

Vertical transport of particles, drops, and microorganisms in density stratified fluids

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Abstract

The vertical motion of particles, drops and organisms through density stratified fluids is ubiquitously found in oceans and lakes. Settling dynamics of marine snow particles, rising motion of drops during oil spills, formation of phytoplankton blooms, and diel vertical migration of organisms are just a few of these examples. Transport properties in these examples are modified when the density of ambient fluid varies over depth due to variation of salinity or temperature. Density stratification directly affects particle settling/rising rates, which impacts particle distribution in natural fluid environments. In this work, we discuss the vertical transport of rigid particles, deformable drops, and swimming organisms in density stratified fluids.

1 Introduction

Marine snow particles play an important role in the vertical transport of nutrients in the ocean. The settling behavior of marine snow particles determines their aggregate size and density, and consequently particles' biogeochemical processes in the water column (Alldredge and Gotschalk, 1988; Fowler and Knauer, 1986). Large marine particles settle at a low to moderate Reynolds number Re , where Reynolds number characterizes the ratio of inertial forces to the viscous forces. In addition to Reynolds number, the settling dynamics of a spherical particle in a linearly stratified fluid depends on Froude and Prandtl numbers, where Froude number $Fr = U/(Nd)$ is the ratio of inertial forces to buoyancy forces and Prandtl number $Pr = \nu/\kappa$ represents the ratio of momentum diffusivity to the diffusivity of the stratifying agent (temperature or salinity). U is the characteristic velocity of the dispersed phase, d is the characteristic size of the dispersed phase, ν is the kinematic viscosity of the fluid, $N = \sqrt{\gamma g/\rho_0}$ is the Brunt-Väisälä frequency, g is the gravitational acceleration, γ is the density gradient, and ρ_0 is the reference density.

During the transient settling process of a spherical particle released from rest in a linearly stratified fluid, the particle velocity first reaches a peak value, and then it monotonically approaches zero for weak stratification or it is followed by velocity oscillations at small Froude numbers. The oscillation frequency of the settling velocity scales well with the Brunt-Väisälä frequency (Doostmohammadi et al., 2014). Studies of settling particles in a stratified fluid have mainly focused on spheres. Natural particles, however, often exhibit striking departures from the spherical shape. Particle elongation affects both the settling orientation and the settling rate of particles in stratified fluids, which have direct consequences on the vertical flux of particulate matter and carbon flux in the ocean. Our results on the effect of anisotropy of elongated objects on the settling dynamics reveal that a change of stability occurs for the ellipsoid orientation in the low Reynolds number regime. In the absence of stratification, the broadside-on settling occurs due to weak inertial effects, whereas the long axis of the particle in a linearly stratified

fluid rotates toward the settling direction (Doostmohammadi and Ardekani, 2014). Similarly, settling dynamics of a pair of spherical particles is modified. In particular, the settling dynamics of two spherical particles initially in tandem is dramatically altered due to the presence of the stratification, and the drafting-kissing-tumbling dynamics in a homogeneous fluid is modified to drafting-kissing-separation or drafting-separation in a linearly stratified fluid depending on the strength of the stratification. The tumbling rate of the particles is significantly reduced (Doostmohammadi and Ardekani, 2013). By fully resolving particle-particle interaction within a suspension of settling particles, our numerical study suggests that the fluid stratification enhances the formation of horizontally aligned clusters (Doostmohammadi and Ardekani, 2015).

The vertical motion of bubbles and drops in a stratified fluid is frequently observed in the aeration of lakes (Hill et al., 2008) and oil spills in oceans (Blumer et al., 1971). Understanding the rising motion of oil drops in a stratified fluid is essential in estimating oil dispersion, and consequently, determining the scale of required remediation efforts. We investigate the rising dynamics of a single deformable drop in a linearly stratified fluid using the finite-volume/front-tracking approach. The first observation is that the fluid stratification enhances the drag force acting on the drop compared to that in a homogeneous fluid. The drop is less deformable in the presence of stratification due to the enhanced drag and smaller rising velocity (Bayareh et al., 2013). For a swarm of drops, fluid stratification enhances horizontal cluster formation compared to that in a homogeneous fluid. Both the averaged rising velocity and velocity fluctuations of the swarm are reduced in a linearly stratified fluid (Dabiri et al., 2015b).

