The relative importance of transient wave-mean-flow interactions and turbulent wave breaking in atmospheric gravity-wave parameterizations

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Abstract

Present-day gravity-wave (GW) parameterizations in atmospheric models use Wentzel-Kramer-Brillouin (WKB) theory in an approximation where the transient interaction of propagating GW fields with a mean flow is simplified to the calculation of steady-state profiles in a prescribed mean flow. In this framework, turbulent wave breaking - after sufficient amplitude growth with height - is the only possibility for the wave to force the mean flow. Transient non-turbulent interactions between GWs and mean-flow are neglected. Using either a fully interactive WKB algorithm or a steady-state limit thereof, we investigate the relative importance of the transient GW-mean-flow interaction and turbulent wave breaking. In idealized test cases describing horizontally homogeneous GW packets, the transient WKB simulations reproduce successfully the most important characteristics of wave-resolving calculations using a Large-Eddy-Simulation (LES) code. It is also found that a wave-breaking parametrization added to the transient WKB model plays a comparatively secondary role in the wave drag. On the contrary, results from steady-state simulations supplemented by a wave-breaking scheme tend to differ considerably from the LES data, arguing for a fully transient WKB approach in GW parameterizations in atmospheric models.

1 Introduction

The parametrization of gravity waves (GW) is of significant importance in atmospheric global circulation models and in numerical weather prediction (Alexander et al., 2010; Butchart, 2014; Lindzen, 1981; Scaife et al., 2005, 2012). Corresponding schemes (Lindzen, 1981; Medvedev and Klaassen, 1995; Hines, 1997a,b; Lott and Miller, 1997; Alexander and Dunkerton, 1999; Warner and McIntyre, 2001; Lott and Guez, 2013) are based on Wentzel-Kramer-Brioullin (WKB) theory. They use it, however, under a steady-state assumption so that, by the non-acceleration theorem, GWs can deposit their momentum only where they break. In theoretical analyzes of this problem in a rotating atmosphere Bühler and McIntyre (1999, 2003, 2005) show that the steady-state assumption can lead to the neglect of important aspects of the interaction between GWs and mean-flow. By wave-resolving numerical simulations and analyses on the basis of a nonlinear Schrödinger equation Dosser and Sutherland (2011) have demonstrated the relevance of GW-mean-flow interactions as well. In line with these predictions, the most important goal of the present work is to study to what extent WKB theory without steady-state assumption and thus enabled transient GW propagation coupled to the mean-flow leads to more realism in the simulated wave drag, and whether or not this transient WKB approach shows potential for improvements as compared to steady-state WKB parametrizations.

2 Theoretical background

It is well known that in 2-dimensional, non-rotating, horizontally homogeneous compressible fluid the GW drag can be expressed within WKB theory as

$$\frac{\partial u_b}{\partial t} = -\frac{1}{\bar{\rho}}\frac{\partial}{\partial z}\left(k c_{gz}\mathcal{A}\right) \tag{1}$$

where u_b is the induced mean wind, $\bar{\rho}$ is the ambient density profile, k and c_{gz} are the horizontal wavenumber, constant under horizontal homogeneity, and vertical group velocity of GWs respectively, and $\mathcal{A} = E_w/\hat{\omega}$ is the wave action density with the intrinsic frequency $\hat{\omega}$ and the wave energy E_w. The latter is predicted by

$$\frac{\partial \mathcal{A}}{\partial t} + \frac{\partial\left(c_{gz}\mathcal{A}\right)}{\partial z} = 0, \tag{2}$$

In current GW parametrization schemes, in atmospheric models, the steady-state assumption is applied (Nappo, 2002; Fritts and Alexander, 2003; Kim et al., 2003; J.Coiffier, 2011), which amounts to supressing the time evolution in the wave action conservation equation and thus assuming a constant equilibrium profile of $c_{gz}(z)\mathcal{A}(z)$ leading to a zero right-hand-side in equation (1). In this framework the wave induced mean-flow will remain unchanged unless wave breaking is assumed - e.g. based on static instability conditions - which reinforces a vertical gradient in $c_{gz}(z)\mathcal{A}(z)$ and thus allows for a tendency. This leads to parametrizations where GWs instantenously deposit their momentum at breaking height, but their propagation is not described and apart from wave breaking no interactions are allowed between the GWs and the mean-flow.

