

Need for Caution in Interpreting Extreme Weather Statistics

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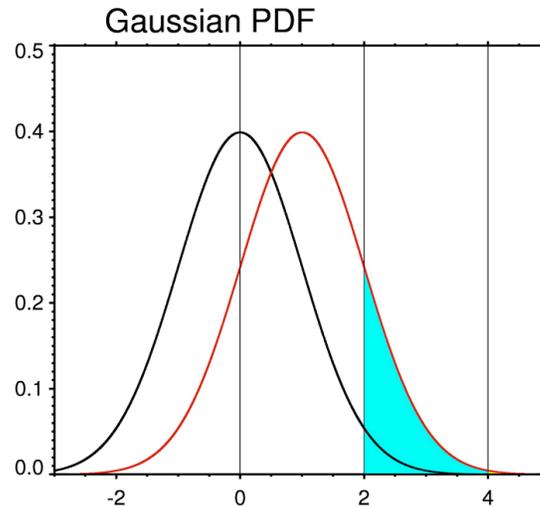
Introduction

1. Are century-long observational datasets and climate model simulations long enough to pin down the statistics of extreme events, and of changes in those statistics? It is difficult to do this directly using the necessarily small samples of extreme events in a decade or a century. This makes some form of modeling of the extreme-anomaly statistics a necessity.
2. There are two fundamentally different approaches to this problem. One can try to fit a theoretical distribution, such as a GEV or a GP distribution, directly to just the extreme values, whose sample size is necessarily limited. This approach has its attractions, but doesn't really get around the sampling issue.
3. Alternatively, one can try to fit a theoretical distribution to *all* values, not just the extreme values, and look at the tails of the PDF. If the PDF is Gaussian, then one need be concerned only with estimating the mean and the variance (the first two statistical moments), which are reasonably reliably estimated from century-long records. One could argue that this approach is preferable to the first approach, certainly from a sampling perspective.
4. The difficulty with the second approach is that many extreme events are associated with the extremes of *daily* weather. The PDFs of daily anomalies are not Gaussian. They are often skewed and heavy tailed. This has enormous implications for extreme statistics. Our study is chiefly concerned with addressing this problem.

Non-Gaussianity has enormous implications for the probabilities of extreme values, and for our ability to estimate their changes using limited samples

