A new theory for downslope windstorms and trapped mountain waves

François Lott
Laboratoire de Météorologie Dynamique, Ecole Normale Supérieure
24 rue Lhomond, 75231 Paris France, E-mail: flott@lmd.ens.fr

Abstract

Mountain gravity wave theories often neglect the observationnal evidence that the large scale winds can become very small near the earth surface. This common shortcoming is related to the fact that mountain waves have a critical level near the surface in this case, making the problem extremely involved. We briefly expose here how Lott (2016) circumvents this difficulty and derive a theory where linear mountain gravity waves are forced by a nonlinear surface boundary condition, where the background wind is null at the surface and increases smoothly to reach a constant value aloft, and where the buoyancy frequency is constant. In this configuration, the critical level just below the surface is dynamically controlled by the surface and minimum Richardson number J. When the flow is very stable ($J \gtrsim 1$), this critical level dynamics easily produces large downslope winds and Foehn. The downslope winds are more intense when the stability increases and much less pronounced when it decreases (when J goes below 1). On the opposite, the trapped lee waves are very small when the flow is very stable, start to appear when $J \approx 1$ and are more intense when the stability decreases further. For the trapped waves, this result is explained by the fact that the critical level absorbs the gravity waves downstream of the ridge when $J > 0.25$, while absorption decreases when J approaches 0.25. Two new examples for $J = 2$ and $J = 0.3$ are given here to illustrate these results.

Download Lott (2016) at: `http://www.lmd.jussieu.fr/~flott/articles/LOTT_JAS15.pdf`

1. Introduction

The interaction between orography and stratified flows is extremely sensitive to the upstream flow conditions "near" the surface, where "near" means up to around the maximum mountain height. Two classical examples can be used to illustrate this sensitivity. The first is that mountain flow regimes are largely controlled by the non dimensional mountain height, $H_N = \frac{HN}{U_0}$, where H is the maximum altitude of the mountain, N the low level flow stability, and U_0 a scale for the low level incident wind (Smith 1979). It compares the vertical wavelength of the stationary gravity waves and the maximum mountain height. When the first is small compared to the second, the gravity waves break near aloft the mountain, and the resulting nonlinear dynamics can yield downslope windstorms and Foehn (see reviews by Smith (1985) and Durran (1990)). A problem is that it is hard to define N or U_0: near the surface the vertical shear of the synoptic flow makes that the incident wind can become quite small and H_N can be made arbitrarily large. From a dynamical point of view, this follows that the vertical wavelength of stationary gravity waves becomes small when the incident wind is small, a near surface behavior which is strongly reminiscent of the concept of critical level introduced by Booker and Bretherton (1967). Importantly also, near a critical level, the horizontal winds due to the gravity waves can become very large. We will see that this can explain the onset of downslope winds and Foehn, even when there is no upper level gravity waves breaking.

The second example is that the low level shears control the onset of trapped waves, which are free modes of oscillations that are resonantly excited by mountains (Scorer (1949); Durran (1990)). According to the conventional theory of trapped lee waves, these free modes develop along a low-level wave guide which often result from the trapping of vertically propagating solutions between a perfectly reflecting surface and a turning point aloft. Again, in most studies, the smallness of the incident wind near the surface and the presence of a wind shear above is not a central ingredient. This has been criticized by Smith et al. (2006) and Jiang et al. (2006) which analyze the reflection of the downward propagating waves by the boundary layer and discuss how it affects the low level wave-guide. It is in this context that Lott (2007) calculates almost exactly the reflection of downward stationary gravity waves by a viscous boundary layer. In this viscous case, the wind at the surface has to be null, constant background wind shear and stratification are exact solutions, and the surface reflection decreases when the surface Richardson number J increases: perfect reflection only occurs in the inviscid limit when $J < 0.25$.