In contrast, without the steady-state assumption the wave action density $\mathcal{A}(z,t)$ and the group velocity $c_{gz}(z,t)$ are transiently changing as described by the prognostic wave action conservation equation (2) and the ray equations (Berethon, 1966; Grimshaw, 1975; Achatz et al., 2010; Rieper et al., 2013a)

$$\frac{dz}{dt} = c_{gz} = \mp\frac{Nkm}{(k^2+m^2)^{3/2}} \tag{3}$$

$$\frac{dm}{dt} = \dot{m} = \mp\frac{k}{(k^2+m^2)^{1/2}}\frac{dN}{dz} - k\frac{\partial u_b}{\partial z} \equiv \dot{m} \tag{4}$$

where $d/dt = \partial/\partial t + c_{gz}\partial/\partial z$, N is the Brunt-Väisälä frequency, and m is the vertical wavenumber. These coupled equations ensure a realistic description of the GW propagation in a transiently changing background flow via equation (1) (Rieper et al., 2013a; Muraschko et al., 2015). Note that in this transient case a mean-flow tendency arises even without the effect of wave breaking, solely via GW-mean-flow interactions described by equations (1)-(4). Nevertheless, besides these interactions, wave breaking - if happening - might also play a role and for this reason its parametrization also makes sense together with the transient WKB theory.

Having presented these conceptual differences in the steady-state and the transient WKB theories, the interesting question arises: what is the relative importance of the transient GW-mean-flow interactions in comparison with wave breaking regarding the GW-drag produced and the evolution of GW energy? Or in other words: how big is the room for improvement by dropping the steady-state assumption?

3 Numerical simulations

In order to answer the above questions, numerical model simulations solving both the steady-state and the transient WKB equations have been compared with fully nonlinear Large Eddy Simulations (LES) (Rieper et al., 2013b) solving the pseudo-incompressible equations (Durran, 1989). Note that in order to avoid numerical instabilities due to caustics in the transient WKB model (Muraschko et al., 2015), the coupled equations are transformed to the phase-space spanned by the vertical position z and the vertical wavenumber (Bühler and McIntyre, 1999; Hertzog et al., 2002). Muraschko et al. (2015) also show that besides numerical stability the phase-space approach allows a Lagrangian formulation to solve the coupled equations (1)-(4) and thus some gain in numerical efficiency. A wave-breaking scheme has been implemented optionally in the transient WKB model, which - by switching it on and off - allows for the separation of the impact of wave breaking and the transient wave-mean-flow interactions. The steady-state variant of the WKB model is a simplification using the steady-state concept briefly explained in the previous section.

The simulations have been performed for idealized test cases with vertically confined GW packets, including the refraction or reflection from a jet and large amplitude wavepackets turning into statically unstable regimes (Bölöni et al., 2016). The most important findings based on the simulations will be presented here through a case, where a non-hydrostatic wavepacket ($\lambda_x = \lambda_z = 1km$) with a Gaussian envelop with a half width of $5km$ is initialized with a large amplitude close to static instability, i.e. the wave action density is set to 90% of the saturation value. The isothermal background temperature was set to $300K$ leading to a Brunt-Väisälä frequency $N \approx 0.018$. The results are shown in Figure 1. If comparing the Hovmöller diagrams (both wave energy and induced mean wind) between the LES (panels e and i) and the transient WKB model without wave breaking parametrization (panels f and j), we find a rather good agreement both in terms of magnitudes and spatio-temporal structures. This suggests that the transient GW-mean-flow interactions alone explain the predominant part of the corresponding dynamics. If comparing the time evolution of the vertically integrated GW and mean-flow energies between the same simulations (panels a and b), it appears that the integrated total energy (sum of the GW and mean-flow energy) reduces with time in the LES, but not in the transient WKB model. This is due to the fact that in the transient WKB theory - without additional wave breaking parametrization - the total integrated energy is conserved, but not in the LES where the fully nonlinear dynamics and an implicit turbulence scheme dissipates energy. This motivates to switch on the wave breaking parametrization in the transient WKB model, which then leads to a rather similar total integrated energy evolution as in the LES (panel c). This shows, that taking into account wave breaking in the transient WKB model helps to reproduce the dissipative energetics of the fully nonlinear reference. However, it is also important to notice that the GW energy and the induced men-flow structure is not very strongly affected by wave breaking (panels g, k compared to panels e,i,f,j). This suggests that wave breaking has a secondary role in the wave drag in comparison with the transient GW-mean-flow interactions.

Notably under the steady-state assumption the one and only source of GW drag would be wave breaking because there transient GW-mean-flow interactions are not described at all. Correspondingly, as a test how much supressing transient GW-mean-flow interactions really matters, numerical simulation results performed with the steady-state variant of the WKB model are also shown (panels d, h, l). They reflect a GW and mean-flow kinetic energy, which is rather over-damped. The upward strengthening induced mean-flow shows well, that here the only source of GW drag is wave breaking, and it leads to a structure, which is rather different from that of the reference LES.