Consider Gaussian vs non-Gaussian PDFs, both $p(0,1)$, and shifted by 1 sigma

Gaussian PDFs



$P(x \geq 2) = 2.3\%$
and increases by
a factor of 7

$P(x \geq 4) = 0.003\%$
and increases by
a factor of 43

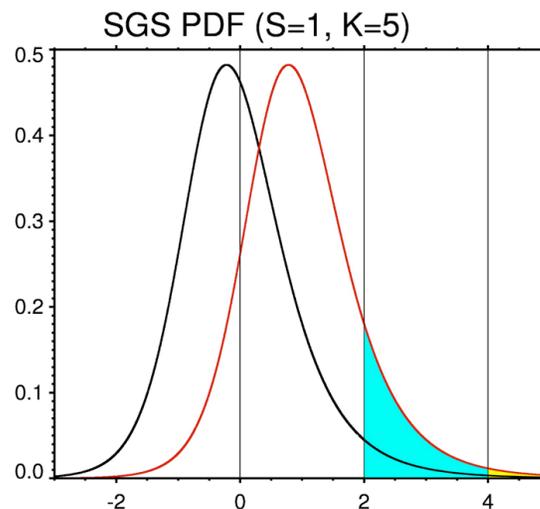
Non-Gaussian PDFs

skewed and heavy-tailed

with

Skewness $S = 1$

Kurtosis $K = 5$



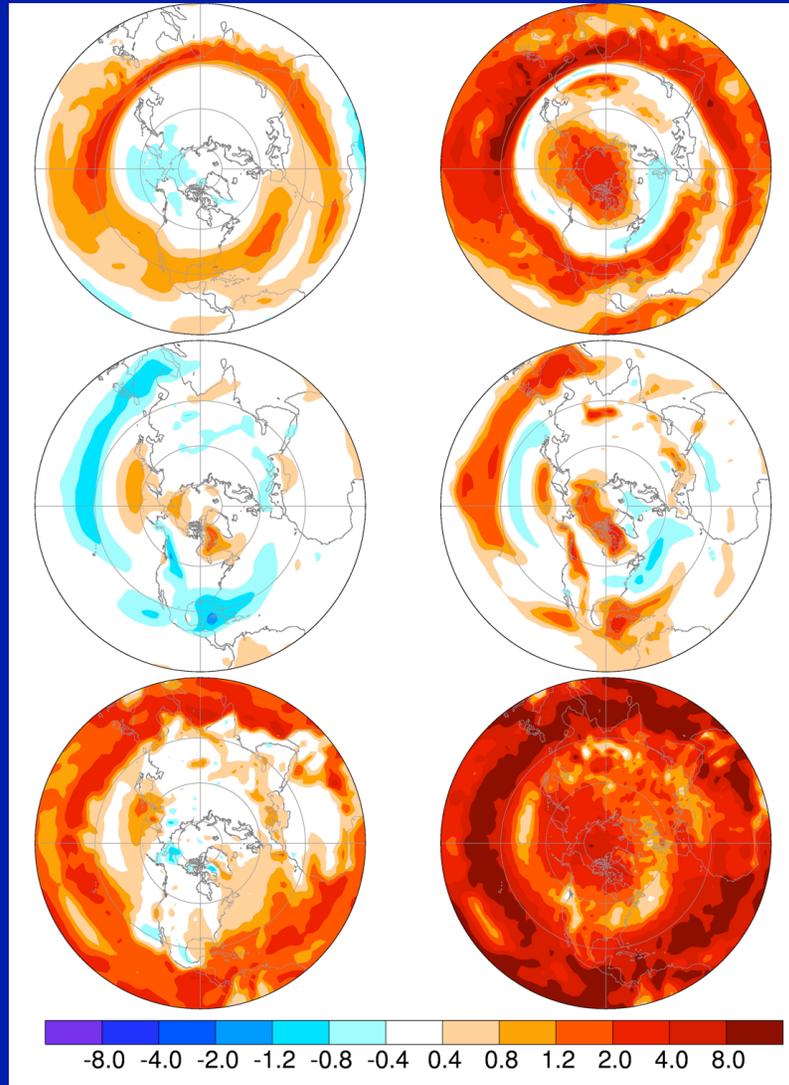
$P(x \geq 2) = 3.4\%$
and increases by
only a factor of 4

$P(x \geq 4) = 0.34\%$
and increases by
only factor of 3

Skewness $S = \langle x^3 \rangle / \sigma^3$ and Kurtosis $K = \langle x^4 \rangle / \sigma^4 - 3$ of daily anomalies in winter computed over 137 winters (1871-2007) in the 20CR dataset (Compo et al 2011)