Basically, the theory developped in Lott (2016) apply these results by analyzing the disturbances produced by a mountain when the incident flow is nul at the surface far upstream. To understand why this has not been done before it is important to recall that there is a fundamental reason that makes this problem very involved: in the inviscid linear case, both (i) the vertical velocity associated with stationary gravity waves and (ii) the mountain forcing are null at $z = 0$. The inclusion of dissipations can help to circumvent the first problem, but as the viscous ones are extremely complex to treat, linear ones are used in Lott (2016). For the second problem, a nonlinear free-slip boundary condition can be taken at $z = h$, the altitude of the mountain, rather than at $z = 0$ only (Long 1953).

2. Formalism

To analyze the mountain gravity waves produced by a stably stratified shear which is null at the surface, we consider the background flow vertical profiles

$$U(z) = U_0 \tanh(z/d), N^2(z) = \text{constant}, \tag{1}$$

incident on a 2-dimensionnal mountain which height follows the Witch of Agnesi profile

$$h(x) = \frac{H}{1 + \frac{x^2}{2L^2}}. \tag{2}$$

Inflow, we then use linear Boussinesq equations, and a dimensionnal analysis shows that the response is controlled by the non-dimensionnal numbers

$$J = \frac{N^2 d^2}{U_0^2},\ H_N = \frac{HN}{U_0},\ \text{ and } F_r = \frac{LN}{U_0}, \tag{3}$$

e.g. the surface and minimum Richardson number, the non-dimensional mountain height, and the Froude number respectively. These 3 parameters are central to our study, the significance of J and H_N have already been discussed before, whereas we know from the literature that the Froude number measures the significance of non-hydrostatic effects: when it decreases more harmonics are evanescent in the vertical when $z \to \infty$, and the relative contribution of the trapped harmonics to the total response increases. We also introduce linear dissipations (raygleigh drag, Newtownian cooling and horizontal diffusion), and can evaluate for them a non-dimensional boundary layer Z_B depth over which they are active (see Lott 2016 for details). These disspations are introduced to regularize the near surface critical level dynamics.

As the inflow solutions are linear they are expressed in terms of Fourier transforms,

$$w(x, z) = \int_{-\infty}^{+\infty} f(k)\hat{w}_c(k, z)e^{ikx}dk, \tag{4}$$

where $\hat{w}_c$ is a characteristic solution of the vertical structure equation. Far above the boundary layer it is solution of the inviscid Taylor-Goldstein equation,

$$\frac{d^2\hat{w}_c}{dz^2} + \left(\frac{N^2}{U^2} - \frac{U_{zz}}{U} - k^2\right)\hat{w}_c = 0 \tag{5}$$

and for the profiles in (1) its solutions can be expressed analytically in terms of Hypergeometric functions (VanDuin and Kelder 1982). We then retain the one that corresponds to a unit amplitude exponentially decaying solution in $z \to \infty$ when $|k| > N/U_0$ or to a unit amplitude upward propagating gravity wave when $|k| < N/U_0$:

$$\hat{w}_c(k,z) \underset{z\to\infty}{\approx} e^{-\sqrt{k^2-\frac{N^2}{U_0^2}}\,z}, \text{ when } |k| > N/U_0, \text{ and } \hat{w}_c(k,z) \underset{z\to\infty}{\approx} e^{i\text{sign}(k)\sqrt{k^2-\frac{N^2}{U_0^2}}\,z} \text{ when } |k| < N/U_0. \quad (6)$$

Near the surface, the dissipations can no longer be neglected but the vertical structure equation can be derived (Booker and Bretherton 1967),

$$\frac{d^2\hat{w}_c}{d\tilde{z}^2} + \frac{J}{\tilde{z}^2}\hat{w}_c = 0, \text{ where } \tilde{z} = z - iz_k, \quad (7)$$

z_k being a small linear dissipation scale, it has for solution

$$\hat{w}_c \approx a_1(k)\tilde{z}^{\frac{1}{2}-i\mu} + a_2(k)\tilde{z}^{\frac{1}{2}+i\mu}. \quad (8)$$

