Dynamics of flow over a sphere at Re = 3700 in moderate to highly stratified environments.

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Abstract

A direct numerical simulation (DNS) study of the turbulent wake of a sphere is performed over a wide range of background stratifications. It is found that, after an intial reduction of turbulence with increasing stratification, fluctuations reappear in the the near wake as the strength of the stratification is increased above a certain extent. The fluid flow escapes from the horizontal direction owing to the inhibition of the vertical velocity by stratification, thereby leading to a new regime of unsteady vortex shedding and increased horizontal shear. Examination of coherent structures identified using the Q-criterion shows that, near and far from the body, the vortical structures are reorganized by stratification.

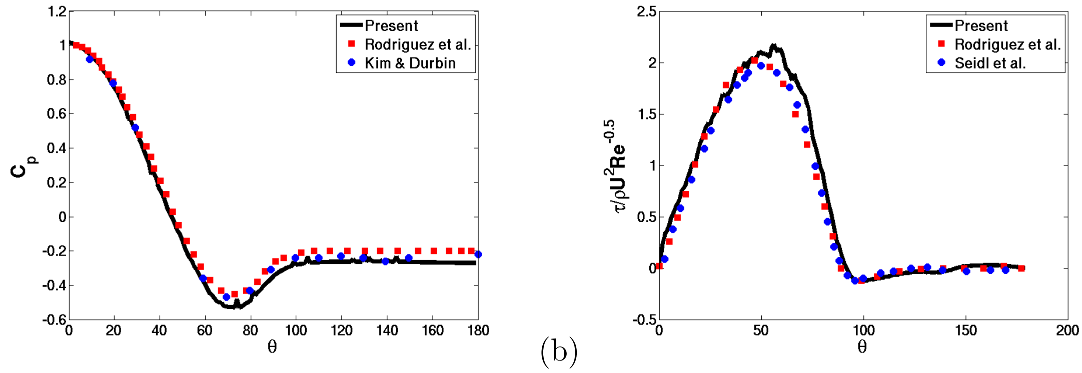
1 Introduction

Wakes of bluff bodies in a stratified environment are common in oceanic and atmospheric flows. Some examples are marine swimmers, underwater submersibles and flow over mountains and past islands. There are scenarios in which these wakes might encounter highly stratified surrounding, such as within a pycnocline. The first numerical simulations investigating laminar ($Re = 200$) flow over a sphere in a strongly stratified environment are reported in [1]. It was found that the flow is restricted from going over the sphere if $Fr_R = U/NR < 1$, and eventually approaches two dimensionality ($2D$) for $Fr_R < 0.4$ and is characterized by quasi $2D$ eddies. Experimental studies by [2] cover $Re \in [5, 10000]$ and $Fr_R \in [0.005, 15]$, however for low Fr_R, the flow patterns are similar to the findings of [1]. Quasi 2D eddies in a strongly stratified medium were studied in [3] and it was concluded that, at high Reynolds number, the vertical shear is able to sustain small scale turbulence until the local Re becomes smaller than a critical value. Experimental studies [2, 4] cover a wide range of Fr and Re, but the values of Re for low-Fr cases were low.

None of the previous studies of laboratory wakes show turbulent fluctuations in the strongly stratified regime of $Fr = U/ND < 0.5$. Since the low-Fr wakes were generally at low Re too, we perform a numerical study of low-Fr wake dynamics at a moderate Reynolds number of $Re = 3700$.

2 Numerical method and simulation parameters

DNS of flow past a sphere in a stratified fluid has been carried out at a sub-critical Reynolds number of 3700 and for a range of Froude numbers $(U/ND) \in [0.025, \infty]$, where U is the free stream velocity, D is the sphere diameter and N is the background stratification. The conservation equations are solved in a cylindrical coordinate system and an immersed boundary method is employed to represent the sphere [5, 6]. The choice

