Stellar Dynamos in 90 minutes or less!

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(with contributions and inspiration from Mark Miesch)
Why are we here?

- Stellar dynamos: Outstanding questions
- A closer look at Solar Magnetism
- Dynamo theory in the solar context
- The big question: convective flow speeds
- Today: emphasize induction
- Tomorrow: more on convection
Question

What even is a dynamo?
Dynamo

The process by which a magnetic field is sustained against decay through the motion of a conducting fluid.

Stellar Dynamo

The process by which a star maintains its magnetic field via the convective motion of its interior plasma.
The Stars

Luminosity (solar units)

The Sun

Bennett, et al. 2003
One Broad Distinction

Luminosity (solar units)

10^4

10^2

10

1

10^-2

10^-4

30,000 K 10,000 K 6,000 K 3,000 K

Low-Mass Stars

High-Mass Stars

10^7 yrs

10^8 yrs

10^10 yrs

Bennett, et al. 2003
As far as dynamo theory is concerned, what do you think is the main difference between massive and low-mass stars?
Massive vs. Low-Mass Stars

The key difference is...
...mass, ...luminosity, ...size ... geometry!

massive stars

low-mass stars

Different magnetic field configurations arise in different geometries.
Massive vs. Low-Mass Stars

Fundamental questions similar, but some differences.

**massive stars**
- radiative envelope
- convective core

**low-mass stars**
- radiative interior
- convective envelope

Different magnetic field configurations arise in different geometries.
Stellar Dynamo Question: The Sun’s Magnetic Cycle

- Magnetic activity increases with integrated sunspot area
- Mean magnetic polarity reverses every 11 years

How do stars generate a magnetic field? What causes the cyclic behavior?
Big Question: Magnetism and Stellar Age?

“Young Sun” simulation (3Ω_{sun})

- Stars rotate more slowly as they age.
- Rapid rotation favors large-scale magnetic fields.
- Is a tachocline necessary for organized field growth?

Brown et al. 2010
Big Question: Is the Sun “normal”?

Survey of solar analogues

Metcalf et al. 2016
Bohm-Vitense (2007)
Big Question: Massive Star Dynamos?

- How is the magnetic field structured?
- Are core-generated fields visible at the surface?
- Can core-generated fields become buoyant?

Featherstone et al. 2010 (A stars)
see also Augustson et al. 2016 (B stars)
**Peculiar Question: Ap and Bp Stars**

- Abnormally strong abundances of Si and various rare earth metals (Hg, etc.)

- Magnetic: typical field strengths of a few hundred Gauss.

- Field strengths and line widths vary periodically: **Oblique Rotator Model**

- Rotation periods from days to decades (magnetic braking?)

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**Source of Magnetic Field?**

- Core-dynamo? (but diffusion time through radiative zone very long)

- Primordial magnetic field?
Magnetic Field

Abundance Maps

Magnetic Doppler Imaging

Ap Star HR3831

Kochukhov et al. 2004
Big Question:
Exoplanets and Host-Star Magnetism

TRAPPIST-1 System

- Exoplanets most easily found around low-mass M dwarfs
- Vigorous dynamo action + fierce flaring
- How does magnetism impact habitable zone?

Illustration
Credit: NASA/JPL-Caltech
Big Question: Relationship between Magnetism and Stellar Rotation Rate?

- Solar-like stars shown
- General trend holds for all low-mass stars
- What causes saturation?
At the end of the day...

- The Sun is a star
- Stellar magnetism is the superset of solar magnetism
- We study stellar dynamos indirectly via the Sun as much as we do directly via observations
- Let’s turn to the Sun, which we can RESOLVE!
The Sun
(9/7/2016)

visible light
617 nm

sunspots

HMI/SDO

HMI/SDO
Detailed sunspot records from 1600s onward (Galileo Galilei, 1612)

Naked-eye observations from China since 23 BCE
Detailed sunspot records from 1600s onward (Galileo Galilei, 1612)

Naked-eye observations from China since 23 BCE
Sunspots: A Closer Look

Swedish Solar Telescope (visible; 430 nm)

photospheric convection “granulation”

Magnetic fields trap gas.

~1 kG

T = 5,800 K

Magnetic fields of sunspots suppress convection and prevent surrounding plasma from sliding sideways into sunspot.

