

Toward Direct Numerical Simulations of the Stratified Turbulence Inertial Range

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Abstract

It has been proposed in the literature that the concept of a stratified turbulence inertial range may explain important features of stratified turbulence in the ocean and atmosphere (Riley and Lindborg, 2008). In this paper, we consider the feasibility of using direct numerical simulations (DNS) on today's computers to address the stratified turbulence inertial range. DNS is a research tool that has proven valuable for better understanding turbulence, but it requires that all dynamically relevant length scales be resolved so that all the inter-scale interactions are correctly accounted for. Preliminary results indicate that it is possible to simulate an inertial range in the strongly stratified regime at high Reynolds number with careful selection of parameters in the Froude-Reynolds number space.

1 Introduction

A widely used conceptual model for stratified turbulence includes an inertial range divided into two subranges as indicated in the sketch of the energy spectrum in figure 1. For length scales larger than the Ozmidov length scale, L_o , there is, in the model, a stratified turbulence inertial range that is affected by buoyancy but not by viscosity. Below the Ozmidov length is a classical, or "Kolmogorov", inertial range. As discussed in Riley and Lindborg (2008), there are important unanswered questions about the stratified turbulence inertial range, in particular whether stratified turbulence explains $\kappa^{-5/3}$ scaling in ocean and atmospheric data at horizontal length scales too large for it to be explained by the theories of Kolmogorov, Oboukhov, and Corrsin.

Using the notation defined in the figure, the total dynamic range available for turbulence is characterized by the Reynolds number $Re = (L/L_k)^{4/3}$, where L is a horizontal length scale characteristic of the energy containing motions and L_k is the Kolmogorov length scale. The part of the total dynamic range not affected by buoyancy is characterized by $Gn = (L_o/L_k)^{4/3}$. We use the symbol Gn for this parameter (sometimes called the activity parameter) in recognition of Gibson's seminal work with this quantity and of Gargett's association of it with the dynamic range available for fully three-dimensional turbulence (Gibson, 1980; Gargett et al., 1984). Justification for the model depicted in the figure typically includes the assumptions that both Re and Gn are sufficiently large and that the Froude number, Fr , is of order one or less. Since Re (the dynamic range) in the ocean and the atmosphere is typically many orders of magnitude higher than what can be resolved

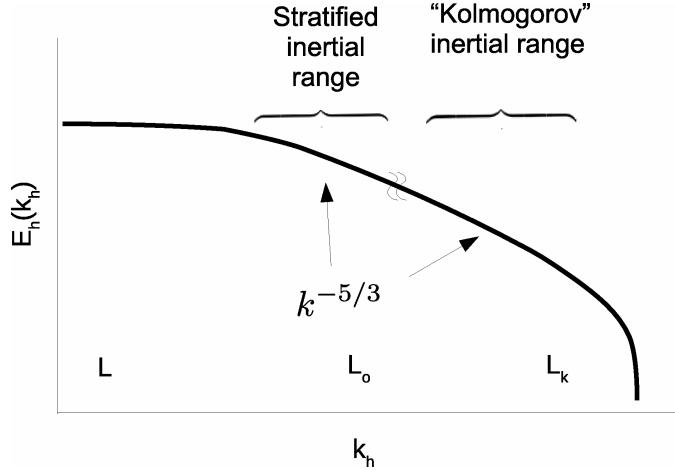


Figure 1: Sketch of stratified turbulence conceptual model in terms of the energy spectrum. E_h is the spectrum of the horizontal component of kinetic energy.

in simulations for the foreseeable future, we narrow our investigation here to identifying the ranges of Gn and Fr we can simulate accurately.