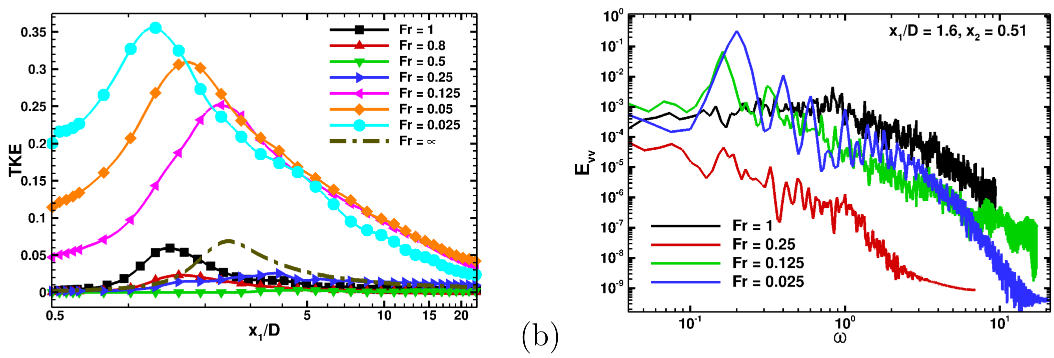


(a) (b)

Figure 1: Validation of (a) pressure coefficient, C_p (b) $\tau/(\rho U^2)Re^{-0.5}$ for $Fr = \infty$ with available literature.

of $Re = 3700$ allows validation against previous unstratified wake simulations including the recent DNS of [7] (figure 1).

3 Results and discussion



(a) (b)

Figure 2: (a) Evolution of integrated turbulent kinetic energy in streamwise direction, (b) Spectra of vertical w fluctuations at a downstream point $(x_1 = 1.6, x_2 = 0.51, x_3 = 0)$ in the horizontal center plane at various Froude numbers.

Figure 2 (b) shows the evolution of integrated turbulent kinetic energy in the downstream direction for different Fr cases. As the Fr decreases from 1 to 0.5 we observe a decrease in TKE. This decrease in TKE is attributed to inhibition of the fluctuations in the vertical direction owing to the effect of buoyancy. There is a significant decrease in TKE for $Fr = 0.5$ and the flow becomes almost laminar in nature. From all the previously reported experimental and numerical investigations [8, 9, 10, 11], it is a general observation that increasing stratification suppresses turbulence. However, a very different behavior is observed in the present cases with Fr lower than 0.5. As Fr decreases to 0.25, an increase in TKE to values similar to those of case $Fr = 0.8$ is noticed. A further decrease of Fr to 0.125, manifests a significant increase in TKE, substantially larger than even the unstratified case with $Fr = \infty$. The increasing trend in TKE continues as Fr reduces to 0.05 and 0.025. It is also worth noticing that the fluctuations for cases $Fr = 0.125, 0.05, 0.025$ at $x/D = 0.5$ are relatively in an increasing order as compared to very low values for the remaining cases at the same streamwise location.

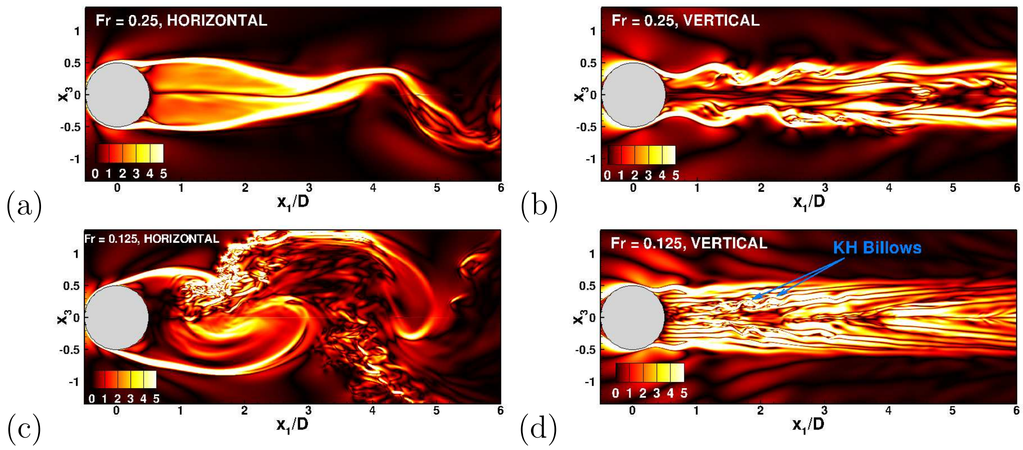


Figure 3: Azimuthal vorticity magnitude on $x_1 - x_2$ ($x_3 = 0$; horizontal) and $x_1 - x_3$ ($x_2 = 0$; vertical) planes.