Cosmic Perspective, 3rd Ed.
The Sun’s Magnetic Cycle

- Magnetic activity increases with integrated sunspot area
- Mean magnetic polarity reverses every 11 years
The Sun’s Magnetic Cycle

- Magnetic activity increases with integrated sunspot area
- Mean magnetic polarity reverses every 11 years

Where does all this magnetism come from?

What makes it so ordered in space and time?

Disk-Integrated Sunspot Area

Equatorward Migration
Solar Dynamo: The Big Questions

Where does solar magnetism originate?
Why is there a solar cycle?

A deep origin for solar magnetism?

Sunspot clusters (active regions)

Line-of-sight magnetic Field (HMI)

large-scale organization
What Lies Beneath: Solar Helioseismology

Dopplergram (radial velocity)

Resonant Acoustic Modes

MDI Medium-\( \ell \) Power Spectrum

Spherical Harmonic Degree \( \ell \)
Helioseismology Key Result: Differential Rotation

- Sun rotates differentially
- 24-day period equator
- 30-day period poles
- Latitudinal Shear
- Radial Shear

North Pole (-240 m/s)

Equator (370 m/s)

Howe et al. 2000; Schou et al. 2002
Helioseismology Key Result: Meridional Circulation

Time Distance

Zhao et al. 2012

Ring Diagrams

Greer et al. 2013

Shallow or Deep reversal? Latitudinal variation?

20 m s$^{-1}$ poleward
Helioseismology Key Result: Solar Structure
Fusion Core

Central Temperature: 15.7 million K
Central Density: 154 g cm$^{-3}$
Expanse: 0 – 0.25 $R_{\text{sun}}$
Radiative Zone

- Convectively **Stable**
- Energy Transport: photon scattering
- Random walk time: 100,000 years

**Temperature:** 2.3 million K
**Density:** 0.2 g cm\(^{-3}\)
**Radius:** 0.7 \(R_{\text{sun}}\)

**Temperature:** 8 million K
**Density:** 20 g cm\(^{-3}\)
**Radius:** 0.25 \(R_{\text{sun}}\)
Convection Zone

Temperature: 5800K
Density: 2x10^{-7} \text{ g cm}^{-3}
Radius: 1 R_{\text{sun}}

Temperature: 2.3 million K
Density: 0.2 \text{ g cm}^{-3}
Radius: 0.7 R_{\text{sun}}

- Convectively unstable
- Convective timescale: months, years?

region of extreme contrasts...
... ionized plasma!
Temperature: 14,400K
Density: \(2 \times 10^{-6} \text{ g cm}^{-3}\)
Radius: \(0.9985 \text{ R}_{\text{sun}}\)

Temperature: 5,800K
Density: \(2 \times 10^{-7} \text{ g cm}^{-3}\)
Radius: \(1 \text{ R}_{\text{sun}}\)

Note:
The convection that we can see occurs in a region thinner than the width of this line (about 1,000 km in depth)

Strong variation with depth
Convection Zone Bulk

Temperature: 14,400K
Density: 2x10^{-6} g cm^{-3}

Temperature: 2.3 million K
Density: 0.2 g cm^{-3}

- 11 density scaleheights
- 17 pressure scaleheights
- Reynolds Number \( \approx 10^{12} - 10^{14} \)
- Rayleigh Number \( \approx 10^{22} - 10^{24} \)
- Magnetic Prandtl Number \( \approx 0.01 \)
- Prandtl Number \( \approx 10^{-7} \)
- Ekman Number \( \approx 10^{-15} \)
Question

Where does the energy in the solar magnetic field come from?
The **Solar Dynamo** generates magnetic fields from flows

- convection
- differential rotation
- meridional circulation

Magnetic energy ultimately comes from the Sun’s own mass

**Fusion**

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \eta \nabla^2 B
\]

Mass ⇒ radiation & thermal energy

**Convection**

Thermal energy ⇒ kinetic energy

**Dynamo**

Kinetic energy ⇒ magnetic energy
More on convection tomorrow.

Today: Induction!
MHD Magnetic Induction equation

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \]

Comes from Maxwell’s equations (Faraday’s Law and Ampere’s Law)

\[ \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \]

\[ \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} \]

And Ohm’s Law

\[ \mathbf{J} = \sigma \mathbf{E} \]

Magnetic diffusivity

\[ \eta = \frac{c^2}{4\pi \sigma} \]

electrical conductivity
Creation and destruction of magnetic fields

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B - \eta \nabla \times B)
\]

How would you demonstrate this?