Skewness S

Kurtosis K



*250 mb
Vorticity*

*850 mb
Air temperature*

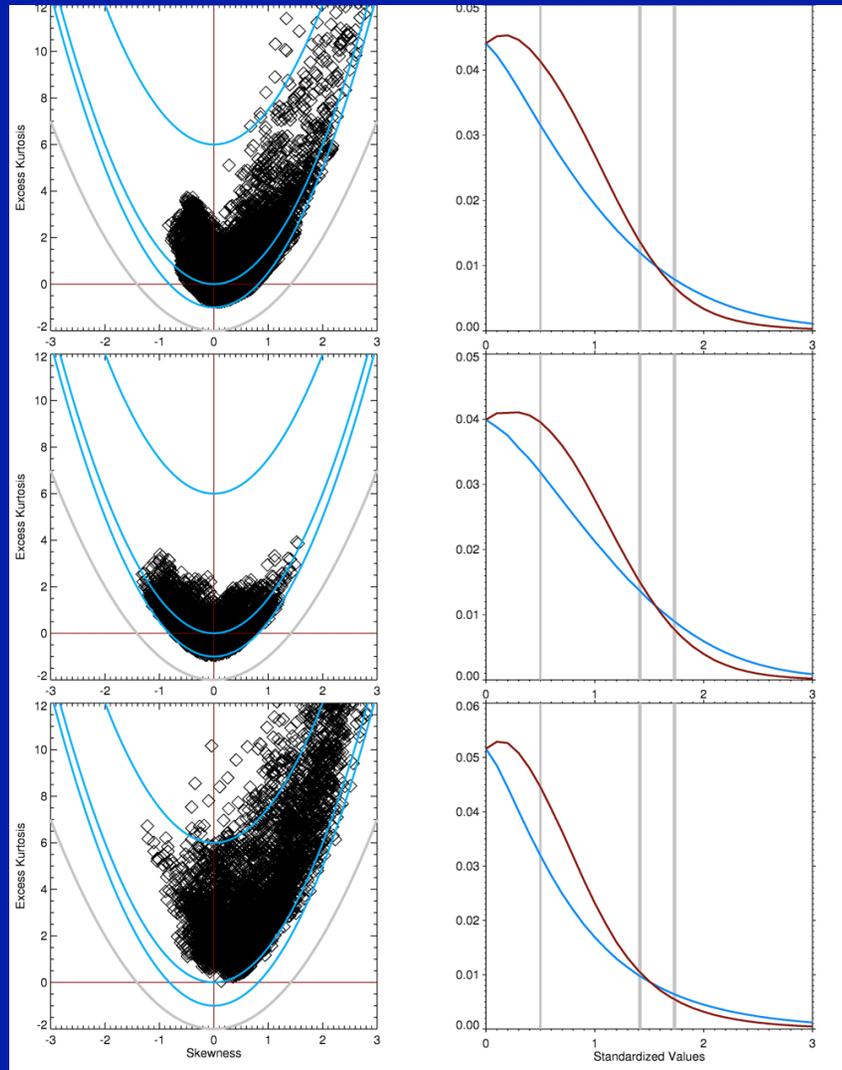
*500 mb
Vertical Velocity*

K vs S

Average Histograms

Some distinctive features of the non-Gaussianity of standardized daily anomalies at all N.H. grid points

computed using 137 winters (1871-2007) of 20CR data



*250 mb
Vorticity*

*850 mb
Air temperature*

*500 mb
Vertical Velocity*

Note the parabolic inequality

$$K \geq 3/2 S^2$$

Note that the crossover point where $p(x) = p(-x)$ lies between 1.4σ and 1.7σ

A generic “Stochastically Generated Skewed” (SGS) probability density function (PDF) suitable for describing non-Gaussian climate variability (Sardeshmukh and Sura *J. Clim* 2009)

$$p(x) = \frac{1}{\mathcal{N}} \left[(Ex + g)^2 + b^2 \right]^{-\left(1 + \frac{\lambda}{E^2}\right)} \exp \left[-\frac{2\lambda g}{E^2 b} \arctan \left(\frac{Ex + g}{b} \right) \right]$$

If $E \rightarrow 0$, then $p(x) \rightarrow$ a Gaussian PDF

$$\begin{aligned} \lambda &> 0 & b &\geq 0 \\ g &\geq 0 \text{ or } g < 0 \\ E &\geq 0 \end{aligned}$$

Such a PDF has power-law tails, its moments satisfy $K \geq (3/2) S^2$, and

$$p(x) = p(-x) \text{ at } \hat{x} \approx \sqrt{3} \sigma$$

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Such a PDF has power-law tails, its moments satisfy $K \geq (3/2) S^2$, and $p(x) = p(-x)$ at $\hat{x} \approx \sqrt{3} \sigma$

This PDF arises naturally as the PDF of the simplest 1-D damped linear Markov process that is perturbed by Correlated Additive and Multiplicative white noise (“CAM noise”)

$$\frac{dx}{dt} = - \left(\lambda + \frac{1}{2} E^2 \right) x + b \eta_1 + (Ex + g) \eta_2 - \frac{1}{2} E g$$

If $E \rightarrow 0$, this is just the evolution equation for Gaussian "red noise"

η_1 and η_2 are Gaussian white noises of unit amplitude.