To ensure matching we then take for a_1 and a_2 the values deduced from the asymptotic form of the inviscid exact solution. As boundary condition, we typically take the nonlinear equation,

$$w(z=h) = U(z=h)\frac{dh}{dx} \quad (9)$$

which permits to calculate the amplitude factor $f(k)$ in (4) through inversion of the equation:

$$\int_{-\infty}^{+\infty} \left(a_1(k)\tilde{h}^{+1/2-i\mu} + a_2(k)\tilde{h}^{+1/2+i\mu}\right) e^{ikx} f(k)dk = U(h)\frac{dh}{dx}, \quad (10)$$

where $\tilde{h}(x,k) = h(x) - iz_k$ (see Lott 2016 for the numerical aspects).

3. Results

To illustrate the conditions favorable to downslope winds, Foehn, and trapped waves we next consider a mountain of moderate elevation compare to the shear layer depth, $H_N = 0.25 * \sqrt{J}$, where the $\sqrt{J}$ term is to insure that when the Richardson number is changed, we do not change the penetration of the mountain into the shear: $H/d = H_N/\sqrt{J} = 0.25$. We also take a boundary layer of moderate depth compared to the mountain height $Z_B = 0.1H_N$. Then, to potentially excite substantial trapped waves, we also take a quite narrow mountain $F_r = 2$. Finally the model grid has an horizontal dimension of $D = 200U_0/N$ spanned by $M = 2048$ points. As the model in Lott (2016) is non-dimensionnal we do not need to specify U_0 and N.

The vertical velocity field produced by the obstacle when $J = 2$ is shown in Fig. 1a. It has the characteristic structure of vertically propagating mountain gravity waves with phase lines tilted in the direction opposite to the incident wind. We also see that the wave field is essentially confined to the lee-side, the upstream disturbances being very small. Comparing this behaviour to the more conventionnal case where U is uniform, Lott (2016) have shown that this asymetry is a direct consquence of the combination between the nonlinear boundary condition and the low level wind shear. Also, it is not related to wave trapping, the same response occuring for large F_r and in the hydrostatic case. Concerning wave trapping, it is quite evident in Fig. 1a that the trapped lee waves are rapidly damped downstream, consistent with the results in Lott (2007) that the stationary gravity waves are strongly absorbed downstream. The asymmetry between the upstream and lee sides of the ridge is even more remarkable on the horizontal wind disturbance near the surface and because for each harmonics the horizontal wind amplitude varies in $\tilde{z}^{-1/2}$ rather than in $\tilde{z}^{1/2}$ for $\hat{w}$ in Eq. (8) (this follows that div $\vec{u} = 0$). The consequences on the total wind vector near the surface are quite spectacular, with a very strong wind flowing