(Hint: have a sheet handy with lots of vector identities!)

\[
E_m = \frac{B^2}{8\pi}
\]
Creation and destruction of magnetic fields

\[
\frac{\partial B}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})
\]

Dot with \( \mathbf{B} \):

\[
E_m = \frac{B^2}{8\pi}
\]

Source of Magnetic Energy

\[
\frac{\partial E_m}{\partial t} = -\nabla \cdot \mathbf{F}_P - \frac{\mathbf{v}}{c} \cdot (\mathbf{J} \times \mathbf{B}) - \Phi_o
\]

Sink of Magnetic Energy

Poynting Flux

\[
\mathbf{F}_P = \mathbf{E} \times \mathbf{B} = \left[ \frac{\eta}{c} \mathbf{J} - \frac{1}{4\pi} (\mathbf{v} \times \mathbf{B}) \right] \times \mathbf{B}
\]

Ohmic Heating

\[
\Phi_o = \frac{4\pi\eta}{c^2} J^2
\]
Creation and destruction of magnetic fields

\[ \frac{\partial B}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \]

Source of Magnetic Energy
\[ \sim UB / D \]
Sink of Magnetic Energy
\[ \sim \eta B / D^2 \]

\[ R_m = \frac{UD}{\eta} \]

If \( R_m >> 1 \) the source term is much bigger than the sink term

\[ \text{ratio of source and sink terms} \]

\[ \text{...Or is it???)} \]
Creation and destruction of magnetic fields

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})
\]

- **Source of Magnetic Energy**
  \[ \sim U B / D \]

- **Sink of Magnetic Energy**
  \[ \sim \eta B / \delta \]

\( \delta \) can get so small that the two terms are comparable

It’s not obvious which term will “win” - it depends on the subtleties of the flow, including geometry & boundary conditions.
Creation and destruction of magnetic fields

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B - \eta \nabla \times B)
\]

Source of Magnetic Energy
\[\sim U B / D\]
Sink of Magnetic Energy
\[\sim \eta B / \delta\]

What is a Dynamo? (A corollary)

A dynamo must sustain the magnetic energy (through the conversion of kinetic energy) against Ohmic dissipation.

How exactly does this work? It depends on the nature of the flow...
Large Scale Dynamos: The Mean Induction Equation

Go back to our basic induction equation

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}
\]

Now just average over longitude and rearrange a bit
(other averages are possible but we’ll stick to this for simplicity)

The equation for the mean field comes out to be

\[
\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \lambda \mathbf{B}_p \cdot \nabla \hat{\phi} + \nabla \times (\langle \mathbf{v}_m \times \mathbf{B} \rangle) + \eta \nabla^2 \langle \mathbf{B} \rangle + \nabla \times \mathbf{\mathcal{E}}
\]

\[
\lambda = r \sin \theta
\]

- **Ω-effect** (large-scale shear)
- **Meridional circulation** (transport)
- **Diffusion** (molecular)
- **Fluctuating emf** (small-scale shear)

No assumptions made up to this point beyond the basic MHD induction equation

Straightforward to show that if \( E=0 \), the dynamo dies (Cowling’s theorem)

Note:
The B field in the Sun is clearly not axisymmetric. Still, the solar cycle does have an axisymmetric component so that’s a good place to start
The MHD Induction Equation: Alternate View

\[
\frac{\partial B}{\partial t} = -B \nabla \cdot \mathbf{v} + B \cdot \nabla \mathbf{v} - \mathbf{v} \cdot \nabla B - \nabla \times (\eta \nabla \times B)
\]

- compression
- advection
- diffusion
- shear production
- Rotation

Large-Scale Shear (differential rotation)
Small-Scale Shear (helical rolls)

Convection Simulations

Non-rotating: red upflow
Rapidly-rotating: blue downflow

Rotating convection naturally generates both small-scale and large-scale shear!
The $\Omega$-effect (large-scale shear)

Converts \textit{poloidal} to \textit{toroidal} field and amplifies it

...by tapping the kinetic energy of the \textit{differential rotation}

$\nabla \nu \phi$

J. Werne
The $\Omega$-effect (large-scale shear via helical convection)

- 24-day period equator
- 30-day period poles

Mean shear:
- Latitudinal (Omega effect)
- Radial (Tachocline; Interface Dynamo)