The parameter range $Gn \in [1, 1000]$ is observed to occur widely in the world's oceans (Gargett et al., 1984; Jackson and Rehmann, 2014; Salehipour et al., 2016), and so it is of geophysical interest to investigate this range. This range is also of particular interest as different dynamical regimes have been identified with different ranges of Gn . Specifically, turbulence is expected to be suppressed leading to quiescent flow when $Gn \sim O(1)$ (Brethouwer et al., 2007). In the flows considered by Bartello and Tobias (2013), they found that the turbulence dynamics are very sensitive to Gn unless Gn is larger than order $O(10)$, which is consistent with some of the earliest scaling analyses on the subject (Gargett et al., 1984; Gibson, 1986; Rohr et al., 1988; Itsweire et al., 1993). Such sensitivity is also observed in the results reported in both Hebert and de Bruyn Kops (2006a) and Hebert and de Bruyn Kops (2006b). More recently, de Bruyn Kops (2015) found that the dynamics of stratified flows are different at $Gn = 48$ and $Gn = 220$, which suggests another threshold at $Gn \sim O(100)$, consistent with the findings of Shih et al. (2005) and Salehipour and Peltier (2015).

In an attempt to understand the dynamics of the stratified turbulence inertial range using direct numerical simulations while guarding against misleading results due to insufficient dynamic range, a new series of simulations has been generated using up to 16384 grid points in each of the horizontal directions. These simulations are related to those reported in de Bruyn Kops (2015), that is, they are forced, homogeneous, and horizontally axisymmetric (in the statistical sense), but are different in an important way. Whereas the published simulations are for a single Re and multiple Froude numbers Fr (and hence multiple Gn), the new simulations are designed to have a given Gn with four different combinations of Fr and Re . This is done for four different values of Gn (14, 42, 220, 1000) for a total of 16 simulations.

Table 1: Simulation metrics. Identifiers are derived from nominal Fr_h and Gn . Computed values are tabulated. Run 1, F1, etc. refer to comparable simulations in Lindborg (2006) and Almalkie and de Bruyn Kops (2012) taking into account the different definitions of the length scale for Fr_h and Re_h in those papers and this paper. Lindborg's simulations are LES at lower resolution and so the comparison is indicated for only one of the simulations in each Fr_h group.

Identifier	Fr_h	Re_h	Gn	Re_b	ϵ_ν/ϵ	N_x, N_y	N_z	Compare
F025Gn0014	0.20	78115	14	3170	1.00	16384	2048	Run 4
F025Gn0050	0.19	250035	39	8810	0.65	16384	2048	
F025Gn0223	0.28	598430	174	45825	0.22	12288	1536	
F025Gn1000	0.16	9411831	584	251241	0.06	15360	1920	
F050Gn0014	0.52	5932	15	1619	1.00	4096	512	Run 3, F3
F050Gn0050	0.43	12667	24	2369	0.99	12288	1536	
F050Gn0223	0.75	96391	138	53510	0.90	12288	1536	
F050Gn1000	0.72	642512	1552	334059	0.06	8192	1024	
F100Gn0014	1.02	1453	11	1498	1.00	4096	1024	Run 2
F100Gn0050	1.17	5307	46	7250	0.99	8192	2048	F2
F100Gn0223	1.31	25429	190	43700	0.96	16384	4096	
F100Gn1000	1.36	130057	691	240099	0.48	10801	2700	
F200Gn0014	2.11	415	14	1849	1.00	2048	1024	Run 1
F200Gn0050	2.19	1299	50	6237	1.00	4096	2048	
F200Gn0223	2.02	7043	207	28740	1.00	8192	4096	F1
F200Gn1000	2.82	31405	829	249251	1.00	12288	6144	

2 Methodology

We consider statistically stationary direct numerical solutions of the Navier Stokes equations subject to the non-hydrostatic Boussinesq assumption and with unity Prandtl number. Periodic boundary conditions are imposed in all directions. The flow has no mean shear, but a uniform background ambient stratification is maintained so that the flow statistics are homogeneous and independent of (horizontal) direction. Quasi-stationarity is maintained by forcing the largest scales of the horizontal velocities to have a prescribed spectrum using the method denoted Rf in Rao and de Bruyn Kops (2011). Local shear is induced by random low-energy perturbations to the horizontal velocity components at small vertical wave numbers. The prescribed spectrum for the forcing is obtained by repeating the simulations of Lindborg (2006) using his forcing method and so the current flows are very similar in structure to those in that paper.