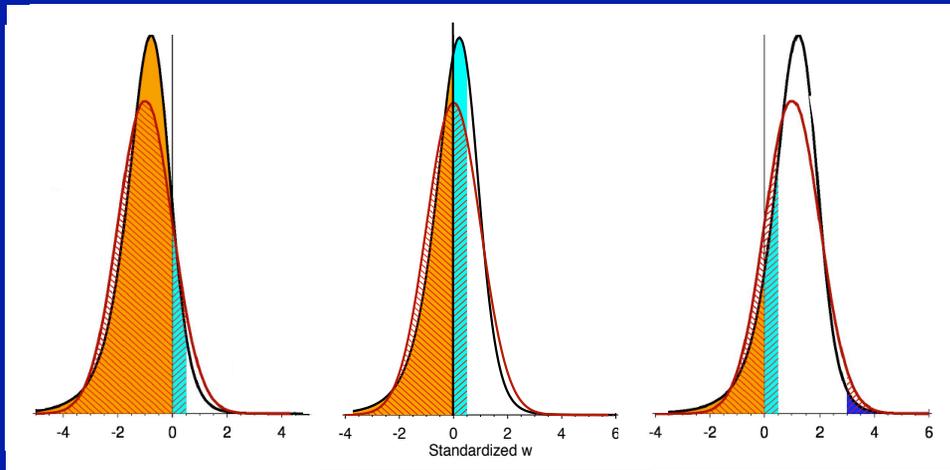
The parameters of this model (and of the PDF) can be estimated using the first four moments of x and its correlation scale. The model can then be run to generate Monte Carlo estimates of extreme statistics

Met. 101: The PDF of vertical velocity w strongly affects the PDF of precipitation

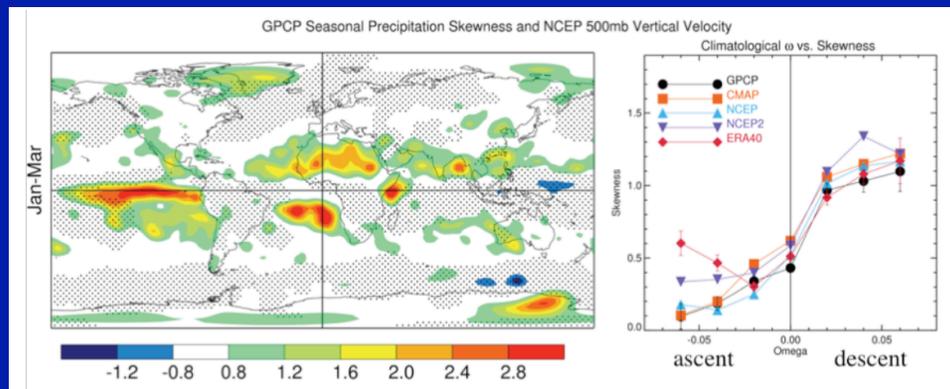
Mean Descent

Mean Zero

Mean Ascent



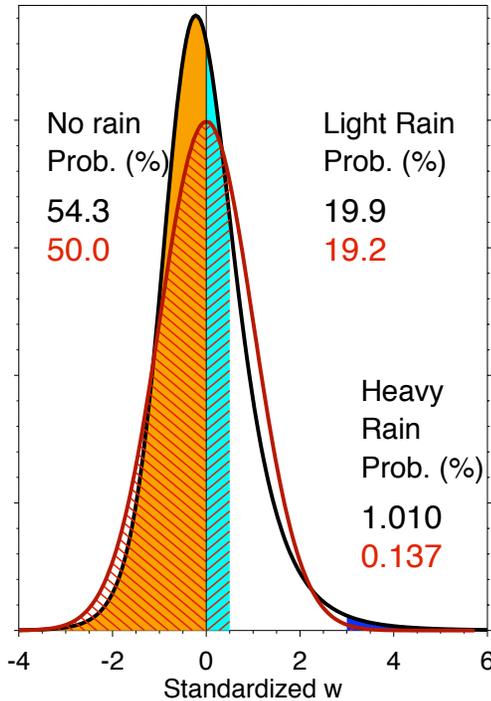
To a first approximation, the precipitation PDF has the same shape as the shape of the w PDF for positive w