Converts *poloidal* to *toroidal* field and amplifies it

...by tapping the kinetic energy of the *differential rotation*
The turbulent $\alpha$-effect (small-scale shear via helical convection)

Helical motions (lift, twist) can induce an emf that is parallel to the mean field

$$\mathbf{E} = \mathbf{v}' \times \mathbf{B}' = \alpha \mathbf{B}$$

This creates mean **poloidal** $(r, \theta)$ field from **toroidal** $(\phi)$ field

which closes the **Dynamo Loop**

**Mean Toroidal Field** (east-west)

**Mean Poloidal Field** $(r, \theta$ plane) $(\text{north/south, up/down})$

Linked to kinetic, magnetic helicity

Linked to large-scale dynamo action

Illustrates the 3D nature of dynamos

Moffat (1978)
Rotation Yields Helical Convection

Non-rotating or Fast Convection

Rapidly-rotating or Slow Convection

"alpha-effect"

Olson et al., 1999, JGR
Helical Convection (Deep Shells)

Northern Hemisphere

Tilted orbits

Streamlines
Helical Convection: Solar-like Simulations

Radial Velocity
Upper Convection Zone

red upflows
blue downflows
Another Way: Starts with Magnetic Buoyancy

If \( T^{(tube)} \approx T^{(ext)} \)

\[
\begin{align*}
P^{(tube)} + P_m^{(tube)} & \approx P^{(ext)} \\
P^{(tube)} & \approx P^{(ext)} - P_m^{(tube)} < P^{(ext)} \\
P & = \mathcal{R} \rho T \\
P_m & = \frac{B^2}{8\pi} \\
\rho^{(tube)} & < \rho^{(ext)}
\end{align*}
\]
The Babcock-Leighton Mechanism

Trailing member of the spot pair is displaced poleward relative to leading edge by the Coriolis force (Joy’s law: the higher the latitude, the more the tilt)

Polarity of trailing spot is opposite to pre-existing polar field

Dispersal of many spots by convection and meridional flow acts to reverse the pre-existing poloidal field

Dikpati & Gilman 2006
Babcock-Leighton Dynamo Models

Poloidal field is generated by the Babcock-Leighton Mechanism

Cycle period is regulated by the equatorward advection of toroidal field by the meridional circulation at the base of the CZ

2-3 m/s gives you about 11 years

For this reason, they are also called Flux-Transport Dynamo Models
Final Thoughts on the Dynamo Process

\[ \frac{\partial B}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \quad \text{• \( \mathbf{v} \) influences \( \mathbf{B} \) …} \]

\[ \frac{\partial \mathbf{v}}{\partial t} = - (\rho \mathbf{v} \cdot \nabla) \mathbf{v} - 2\rho (\boldsymbol{\Omega} \times \mathbf{v}) - \nabla P + \rho \mathbf{g} + \frac{1}{c^2} \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{D} \quad \text{… AND vice versa} \]
Final Thoughts on the Dynamo Process

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \eta \nabla^2 B \quad \text{• } v \text{ influences } B \ldots
\]

\[
\rho \frac{\partial v}{\partial t} = - (\rho v \cdot \nabla) v - 2\rho (\Omega \times v) - \nabla P + \rho g + c^{-1} J \times B - \nabla \cdot D \quad \text{• } \ldots \text{ AND vice versa}
\]

This suggests two classes of dynamos:

**Essentially Kinematic:**

Small seed field that is initially kinematic (too weak to induce a significant Lorentz force) grows exponentially until it becomes big enough to modify the velocity field

This brings up the crucial issue of: Dynamo Saturation

**Essentially Nonlinear:**

The velocity field that gives rise to the dynamo mechanism depends on the existence of the field

The focus then shifts toward: Dynamo Excitation + Dynamics
Ultimately, convective flows drive a stellar dynamo.

How fast are those flows?

What is their structure?