The spectra of the vertical velocity fluctuations are presented in figure 2(b). The energy content in the vertical fluctuations show a significant decrease from $Fr = 1$ to 0.25. However, with a further increase in stratification (decrease in Fr) re-energizing of low and high-frequency modes is observed. This reappearance of turbulence at low Fr is explained in the upcoming paragraphs.

A recirculation bubble attached to the sphere is observed for $Fr = 0.25$ in the horizontal plane (figure 3(a)). At the end of the recirculation zone, the wake undergoes an undulation with the shedding of vortices further downstream. The shear layer in the vertical direction (figure 3(b)) manifests waviness, but does not break down into small scales. This shear layer forms a barrier in the vertical and prevents the interaction of the fluid in the wake region with the ambient fluid. It is also worth noting that buoyancy organizes the wake into thin vortical layers. Further decrease of Fr to 0.125 leads to small-scale patches in the horizontal plane (figure 3(c)) as compared to the smooth recirculation bubble at $Fr = 0.25$. This reappearance of small scale fluctuations in the $Fr = 0.125$ case can be attributed to unsteady vortex shedding in the horizontal from the sphere and vortex interactions which results in thinning and eventual destabilization of the shear layer. The high stratification also leads to vertically thin layers of horizontal motion with differences in velocity. The oscillations in the shear layer barricading the wake regime and the surrounding fluid, perturbs the organized layered structures beneath it. These perturbed layers manifest $K - H$ billows (figure 3 (d)). These $K - H$ rolls have been noticed in previous temporal simulations of the far wake [10] and a horizontal shear layer [12] in a stratified fluid.

We examine vortical structures using the Q-criterion [13], that identifies vortices by the region where the rate of rotation tensor, Ω_{ij}, exceeds the strain rate tensor, S_{ij}. Q is defined by

$$Q = \frac{1}{2}(|\mathbf{\Omega}|^2 - |\mathbf{S}|^2);\ \Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right),\ S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{1}$$

Large positive Q implies strong swirling motion. Figure 4 shows instantaneous vortical structures in the wake using the Q-criterion at Q = 1 for the unstratified case. Near the body, the vortex rings shed from the sphere stay circular before transitioning to turbulence via Kelvin-Helmholtz instability at $x/D \sim 2.4$. Immediately downstream of the transition, a bundle of entangled vortical structures emerges. These vortices are tube-like structures with high length-to-diameter aspect ratio, the so-called vortex tube. These vortex tubes lack directional preference and the density of these vortex tubes per unit volume based on $Q = 1$ decreases significantly for $x/D > 7$.