The simulation methodology is the same as that reported in Almalkie and de Bruyn Kops (2012) and de Bruyn Kops (2015) except here the equations of motion include fourth-order hyperviscous and hyperdiffusive terms as in Lindborg (2006). In many of the the simulations these terms have negligible effect, but they are included in all cases for consistency. The importance of these terms can be quantified by defining ϵ_ν as the dissipation rate of turbulence kinetic energy due Newtonian viscosity and compare it with ϵ , which is the dissipation rate due to Newtonian and hyperviscous effects. The ratio ϵ_ν/ϵ , along with

other simulation parameters, is tabulated in table 1. Fr_h is a horizontal Froude number defined in terms of the horizontal integral length scale and the r.m.s. horizontal velocity, Re_h is the corresponding horizontal Reynolds number, and $Re_b \equiv Fr_h^2 Re_h$. N_x , N_y , and N_z are the number of grid points in each direction. Note that, except for differences in the details of the forcing schema and the resolution of the smaller scales, these simulations correspond to those of Lindborg (2006) and his notation for the simulations is included in the table for reference.

3 Discussion

The dynamic range requirements of the simulations are apparent from the table. At very low Froude number, $Fr_h \approx 0.25$, a fully-resolved simulation is feasible only at low Gn . Note that this Froude number corresponds to a value of about 6×10^{-3} in terms of the definition used by, e.g., Maffioli et al. (2016). At higher Fr_h , it is possible to fully resolve, or almost fully resolve, simulations with Gn up to 1000. We conclude that current computers may enable simulations that closely approximate the model flow illustrated in figure 1.

With the parameter range identified for this type of simulation on existing computers, our next step is to identify simulations that may exhibit stratified turbulence inertial ranges. The important length scale for identifying the potential for a stratified turbulence inertial range in a simulation is L_o . The usual definition of the Ozmidov length scale, based on dimensional reasoning, is $L_o \equiv (\epsilon/N^3)^{1/2}$, where ϵ is the dissipation rate of turbulence kinetic energy and N is the buoyancy frequency. Using this definition and noting that in unstratified turbulence the dissipation range extends to about 100 L_k , one might deduce that L_o is in the dissipation range unless $Gn \geq O(100)$. In several papers, an $O(10)$ multiplicative constant is included in the definition of L_o (Waite, 2011; Almalkie and de Bruyn Kops, 2012). This ad hoc approach increases L_o to above the dissipation range for all but flows with very low Gn .

An alternative method for quantifying the Ozmidov length in a simulation is to extend the approach of Riley and Lindborg (2008). In the appendix of that paper, the Ozmidov length is derived in terms of spectra. The implication is that L_o can be determined by observing the spectra of the horizontal and vertical velocities. These are plotted for several cases in figure 2. The notation is based on a coordinate system in which the velocity components (u, v, w) correspond with the Cartesian coordinates (x, y, z) . So $E_u(\kappa_x)$ is a longitudinal spectrum and $E_v(\kappa_x)$ and $E_w(\kappa_x)$ are transverse spectra. If there exists a dissipation range that is not significantly affected by buoyancy forces, it is expected that the two transverse spectra will coincide and that E_v/E_u will increase with increasing κ_x . In the Kolmogorov inertial range, if it exists, $E_v/E_u = 4/3$. At larger horizontal length scales in a stratified flow, it is expected that $E_w(\kappa_x) < E_v(\kappa_x)$ and $E_w(\kappa_x) < E_u(\kappa_x)$. Therefore an estimate of the Ozmidov scale and the existence of Kolmogorov inertial ranges might be deduced by observing the spectral ratios.

Turning now to figure 2, the spectral relationships outlined in the preceding paragraph are observed in several simulations. From the results of de Bruyn Kops (2015), it is not expected that a true classical inertial range will exist at $Gn \approx 200$, but the figure suggests that an approximate classical inertial range occurs in the vicinity of $\kappa_h = 80$ when $Fr_h \approx 1$ and so more detailed analyses may reveal a stratified inertial range at lower wave numbers. Some of the simulations not shown in the figure also appear to be consistent with the model described by figure 1. We are examining the flows to determine the