These are NOT easy questions to answer.
Granulation in the Quiet Sun

$L \sim 1-2 \text{ Mm}$
$U \sim 1 \text{ km s}^{-1}$
$\tau_c \sim 10-15 \text{ min}$

Dominant size scale of solar convection
The Magnetic Network

CaIIK narrow-band core filter
PSPT/MLSO

Supergranulation
$L \sim 30-35 \text{ Mm}$
$U \sim 500 \text{ m s}^{-1}$
$\tau_c \sim 20 \text{ hr}$
Supergranulation in Filtered Dopplergrams

Most prominent in horizontal velocities near the limb.
A hierarchy of convective scales

Density increases dramatically with depth below the solar surface

Fast, narrow down flows (plumes)
Slow, broad upflows

Most of the mass flowing upward does not make it to the surface

Downward plumes merge into superplumes that penetrate deeper

Supergranulation and mesogranulation are part of a continuous (self-similar?) spectrum of convective motions

Spruit, Nordlund & Title (1990)

Nordlund, Stein & Asplund (2009)
But still stops at 0.97R! What lies deeper still?


Size, time scales of convection cells increases with depth.
Giant Cells
(Loosely, anything bigger than supergranulation)

Eventually the heirarchy must culminate in motions large enough to sense the spherical geometry and rotation.

\[ L \sim 100 \text{ Mm} \]
\[ U \sim 100 \text{ m s}^{-1} \]
\[ \tau_c \sim \text{days - months} \]

Miesch et al (2008)
Giant cells are notoriously difficult to detect (possibly masked by more vigorous surface convection).

How do we know they are there?
We don’t.

This is a major puzzle for solar (and also stellar) dynamo theory and brings us to one of the major outstanding questions.
What is the solar Rossby Number?

\[ Ro = \frac{v}{2\Omega L} = \]

- typical velocity
- rotation rate
- typical length scale

Rotational Timescale
Convective Timescale

What is Ro in the Sun?
What is v?

Aside
1. Consider system-scale Ro
2. \( v = \text{rms convective flow speed} \)
3. \( L = \text{shell depth} \)

Ro influences the structure of the convection and so too MANY aspects of the dynamo!
Ro determines differential rotation

Aside: differential rotation may be indirect evidence of giant cells, but see tomorrow's discussion!
Low Ro Promotes Magnetic Cycles (models)

Ghizaru, Charbonneau & Schmolkiewicz 2010

Ro \approx 0.01
... and Equatorward Propagation of Magnetic Features (models)

$Ro \approx 0.02$

Käpylä et al. 2013
... and Large-Scale Magnetic Structure (models)

Brown et al. 2010

“wreathy” dynamos

Ro ≈ 0.03
Ring-Diagram Analysis (Greer et al. 2015)

Convection Models

What do measurements tell us?

Disagreeing Observations (0.96 $R_{\text{sun}}$)

Open Question: How fast is deep solar convection?
What we know: at Depth ... Ro is LOW

But how low?
Open Question!

Possibly even lower:
See non-detection of Hanasoge et al. 2012, PNAS, 109, 11928
What we know: at Depth ... Ro is LOW

Helioseismology (Greer et al. 2016)

Challenge:
We can directly image convective flows in only the upper 15% of the convection zone.

But how low?
Open Question!

Possibly even lower:
See non-detection of Hanasoge et al. 2012, PNAS, 109, 11928
What do we expect at the solar surface?

We might expect to see two types of convection...

1. Small-scale motions:
   - Near-surface features
   - high-Ro
   - Spatial-scale depends on Ro-transition depth

   Observed: 30 Mm size Ro = 3

2. Large-scale motions:
   - deep-seated, global
   - low-Ro

   Unobserved...

Lack of #2 may be telling us something...
Deep Motions & Their Spectra

Cellular Structure

Classic Cartoon (not observed)

Deep Motions (marginal Ro)

Near-Surface Motions (high-Ro)

Photospheric Power

Total Power

Velocity Power vs. Wavenumber

Alternative Possibility

Deep Motions (Low-Ro)

Featherstone & Hindman 2016
Granulation (1 Mm; photospheric convection)

Giant Cells (200 Mm, Not here)

Supergranulation (30 Mm)

Photospheric Doppler Velocity Spectra

Granulation (1 Mm; photospheric convection)

Hathaway et al. 2015
Supergranulation

The largest, distinctly visible mode of solar convection is not rotationally constrained at the surface.

L ≈ 30 Mm
U ≈ 400 m s⁻¹
Ro ≈ 2.7

AIA 1700
Where is the Sun?

Until we know this, it is very difficult to make meaningful statements about how the solar dynamo works.