Motility affects trophic dynamics and biogeochemistry of ocean ecosystem. At a low Reynolds number, our study shows that self-propulsion generated by an organism alters the stable density field, and consequently, its own swimming velocity. In addition, the stratification reduces both detectability and nutrient uptake of a motile organism (Doostmohammadi et al., 2012). At moderate Reynolds numbers, we evaluate the biogenic mixing generated by interacting swimmers in a stratified fluid in the absence and presence of the background turbulence. We quantify the vertical mass transport drifted by the migrating organisms by evaluating mixing efficiency, diapycnal eddy diffusivity, and Cox number (Wang and Ardekani, 2015). The mixing efficiency is in the range of $O(0.0001-0.04)$ when the swimming Reynolds number is in the range of $O(0.1-100)$.

2 Governing equations

Let us consider the incompressible, viscous flow around drops/particles/swimmers moving in density stratified fluids. Note that the ambient fluid and drops/particles/swimmers are referred to as continuous and dispersed phases, respectively. The governing equations in the entire domain are given as

$$\nabla \cdot \mathbf{u} = 0, \quad (1)$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + (\rho - \bar{\rho}) \mathbf{g} + \mathbf{f}, \quad (2)$$

where t is the time, \mathbf{u} is the velocity vector, p is the hydrodynamic pressure, μ is the fluid's dynamic viscosity, \mathbf{g} is the gravitational acceleration, ρ_0 is the reference fluid density, and $\bar{\rho}$ is the volumetric average of the density over the entire computational domain. $D(\cdot)/Dt$ is the material derivative. The density ρ can be written as $\rho = \rho_f + \phi(\rho_d - \rho_f)$, where ρ_f is the fluid density that depends on the fluid temperature or salinity, and ρ_d is the density of

the dispersed phase. The indicator function ϕ is a phase indicator to identify both phases with $\phi = 1$ for the dispersed phase and $\phi = 0$ for the continuous phase. The body force \mathbf{f} accounts for the hydrodynamic interaction between the continuous and dispersed phases. Both particles and swimmers are modeled as rigid objects. The velocity on the surface of particles satisfies no-slip boundary condition, but the velocity on the surface of swimmers is equal to a prescribed slip velocity. Here, we use the squirmer model widely used in the literature to study motion of microorganisms (Ishikawa et al., 2006; Li et al., 2014). The squirmer model is introduced by Lighthill (1952) and Blake (1971). We consider the first two squirring modes and consequently the magnitude of the tangential velocity on the squirmer surface is written as

$$u_{\theta}^s(\theta) = B_1 \sin \theta + B_2 \sin \theta \cos \theta, \quad (3)$$

where θ is the polar angle measured from the swimming direction, B_1 and B_2 are the first two squirring modes. The parameter $\beta = B_2/B_1$ distinguishes pullers ($\beta > 0$, generating thrust in front of the cell) and pushers ($\beta < 0$, generating thrust behind the cell). In the Stokes regime, the swimming speed of a squirmer in an unbounded domain is $U_0 = 2B_1/3$.

The temporal evolution of the fluid density field is governed by a convection-diffusion process described by

$$\frac{D\rho_f}{Dt} = \nabla \cdot (\kappa \nabla \rho_f), \quad (4)$$

where κ is the diffusivity of the stratifying agent (temperature or salinity).

3 Numerical implementation

Simulations are conducted using a finite volume method on a fixed staggered grid (Dabiri et al., 2013; Dabiri and Tryggvason, 2015; Dabiri et al., 2015a). The time discretization is obtained using a second-order Runge-Kutta method. The convection and diffusion terms in equations (2) and (4) are solved using the QUICK (quadratic upstream interpolation for convective kinetics) and central-difference schemes, respectively. A distributed Lagrange multiplier-based computational method is used to obtain \mathbf{f} for squirmers and particles to satisfy the boundary condition, the details of which are given in Ardekani et al. (2008); Li and Ardekani (2014); Doostmohammadi et al. (2014). For drops, \mathbf{f} is the force distributed on the surface of the droplet to account for the surface tension force (Dabiri et al., 2013, 2015b; Bayareh et al., 2013).