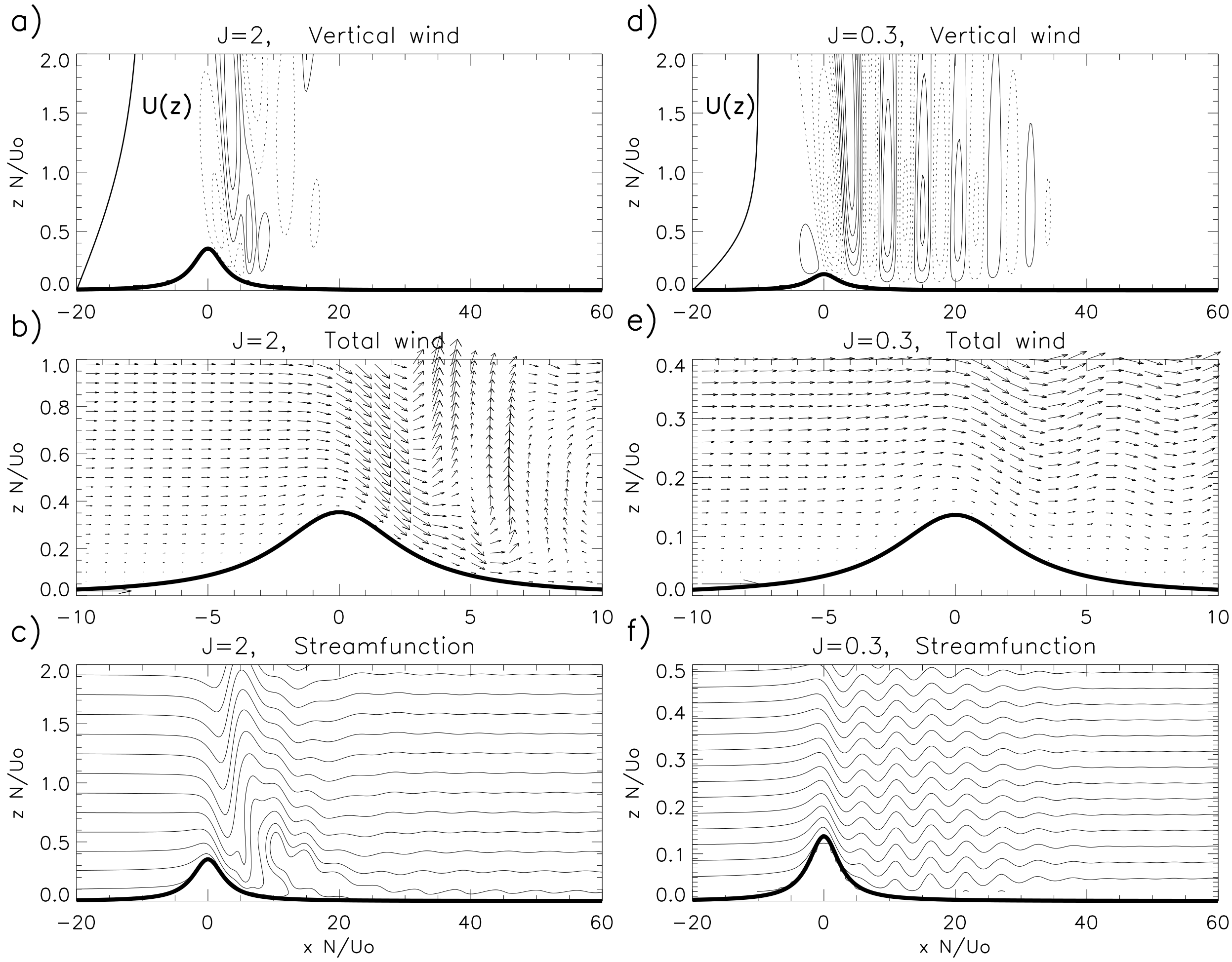

FIG. 1. Flow field produced when the minimum Richardson number $J = 2$ (a, b, c) and $J = 0.3$ (d, e, f). In the two cases, the Froude number $F_r = 2$, the penetration of the mountain into the shear is conserved $H/d = 0.25$ so $H_N = 0.25\sqrt{J}$, and the boundary layer depth $Z_B = 0.1H_N$. In all panels the horizontal and vertical scales are normalized by the characteristic gravity waves vertical wavenumber N/U_0.

downslope of the ridge, whereas the total wind upslope is quite small (see Fig. 1b). The upslope/downslope asymmetry is also very pronounced on streamfunction in Fig. 1c, where the Foehn effect is very pronounced, resulting also in convective overturning on the lee-side of the mountain. Importantly, low level Foehn and strong downslope winds here are not related to upper level wave breaking, as witnesses again the streamlines in Fig. 1c.