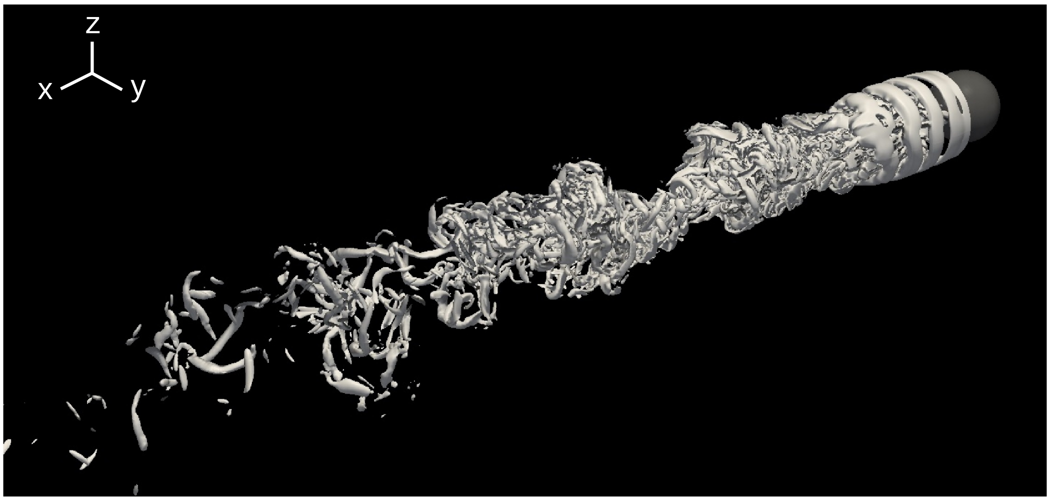


Figure 4: Iso-surface of Q criterion at Q = 1 for $Fr = \infty$.

Figure 5 presents the structural organization in the wake by a isocontour of Q = 50. The vortices for the unstratified $Fr = \infty$ wake at this high level of Q are present in the region $1.5 < x_1/D < 5$ while they are absent elsewhere. The fact that the strength (Q) of the vortex tubes spanning $1.5 < x/D < 5$ is higher than that of vortices shed from the body indicates that vortex shedding from the body is not the only source of vorticity. The vortex tubes at $Fr = 1$ (figure 5 (right)) have a vertical undulation owing to a lee wave pattern behind the body. The number density of vortical structures is significantly smaller than that in the unstratified wake at this level of stratification. The vortex tubes are confined to streamwise-oriented regions when Fr decreases to 0.25 (not shown).

The quasi-2D regime is a feature of the far wake, appearing at $x/D \approx 1000$ or $Nt \approx 250$ in the moderately stratified $Fr = 4$ wake simulated by [9] using a temporal flow model. When the stratification is very high, pancake vortices emerge in the near wake. Figure 6 (top) shows the isosurface of Q = 0.25 in a perspective view for $Fr = 0.125$ and reveals two types of organized structures. Pancake vortices are clear and the first pancake eddy is seen in the perspective view of the top panel at $x/D \approx 5.8$ which corresponds to $Nt = 5.8/Fr = 232$ which is close to the value of $Nt \approx 250$ quoted by [9]. While the pancake vortices are located off the center line, there are "surfboard" like structures located sequentially closer to the center. The side view (figure 6 middle) shows that the surfboard-like structures are not oriented horizontally and their leading edges are located at the same x/D location as of the pancake eddies. In the side view, each surfboard pair appears as a V with an arm of the V emerging from the top or the bottom of the pancake. Figure 6 (bottom) shows pancake eddies are approximately equispaced with a wavelength of $\lambda/D \approx 5$. Assuming that the coherent structures are convected with the freestream velocity allows conversion of the wavelength to a temporal frequency with Strouhal number of $St = fD/U \approx 0.2$. The primary peak in the vertical velocity

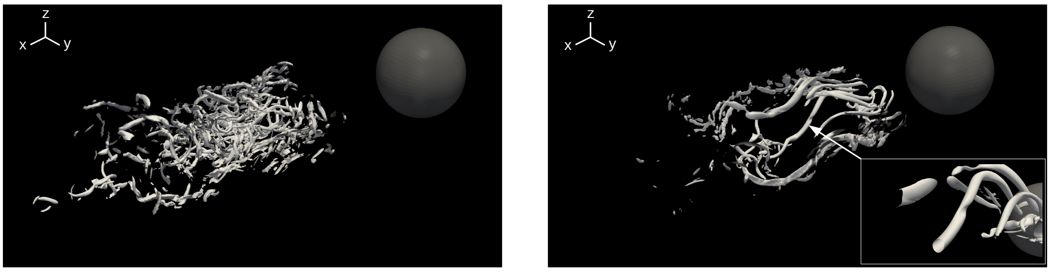


Figure 5: Iso-surface of Q criterion at Q = 50 for $Fr = \infty$ (left) and $Fr = 1$ (right). Inset on right panel shows the circular cross-section of the vortex tube.

frequency spectra (figure 2) is at $St = 0.2$. Therefore, the origin of these pancake eddies is vortex shedding from the sides of the sphere (similar to the Karman vortex street of a cylinder) rather than from an instability of the wake flow profile. The surfboard structures are shed from the sphere with the same frequency as of the pancakes.