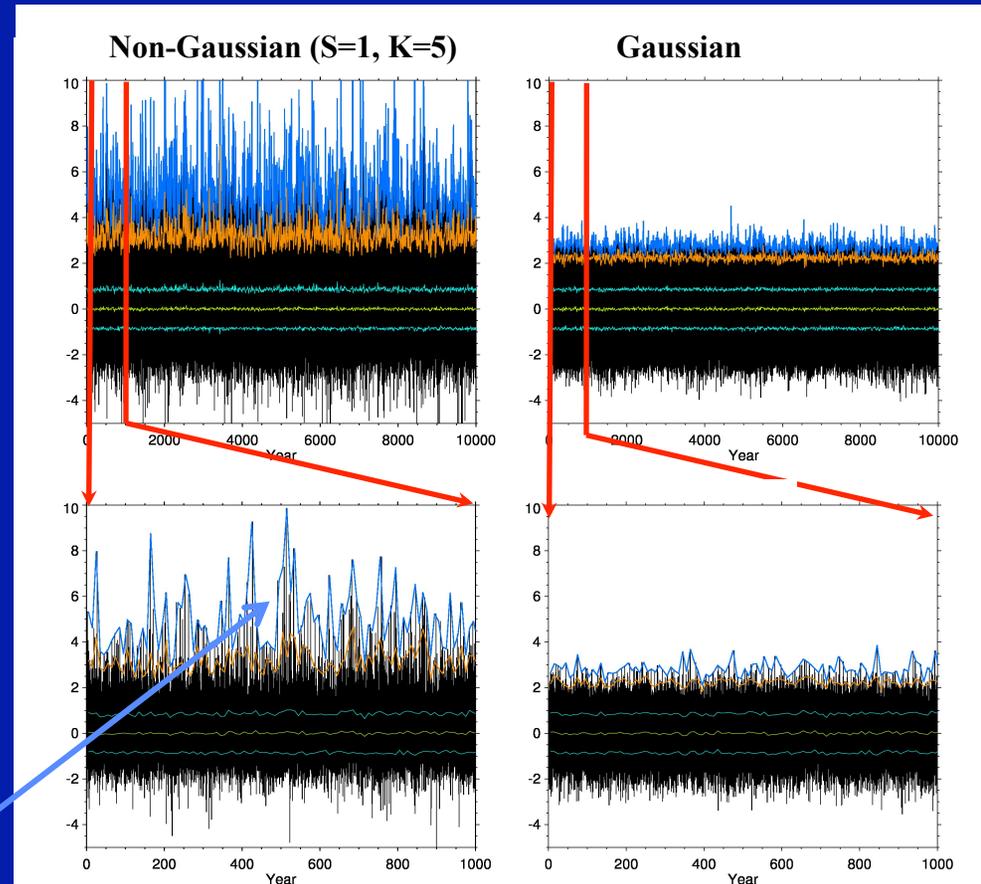
This is the basic reason why the PDFs of even seasonal mean precipitation are generally positively skewed, and are more skewed in drought-prone regions of mean descent

Sharply contrasting behavior of extreme w anomalies (and by implication, of extreme precipitation anomalies) obtained in 10^8 -day runs (equivalent to 10^6 100-day winters) of the Gaussian and non-Gaussian Markov models

Gaussian (red) and non-Gaussian (black, $S=1, K=5$) PDFs with same mean and variance



Note how one can obtain spurious 100-yr trends of decadal extremes in the non-Gaussian case *even in this statistically stationary world.*



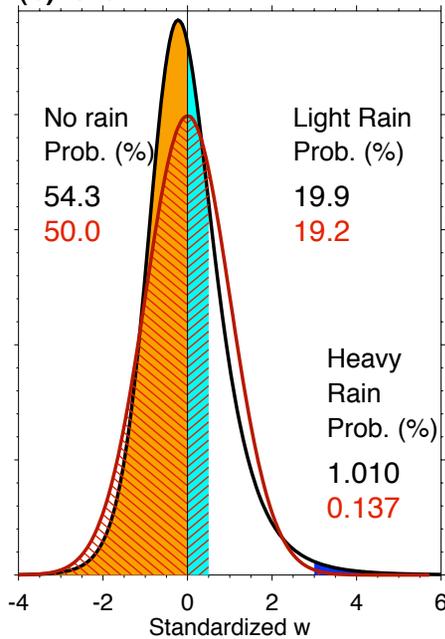
Blue curves: Time series of decadal maxima (i.e. the largest daily anomaly in each decade = 1000 days = 10 100-day winters)

Orange curves: Time series of 99.5th decadal percentile (i.e. the 5th largest daily anomaly in each decade)