4 Discussions

4.1 Single particle settling in a stratified fluid

We investigate the effect of stratification on the particle settling dynamics. The rigid particle begins from rest. The heat/salinity flux on the surface of the particle is set to zero. Since the particle density is larger than the ambient fluid density, it first accelerates to a peak velocity. Its velocity then decreases as the particle encounters denser fluids and finally, its velocity approaches zero. As we decrease the Froude number, the particle's deceleration process exhibits oscillations in particle velocity. A further decrease of Froude number changes the sign of particle's velocity, causing a levitation. By changing the density ratio and Froude number, we characterize the deceleration process as four different phases: levitation-levitation, levitation-oscillation, oscillation, and monotonic deceleration (see Fig. 1a). We observe the occurrence of the levitation at small density ratios and

small Froude numbers. At a large Froude number, the particle experiences a monotonic deceleration.

In oceans and lakes, the density stratification is caused by either fluid temperature or salinity gradient. On the other hand, the density stratification in the atmosphere is caused by variation in air temperature. We investigate the effect of diffusivity on the particle settling. The value of $Pr = 0.7$ corresponds to the temperature stratification in the atmosphere, and the Prandtl number in salinity-induced stratification is about 700. In Fig. 1b, we examine the effect of Prandtl number on the particle's settling velocity. The rigid particle is less affected by the diffusion before reaching the peak velocity. However, the diffusivity strongly affects the deceleration process. At a large Prandtl number, a small diffusion coefficient slowly restores deflected density layers, and therefore, the particle velocity decreases quickly.

4.2 Suspension of solid spheres

Despite extensive research investigating settling particles in a homogeneous fluid, the settling dynamics of a suspension of particles in a stratified fluid is poorly understood. We study the settling dynamics of a suspension of particles in a linearly stratified fluid in a periodic box. Monodisperse particles are initialized in a regular array. Particle volume fraction ranges from $\phi = 0.05$ to $\phi = 0.1$, corresponding to a semi-dilute regime. In this study, the diffusivity coefficients are assumed to be uniform and the same for the dispersed phase and the background fluid. Here, we use the drag coefficient to quantify the effect of stratification on the particle settling dynamics. The normalized drag acting on particles in a suspension settling in a stratified fluid with its homogeneous counterpart is independent of the volume fraction in the semi-dilute regime ($\phi < 0.1$), and the best fit follows $Cd_S/Cd_H = 1 + 8.9Fr^{-1.80}$ in the range of $1 < Fr < 10$, where subscripts S and H correspond to the stratified and homogeneous fluids, respectively. At a low Reynolds number, the quasi-steady drag acting on a single particle scales as $Cd_S/Cd_H = 1 + 1.9Ri^{1/2}$ (Yick et al., 2009), where Richardson number is defined as $Ri = Re/Fr^2$. Torres et al. (2000) numerically calculates the enhanced drag for a single particle in an inertial regime, and the best fit to their data in the range of $1 < Fr < 10$ follows

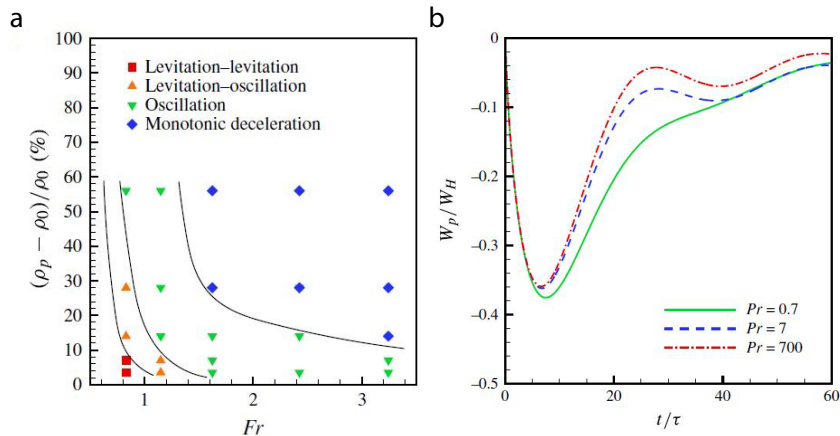


Figure 1: Single rigid particle settling in a linearly stratified fluid. (a) Classification of dynamic behavior of a particle settling in a linearly stratified fluid for $Re = 14.1$ and $Pr = 700$. Particle density is shown by ρ_p . (b) the temporal evolution of particle's settling velocity in a stratified fluid normalized with its homogenous counterpart for different Prandtl numbers for $Re = 14.1$ and $Fr = 1.62$. Reproduced from Doostmohammadi et al. (2014)