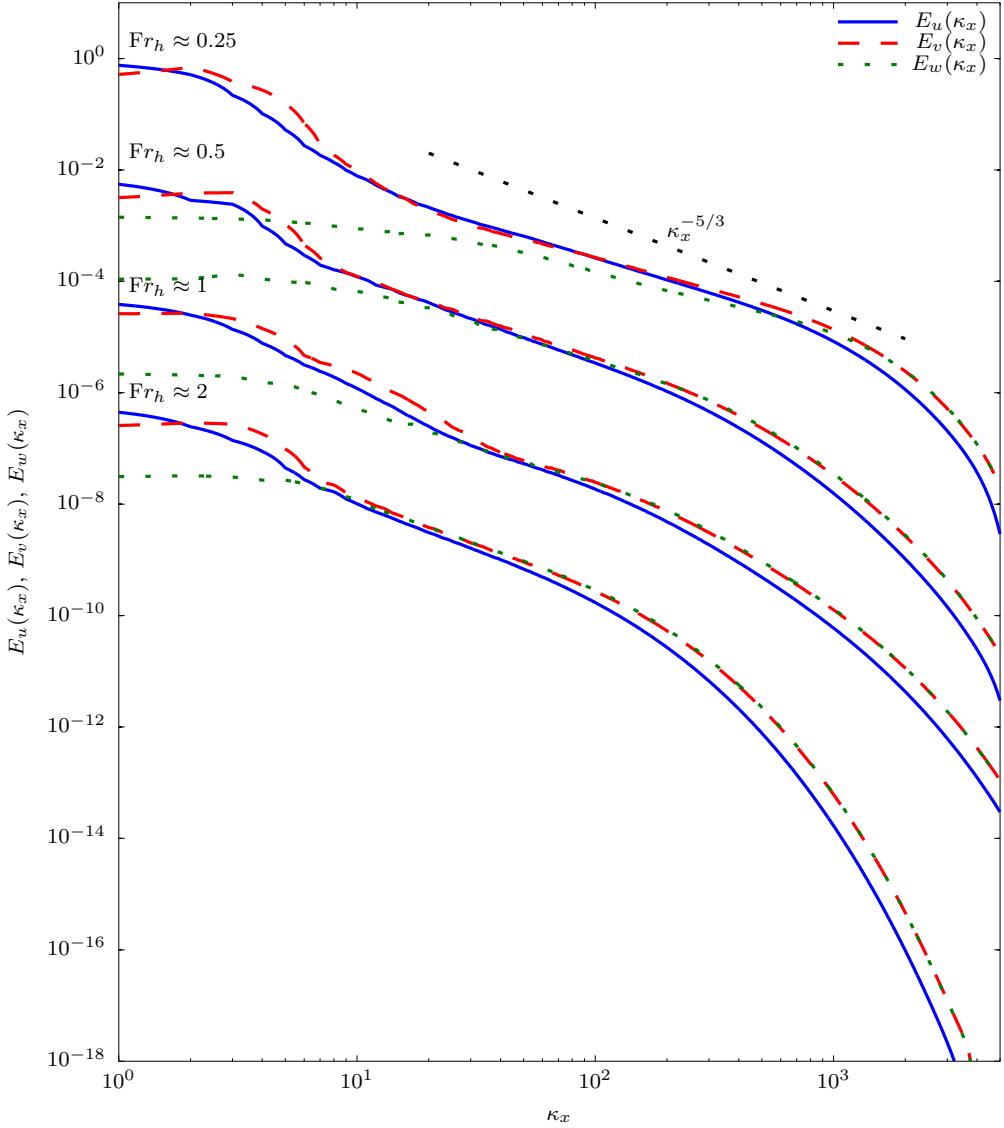


Figure 2: One dimensional x -direction spectra of all velocities, $Gn \approx 200$. The curves for $Fr_h \approx 0.25$ are plotted true and the rest are offset in increments of two decades.

existence and properties of a stratified turbulence inertial range, especially the properties of the spectra of both vertical and horizontal velocities.

In conclusion, the range of Fr_h and Gn that can currently be simulated has been identified for this simulation configuration. Conducting 16 simulations that span the parameter space enables the identification of trends as the parameters are varied, including the limitations on well-resolved simulations. Preliminary results indicate that some of the simulations may exhibit stratified turbulence inertial ranges with sufficient dynamic range to warrant detailed investigations.