4 Summary and Conclusions

Results from DNS of flow over a sphere [14] at a moderate $Re = 3700$ and a range of $Fr \in [0.025, \infty]$ are discussed. Regeneration of fluctuations is observed as Fr is decreased below 0.5. This reappearance of fluctuations at $Fr < 0.5$ occurs because the suppression of vertical motion by buoyancy leads to the escape of approaching fluid by horizontal motion around the sphere which in turn results in high shear in the horizontal and unsteady vortex shedding in horizontal planes. Coherent structures found in the study of vortex dynamics [15] in stratified wakes are also discussed. The isosurfaces of Q in the unstratified wake reveal highly-rotational vortex tubes (Q larger than that of the vortex rings shed off the sphere) in the $1.5 < x/D < 5$ region behind the sphere whose rotational strength gradually decreases with increasing downstream distance. Moderate stratification with $Fr = O(1)$ preferentially orients vortex tubes in the streamwise direction but does not change their tube-like shape whereas high stratification ($Fr \leq 0.125$) changes the cross-section of vortex tubes from circular to flattened shapes. At $Fr = 0.025$, the isosurface of Q shows distinct pancake eddies and inclined surfboard structures. The spacing of the two consecutive pancake eddies on the same side corresponds to the frequency of vortex shedding from the sphere.

References

[1] H. Hanazaki. A numerical study of three-dimensional stratified flow past a sphere. *J. Fluid Mech.*, 192:393–419, 1988.

[2] Q. Lin, W. R. Lindberg, D. L. Boyer, and H. J. S. Fernando. Stratified flow past a sphere. *J. Fluid Mech.*, 240:315–354, 1992.

[3] J. J. Riley and S. M. deBruynKops. Dynamics of turbulence strongly influenced by buoyancy. *Phys. Fluids*, 15(7):2047–2059, 2003.

[4] J. M. Chomaz, P. Bonneton, and E. J. Hopfinger. The structure of the near wake of a sphere moving horizontally in a stratified fluid. *J. Fluid Mech.*, 254(1):1–21, 1993a.

