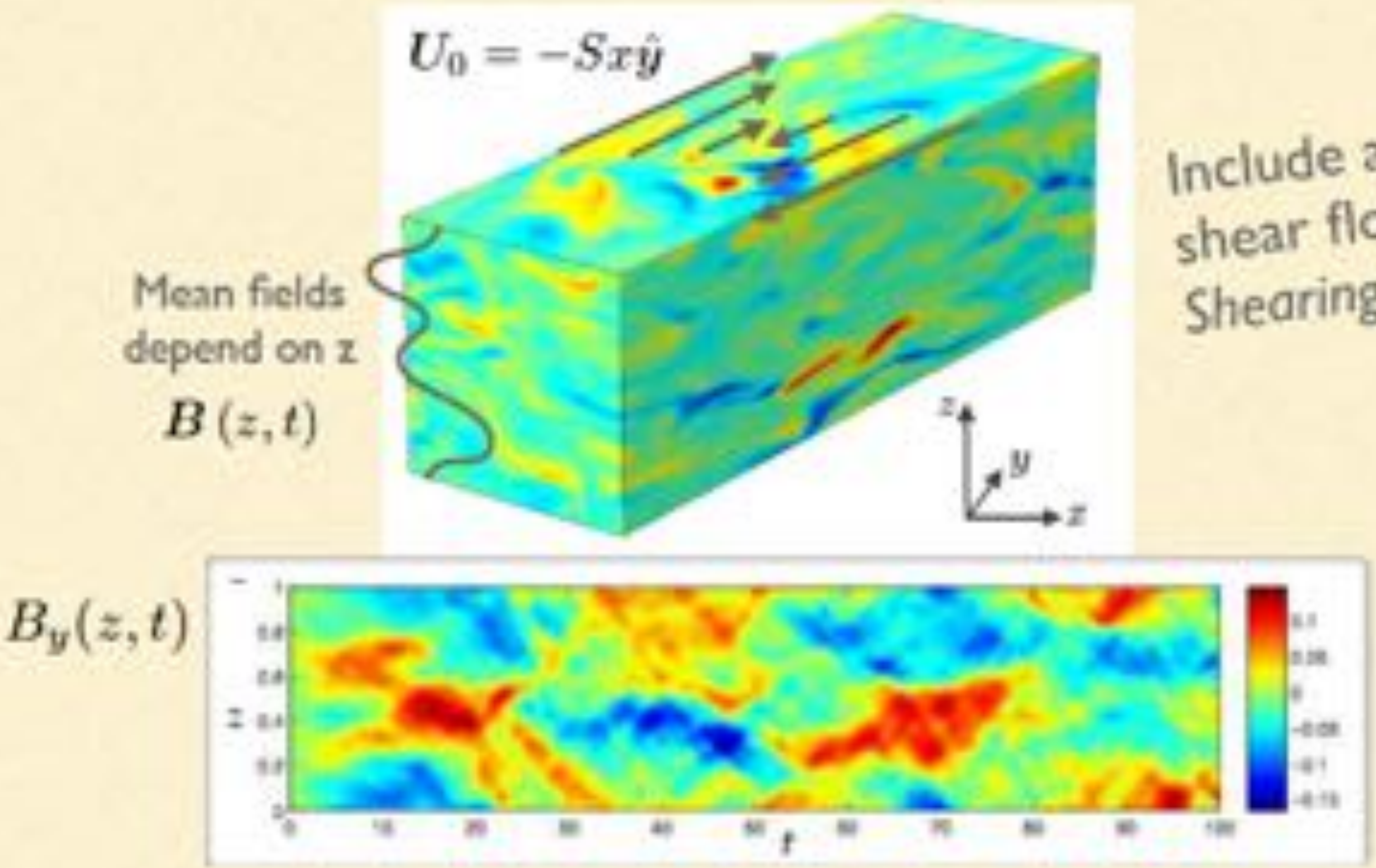


Preliminary – local MRI



- The simplest relevant system exhibiting MRI turbulence is the local incompressible MHD equations — remove global curvature
- In the *shearing box*, boundary conditions are periodic in y (azimuthal) and z (vertical), and shearing periodic in x (radial).

- We use the horizontal average for the mean-field average.



- Study the dynamo by studying $\mathcal{E}(B,U)$.

PRIMARY RESULT

New dynamo mechanism
— *the magnetic shear-current effect* —
small-scale magnetic fields have a *positive* effect on the large-scale dynamo.

Effect requires velocity shear (e.g., Keplerian).

No α effect required.

Off-diagonal component of β couples with the shear.