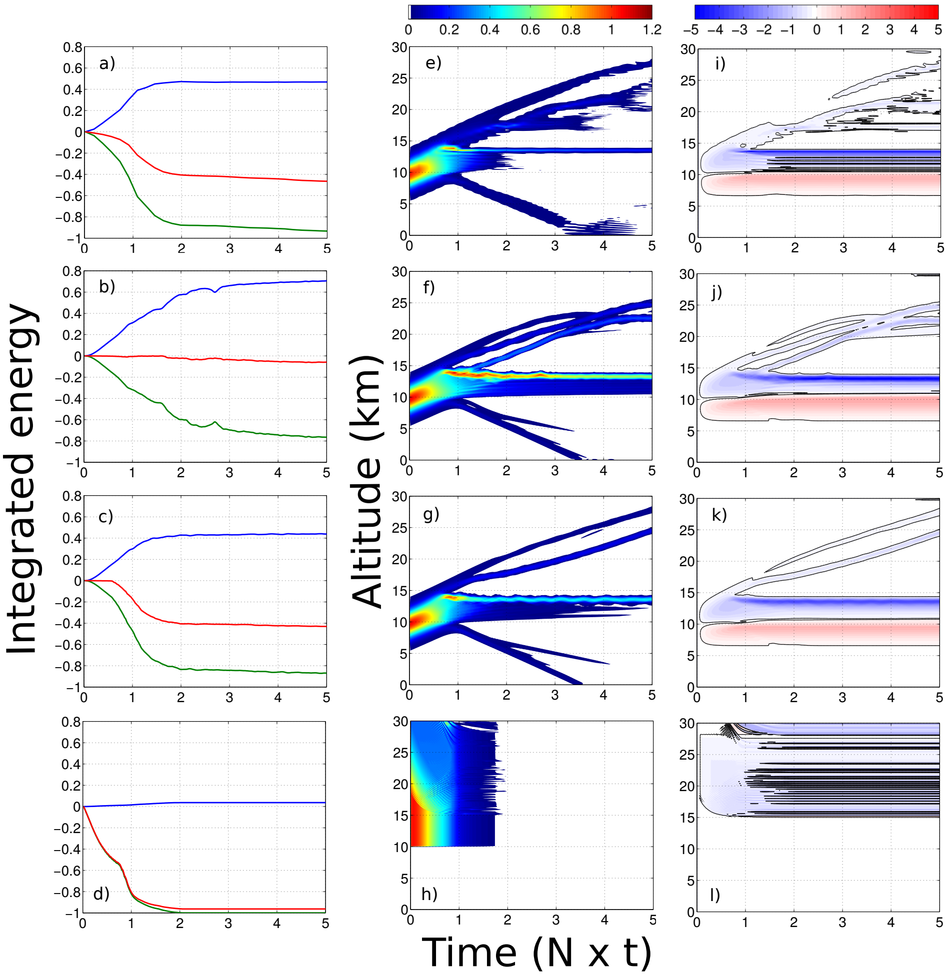

Figure 1: Time evolution of normalized vertically integrated energy (non-dimensional) of the GW packet (green), the mean-flow (blue) and their sum (red) (a)-(d); Hovmöller diagrams of the wave energy (m^2s^{-2}) (e)-(h) and the induced mean wind (ms^{-1}) (i)-(l); LES: (a),(e),(i), transient WKB model: (b),(f),(j), transient WKB model with wave breaking parametrization: (c),(g),(k); steady-state WKB model: (d), (h), (l). The solid black contours (values: -0.1, 0.1) in panels (i)-(l) are added to help the visual comparison.

4 Conclusions

In all investigated cases numerical simulations based on a transient WKB theory reveal a good agreement with Large Eddy Simulations, i.e. the spatio-temporal structure and magnitude of GW energy and the induced mean-flow are rather similar in the reference and the transient WKB simulation. Considering that the LES (transient WKB model) is fully nonlinear (neglects wave-wave interactions) and describes (does not account for) wave breaking, these results suggest that in the underlying dynamics here, wave-wave interactions and wave breaking are less important. A supplementary wave breaking parametrization in the transient WKB model helps to get an integrated energy, which is closer to the reference LES. However, this dissipative "correction" is not very large in magnitude and it does not change the spatial structure of the energy and momentum flux significantly, i.e. the GW drag is not affected strongly either. It thus appears that GW-mean-flow interactions following WKB theory can explain the predominant part of the dynamics in this study and that wave breaking has a secondary role in comparison with that.

In contrast, corresponding simulations using the steady-state assumption lead to too strongly damped wave fields and underestimated-, structurally oversimplified induced mean wind profiles. Based on the above, this failure of the steady-state approach is due to the the negligence of the transient GW-mean-flow interactions in play.

All this suggests that dropping the steady-state assumption in GW parametrization schemes might be a promising direction for developments.

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