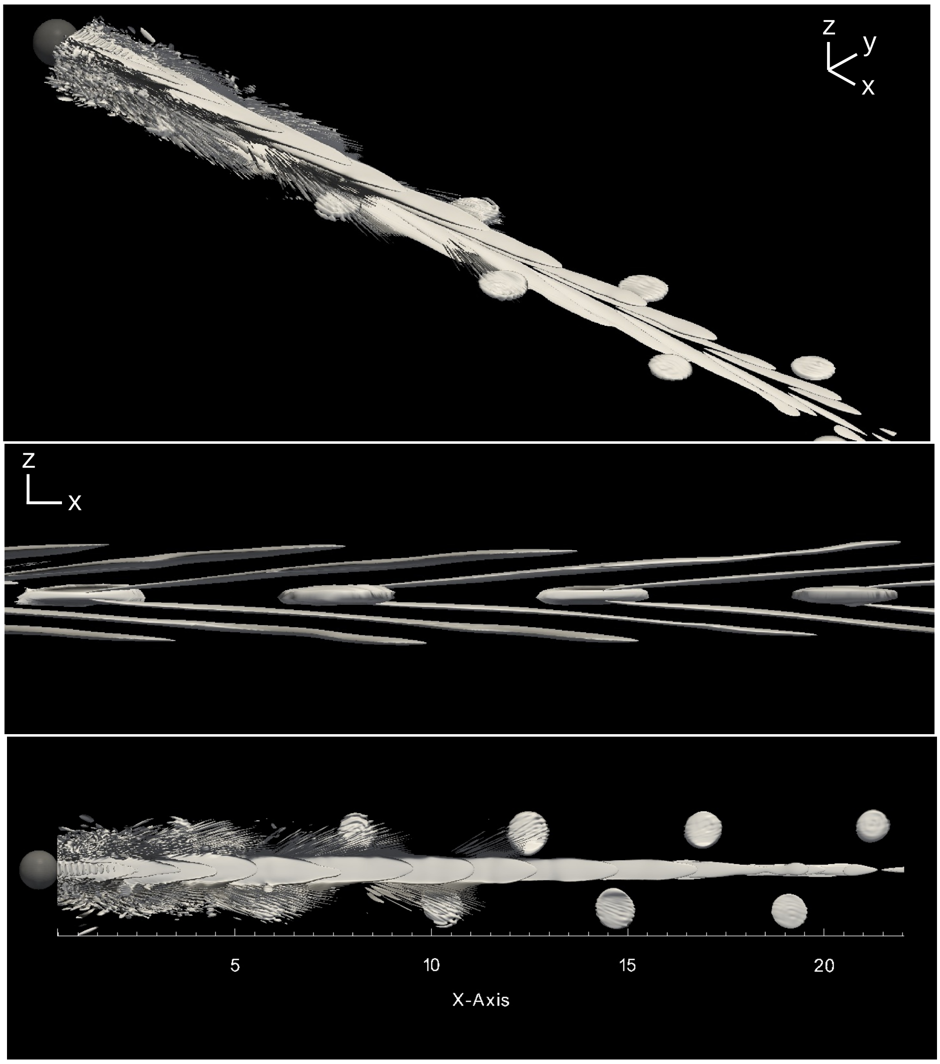


Figure 6: Coherent structures in a strongly stratified wake ($Fr = 0.025$) visualized with the isosurface of Q = 0.25. Top panel is a perspective of the wake with the sphere at the upper left corner. Middle pane is a side view (flow from left to right) and bottom panel is a top view.

[5] E. Balaras. Modeling complex boundaries using an external force field on fixed cartesian grids in large-eddy simulations. *Comput. Fluids*, 33(3):375–404, 2004.

[6] J. Yang and E. Balaras. An embedded-boundary formulation for large-eddy simulation of turbulent flows interacting with moving boundaries. *J. Comput. Phys.*, 215(1):12–40, 2006.

[7] I. Rodriguez, Y. Borelli, O. Lehmkuhl, C. D. Perez Segarra, and O. Assensi. Direct numerical simulation of the flow over a sphere at re = 3700. *J. Fluid Mech.*, 679:263–287, 2011.

[8] J. T. Lin and Y. H. Pao. Wakes in stratified fluids. *Ann. Rev. Fluid Mech.*, 11:317–338, 1979.

[9] K. A. Brucker and S. Sarkar. A comparative study of self-propelled and towed wakes in a stratified fluid. *J. Fluid Mech.*, 652:373–404, 2010.

[10] P. J. Diamessis, G. R. Spedding, and J. A. Domaradzki. Similarity scaling and

vorticity structure in high Reynolds number stably stratified turbulent wakes. *J. Fluid Mech.*, 671:52–95, 2011.

[11] G. R. Spedding. Wake signature detection. *Ann. Rev. Fluid Mech.*, 46:273–302, 2014.

[12] E. Arobone and S. Sarkar. The statistical evolution of a stratified mixing layer with horizontal shear invoking feature extraction. *Phys. Fluids*, 22(11):115108, 2010.

[13] J.C.R Hunt, A.A. Wray, and P. Moin. Eddies, streams, and convergence zones in turbulent flows. Technical report, CTR, 1988.

[14] A. Pal, S. Sarkar, A. Posa, and E. Balaras. Regeneration of turbulent fluctuations in low Froude number flow over a sphere at Re = 3700 (under revision). *J. Fluid Mech.*

[15] K. Chongsiripinyo, A. Pal, and S. Sarkar. On the vortex dynamics of flow past a sphere at Re = 3700 in a uniformly stratified fluid (submitted). *Phys. Fluids.*