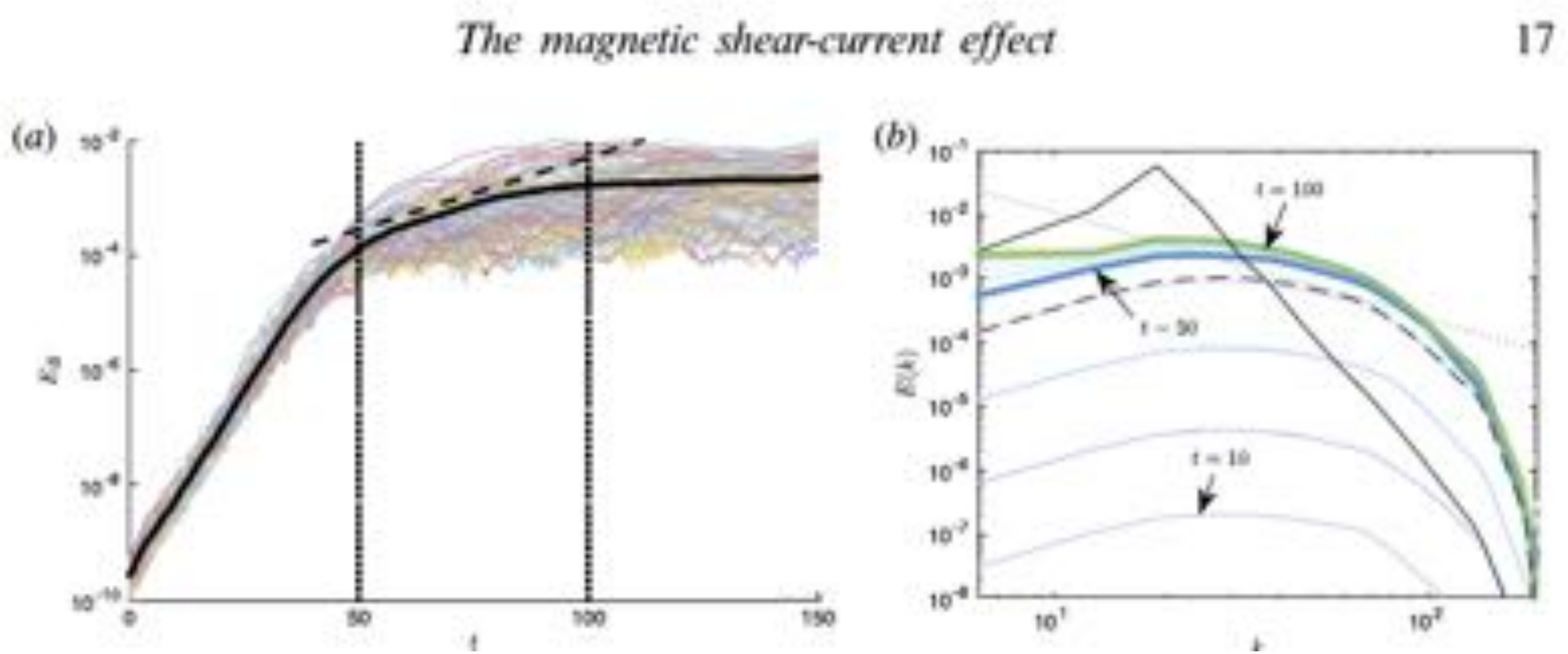
Squire & Bhattacharjee PRL (2015) *Squire & Bhattacharjee ApJ (2015)* *Squire & Bhattacharjee PRE (2015)*

2017 APS M. N. Rosenbluth Dissertation Award

Jonathan Squire

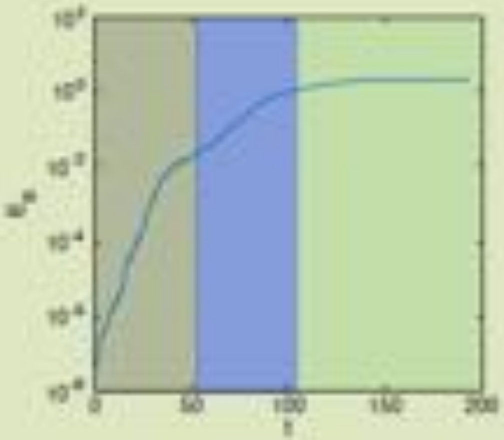
Sherman Fairchild Postdoctoral Scholar, Caltech (PhD, Princeton, 2016)