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References

Almalkie, S. and de Bruyn Kops, S. M. (2012). Kinetic energy dynamics in forced, homogeneous, and axisymmetric stably stratified turbulence. *J. Turbulence*, 13(29):1–29.

Bartello, P. and Tobias, S. M. (2013). Sensitivity of stratified turbulence to buoyancy Reynolds number. *J. Fluid Mech.*, 725:1–22.

Brethouwer, G., Billant, P., Lindborg, E., and Chomaz, J.-M. (2007). Scaling analysis and simulation of strongly stratified turbulent flows. *J. Fluid Mech.*, 585:343–368.

de Bruyn Kops, S. M. (2015). Classical turbulence scaling and intermittency in stably stratified Boussinesq turbulence. *J. Fluid Mech.*, 775:436–463.

Gargett, A., Osborn, T., and Nasmyth, P. (1984). Local isotropy and the decay of turbulence in a stratified fluid. *J. Fluid Mech.*, 144:231.

Gibson, C. H. (1980). Fossil turbulence, salinity, and vorticity turbulence in the ocean. In Nihous, J. C., editor, *Marine Turbulence*, pages 221–257. Elsevier.

Gibson, C. H. (1986). Internal waves, fossil turbulence, and composite ocean microstructure spectra. *J. Fluid Mech.*, 168:89–117.

Hebert, D. A. and de Bruyn Kops, S. M. (2006a). Predicting turbulence in flows with strong stable stratification. *Phys. Fluids*, 18(6):1–10.

Hebert, D. A. and de Bruyn Kops, S. M. (2006b). Relationship between vertical shear rate and kinetic energy dissipation rate in stably stratified flows. *Geophys. Res. Lett.*, 33:L06602.

Itsweire, E. C., Koseff, J. R., Briggs, D. A., and Ferziger, J. H. (1993). Turbulence in stratified shear flows: Implications for interpreting shear-induced mixing in the ocean. *J. Phys. Oceanogr.*, 23:1508–1522.

Jackson, P. R. and Rehmann, C. R. (2014). Experiments on Differential Scalar Mixing in Turbulence in a Sheared, Stratified Flow. *J. Phys. Oceanogr.*, 44(10):2661–2680.

Lindborg, E. (2006). The energy cascade in a strongly stratified fluid. *J. Fluid Mech.*, 550:207–242.

Maffioli, A., Brethouwer, G., and Lindborg, E. (2016). Mixing efficiency in stratified turbulence. *J. Fluid Mech.*, 794:R3.

Rao, K. J. and de Bruyn Kops, S. M. (2011). A mathematical framework for forcing turbulence applied to horizontally homogeneous stratified flow. *Phys. Fluids*, 23:065110.

Riley, J. J. and Lindborg, E. (2008). Stratified turbulence: A possible interpretation of some geophysical turbulence measurements. *J. Atmos. Sci.*, 65(7):2416–2424.

Rohr, J. J., Itsweire, E. C., Helland, K. N., and Atta, C. W. V. (1988). Growth and decay of turbulence in a stably stratified shear flow. *J. Fluid Mech.*, 195:77–111.

Salehipour, H. and Peltier, W. (2015). Diapycnal diffusivity, turbulent Prandtl number and mixing efficiency in Boussinesq stratified turbulence. *J. Fluid Mech.*, 775:464–500.

Salehipour, H., Peltier, W. R., Whalen, C. B., and MacKinnon, J. A. (2016). A new characterization of the turbulent diapycnal diffusivities of mass and momentum in the ocean. *Geophys. Res. Lett.*, 43(7):3370–3379.

Shih, L. H., Koseff, J. R., Ivey, G. N., and Ferziger, J. H. (2005). Parameterization of turbulent fluxes and scales using homogeneous sheared stably stratified turbulence simulations. *J. Fluid Mech.*, 525:193–214.

Waite, M. L. (2011). Stratified turbulence at the buoyancy scale. *Phys. Fluids*, 23(6):066602.