The behaviour when $J = 0.3$ is quite significantly different. The wave field now is still confined downstream, and we clearly see on the vertical velocity in Fig. 1d that the trapped lee-waves are much more substantial than when $J = 2$. Again, Lott (2007) and Lott (2016) have shown that this is because for $J \approx 0.25$ the near-surface critical level absorption is small downstream, the trapped lee-waves can easily develop. Also, we see in Fig. 1d that the downslope winds are not very stong, and on Fig. 1e that there is almost no Foehn.

These figures therefore summarize well the central result in Lott (2016) that trapped lee waves and downslope winds are two regimes of mountain flow dynamics that are clearly separated by the value of the surface Richardson number.

4. Summary

The central result of the paper is that the critical level dynamics near the surface can play a crucial role in mountain flow dynamics. It produces large horizontal wind and buoyancy disturbances at low level that result in intense downslope winds and Foehn. These phenomena occur almost systematically when the flow is stable ($J \gtrsim 1$). The downslope winds intensity also rapidly increases when the flow stability increases, a behavior consistent with the fact that in nature downslope winds more likely occurs during night, e.g. during more statically stable situations (Jiang and Doyle 2008). Interestingly, the mechanism proposed here do not call for upper level wave breaking or trapped waves resonance as often suggest other theories (Smith 1985; Durran 1990). The critical level dynamics also impacts the onset of trapped lee waves. Their downstream extension also becomes conditional to the value of the Richardson number at the surface, with trapped lee waves occurring more easily when $J < 1$ and decreases. This is again consistent with the observationnal fact that trapped waves are favored when the low level flow is more unstable.

Acknowledgments.

This work was supported by the European Commission's project ARISE2, (Grant agreement 653980).

REFERENCES

Booker, J. R., and F. P. Bretherton, 1967: The critical layer for internal gravity waves in a shear flow. *J. Fluid Mech.*, **27**, 102–109, doi:http://dx.doi.org/10.1017/S0022112067000515.

Durran, D. R., 1990: Mountain waves and downslope winds. *AMS Meteorological Monographs*, **23**, 59–83.

Jiang, Q., and J. D. Doyle, 2008: On the diurnal variation of mountain waves. *J. Atmos. Sci.*, **65**, 1360–1377, doi:10.1175/2007JAS2460.1.

Jiang, Q., J. D. Doyle, and R. B. Smith, 2006: Interaction between trapped waves and boundary layers. *J. Atmos. Sci.*, **63**, 617–633, doi:http://dx.doi.org/10.1175/JAS3640.1.

Long, R. R., 1953: Some aspects of the flow of stratified fluids; 1. a theoretical investigation. *Tellus*, **5**, 42–58.

Lott, F., 2007: The reflection of a stationary gravity wave by a viscous boundary layer. *J. Atmos. Sci.*, **139**, 3363–3371, doi:http://dx.doi.org/10.1175/JAS4020.1.

Lott, F., 2016: A new theory for dowslope windstorms and trapped mountain waves. *J. Atmos. Sci.*, **In press**, 0000–0000.

Scorer, R. S., 1949: Theory of waves in the lee of mountains. *Quart. J. Roy. Meteor. Soc.*, **75**, 41–56.

Smith, R. B., 1979: The influence of mountains on the atmosphere. *Advances in Geophys.*, **21**, 87–230.

Smith, R. B., 1985: On severe downslope winds. *J. Atmos. Sci.*, **42**, 2597–2603, doi:http://dx.doi.org/10.1175/1520-0469(1985)042⟨2597:OSDW⟩2.0.CO;2.

Smith, R. B., Q. Jiang, and J. D. Doyle, 2006: A theory of gravity wave absorption by a boundary layer. *J. Atmos. Sci.*, **63**, 774–781, doi:http://dx.doi.org/10.1175/JAS3631.1.

VanDuin, C. A., and H. Kelder, 1982: Reflection properties of internal gravity waves incident upon an hyperbolic tangent shear layer. *J. Fluid Mech.*, **120**, 505–521.