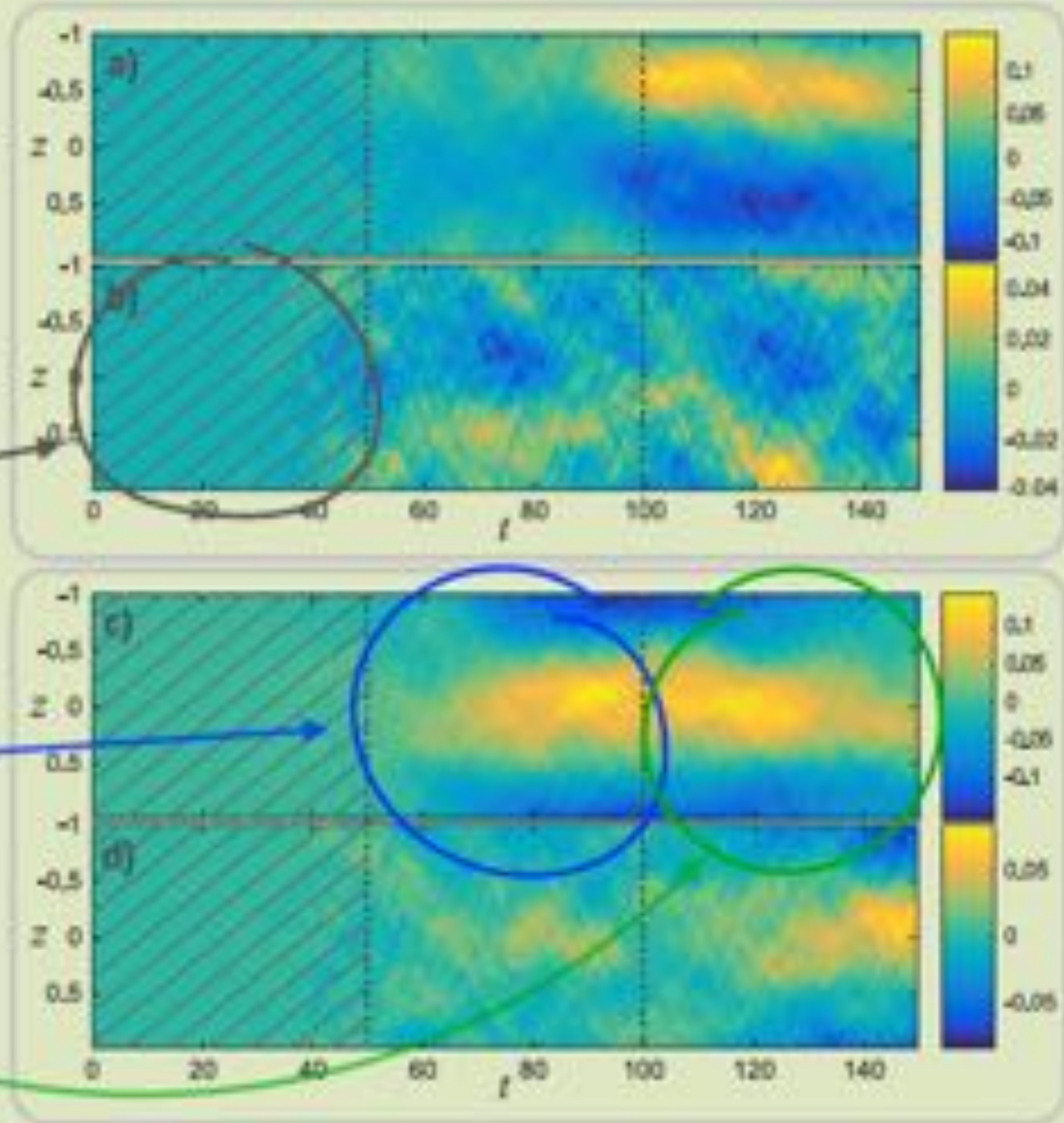
"For fundamental contributions to dynamo theory, and in particular for analytical and computational elucidation of the magnetic shear current effect"



Squire and Bhattacharjee 2016

Rm=2000 — 100 realizations





$\Omega = 0$

Fast growth of small-scale dynamo, saturates t≈40.

Large-scale dynamo driven by small-scale *b* fluctuations?

Large-scale dynamo saturates — change in η?

Keplerian

- Magnetic shear-current effect is like *inverse* quenching — small-scale dynamo can *drive* a large-scale dynamo.

Agreement between simulation and analytic results.

- Good evidence that magnetic shear-current effect is responsible for unstratified MRI dynamo (Shi, Stone, and Huang 2016)

Shear flows being ubiquitous, is the magnetic shear-current effect important for the Sun? (Hotta, Rempel, and Yokoyama, 2016)