$Cd_S/Cd_H = 1 + 4.4Fr^{-1.25}$. We should note, however, the boundary condition for the stratifying agent on the surface of the particle in Torres et al. (2000); Yick et al. (2009) is different from the one used in our study of suspension of particles, where the particles are not impermeable.

4.3 Transport of a semi-dilute suspension of swimmers in a linearly stratified fluid

In the aphotic ocean zone (i.e., regions that are 200 m beneath the sea surface), zooplankton are the most abundant organisms. Their body size ranges from millimeter to centimeter, and their Reynolds number is in the range of $O(1 - 100)$. Therefore, it is important to examine their transport and induced mixing in this inertial regime.

We use “squirmers” model to study fully resolved motion of interacting swimmers in a density stratified fluid. Our study shows that the mixing efficiency and the diapycnal eddy diffusivity, a measure of vertical mass flux, within a suspension of squirmers increase with increase in Reynolds number. The mixing efficiency is in the range of $O(0.0001 - 0.04)$ when the swimming Reynolds number is in the range of $O(0.1 - 100)$. The mixing efficiency generated by a suspension of squirmers in a stratified fluid decrease as the Froude number decreases. On the other hand, the overall vertical mass flux are nearly independent of the density stratification for large Froude numbers (i.e., $Fr > 20$). For a suspension of squirmers in a decaying isotropic turbulence (see Figure 2a), we found that the diapycnal eddy diffusivity enhances due to the strong viscous dissipation generated by squirmers and due to the interaction of squirmers with the background turbulence. Pushers more strongly enhance the overall mixing compared to pullers. The strong mixing generated by pushers compared to pullers can be explained by their swimming trajectories. Pushers (Fig. 2b) rectilinearly swim with infrequent changes in their swimming direction, while pullers (Fig. 2c) swim in helical paths.

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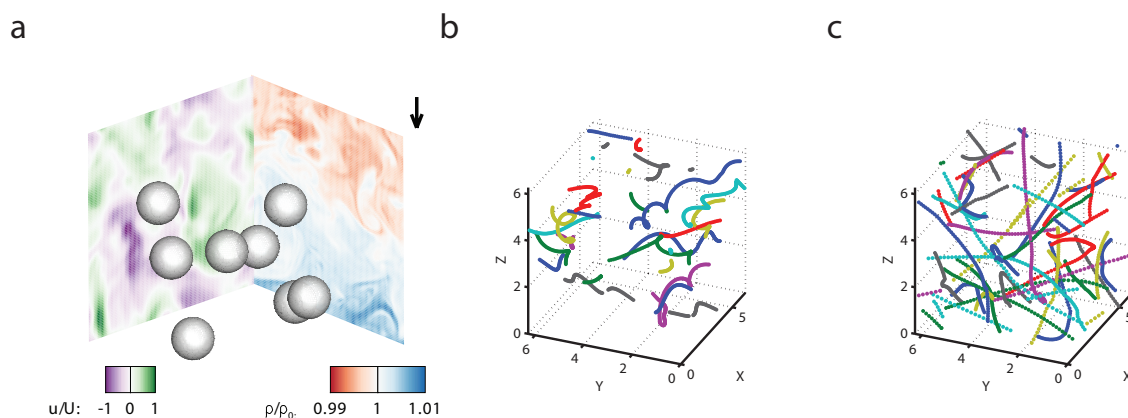


Figure 2: A decaying stratified turbulence is modulated by squirmers of Taylor length-scale size. (a) A snapshot of a suspension of 8 pushers is shown; (b) and (c) correspond to trajectories of pullers and pushers, respectively, where different colors distinguish individual squirmers. Reproduced from Wang and Ardekani (2015)

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