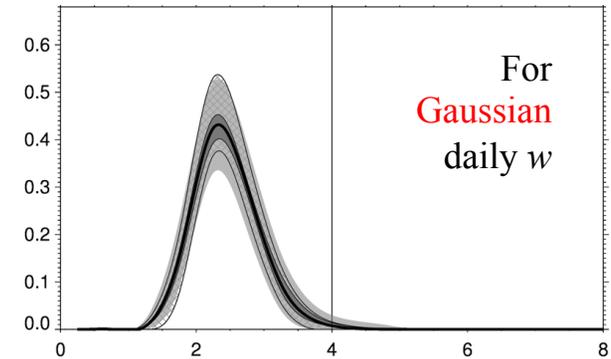
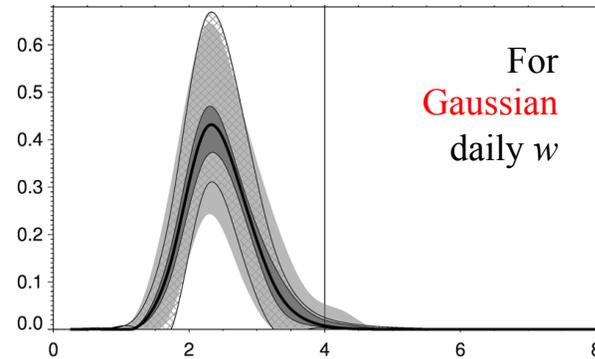
The PDFs of winter maxima are **VERY DIFFERENT** if the PDFs of the daily values are Gaussian or “SGS”. They are also more accurately estimated by fitting SGS distributions to *all* daily values than by fitting GEV distributions to just maximum values

PDFs of daily w :
Gaussian
 and non-Gaussian
 (S=1, K=5)

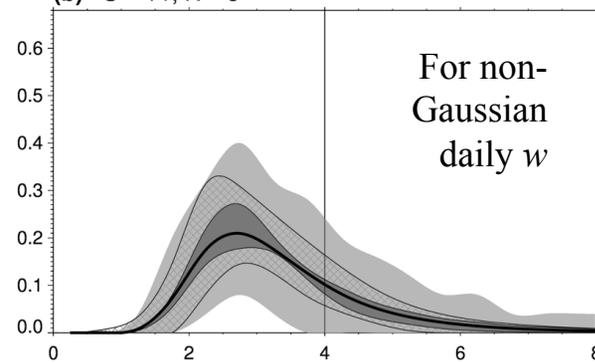
(b) skew = +1



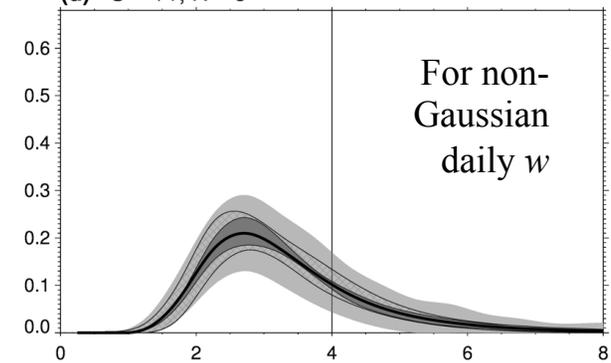
PDFs of daily winter *maxima* (Extreme Value PDFs) estimated using
 25-winter records 100-winter records



(b) S = +1, K = 5



(d) S = +1, K = 5



Standardized w or “precipitation”

Standardized w or “precipitation”

Black Curves:

Shaded bands:

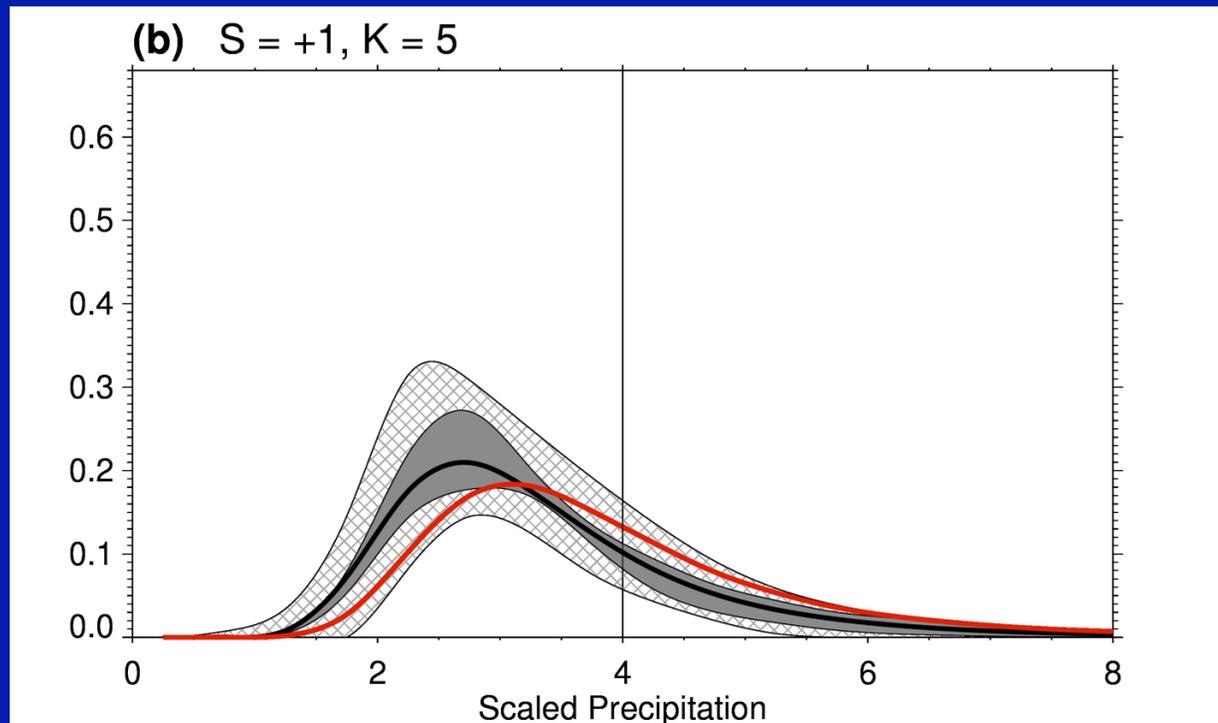
Outer bands:

Inner grey bands:

Darkest grey bands:

Extreme Value PDFs of winter w (or “precipitation”) maxima estimated from 10^6 model winters, when the PDF of daily w is Gaussian or non-Gaussian
 95% intervals of Extreme Value PDFs estimated using 25- or 100- winter records
 95% intervals of raw histogram-based estimates using the 25 or 100 winter maxima
 95% intervals of GEV PDFs fitted to the 25 or 100 winter maxima
 95% intervals of Extreme Value PDFs derived from SGS distributions fitted to all daily values in the 25 or 100 winters

The sampling uncertainties of GEV-based Extreme Value PDFs estimated from 25-yr records are larger than the modest changes expected in the true PDF from, say, a 15% increase in precipitation (red curve). The corresponding uncertainties from an SGS-theory based estimation are much smaller.



Summary

- 1. The PDFs of daily anomalies are significantly skewed and heavy-tailed. This fact has enormous implications both for the probabilities of extremes and for estimating changes in those probabilities.**
Direct estimations from raw histograms or GEV distributions become even more prone to sampling errors than in the Gaussian case.
2. We have demonstrated the relevance of “stochastically generated skewed” (SGS) distributions for describing daily atmospheric variability, that arise from simple extensions of a linear Markov “red noise” process. The parameters of these SGS distributions, and of the associated Markov model, can be estimated from the first four moments of the data (mean, variance, skewness, and kurtosis). The model can then be run to generate not only the appropriate SGS distribution, but also to estimate sampling uncertainties through extensive Monte Carlo integrations.
3. We have shown that extreme-value distributions can be estimated more accurately from limited-length records using such a Markov model than through direct GEV approaches.
- 4. To accurately represent extreme weather statistics and their changes, it is necessary for climate models to accurately represent the first four moments of daily variability. The good news is that for many purposes this may also be sufficient. The bad news is that currently they do not adequately capture the changes of even the first moment (the mean), especially on the regional scales of direct relevance for climate policy.**