Particle transport due to trapped cores

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Abstract

We study the transport and dispersion of particles due to internal solitary waves with trapped cores using a particle tracking numerical model coupled with a vertical random walk to represent vertical turbulent mixing. We initialize a particle cloud ahead of a wave with a trapped core and demonstrate that, via vertical turbulent diffusion, as the trapped core propagates past the cloud, particles are entrained into the core and become trapped. As the wave propagates, some of the particles are transported within the core at the speed of the wave while the remainder exit the core due to turbulent diffusion. We show that the magnitude of the vertical diffusivity plays a direct role in the transport/dispersion process by studying the effects of the Péclet number, which is given by a ratio of the relative effect of horizontal transport to that of vertical diffusion. When the Péclet number is large, particles are unlikely to leave the core and can potentially be transported over very large distances. On the other hand, for smaller Péclet numbers, particles are ejected from the core more rapidly resulting in weak horizontal transport.

1 Introduction

Large amplitude internal solitary waves are ubiquitous and persistent in the Earth's atmosphere and ocean. For particular stratifications, if the maximum horizontal fluid velocity exceeds the wave speed, regions of the flow characterized by closed isopycnals may form (i.e. trapped cores). An atmospheric example of internal waves with trapped cores is the Morning Glory in northeast Australia (Ouazzani et al., 2014). Trapped cores in mode-1 internal solitary waves have also been observed in a laboratory setting, notably in the studies by Grue et al. (2000) and Carr et al. (2008). Numerical studies typically solve the fully nonlinear Dubreil-Jacotin-Long (DJL) equation to generate internal solitary waves with trapped cores in continuous stratification. Instabilities in internal waves with trapped cores were studied by Carr et al. (2012) and their formation via shoaling was studied by Lamb (2002). It is well known that internal solitary waves of large amplitude are able to transport mass over large distances (Lamb, 1997). In the present paper, we conduct a numerical study to assess the potential for particulate matter to entrain into the core and investigate the decay rate of detrainment from the core for different values of the turbulent diffusivity.

2 Particle tracking model

We consider an internal solitary wave propagating in a stratification given by

$$\frac{\overline{\rho}(z)}{\rho_0} = 1 + \frac{1}{2}\frac{\Delta\rho}{\rho_0}\left[1 - \tanh\left(\frac{z+h_1}{\delta}\right)\right] \tag{1}$$

where z is the vertical coordinate measured upward from the free surface. The reference density is $\rho_0 = 1000$ kg m^{-3}, the density difference is $\Delta\rho = 2.4$ kg m $^{-3}$ and the total fluid depth is $D = 60$ m, the upper layer depth is $h_1 = 3$ m and the pycnocline thickness is $\delta = 6$ m.

To compute the velocity and density field in a solitary wave with a trapped core, we solve the DJL equation

$$c^2\nabla^2\eta(x,z) = -\eta(x,z)N^2[z-\eta(x,z)]\,, \tag{2}$$

where x is the horizontal coordinate, $\eta(x,z)$ is the displacement of an isopycnal from its background state and N is the buoyancy frequency. The wave-induced fluid velocity in fixed coordinates is given by

$$\mathbf{u_w} = c\left(\frac{\partial\eta}{\partial z}\,\mathbf{e}_x - \frac{\partial\eta}{\partial x}\mathbf{e}_z\right)\,, \tag{3}$$

where c is the wave speed and $\mathbf{e}_x$ and $\mathbf{e}_z$ are unit vectors indicating the horizontal and vertical directions, respectively. The wavelength λ is defined as twice the horizontal distance from the crest at $x/\lambda = 0$ to a location $|x|/\lambda = 1/2$ where the isopycnal displacement is less than 0.1% of its maximum value. The wave period is given by $T = \lambda/c$ and the wave nonlinearity is measured with the Froude number $F = U/c$, where U is the maximum horizontal fluid velocity. We study a wave with a wavelength $\lambda = 388$ m, a period $T = 760$ s, and a Froude number $F = 1.5$, sufficient to generate a clear trapped core.

We employ the eight-order accurate Runge-Kutta type Dormand-Prince or DOP853 method with added step size control and dense output for particle tracking (Dormand and Prince, 1980). The discrete time-evolution equation for the particle positions at a discrete time-step n is

$$\mathbf{X}^{n+1} = \mathbf{X}^n + \int_{t^n}^{t^n+\Delta t}\mathbf{u_w}dt + R\sqrt{2\mathbf{K}\Delta t}\,, \tag{4}$$

where t is the time, Δt is the time step, $\mathbf{K} = (0,\kappa)$ is the turbulent diffusivity vector and R is a Gaussian random number with zero mean and unit standard deviation. The second term on the right hand side of (4) represents the displacement of a particle due to the wave-induced velocity $\mathbf{u_w}$, which is interpolated with a bivariate cubic spline interpolation method to obtain the velocity at the particle location $\mathbf{X}^n$. The last term in (4) represents a random spatial increment $R\sqrt{2\mathbf{K}\Delta t}$.

To measure the relative effect of vertical diffusivity and longitudinal dispersion, we define the Péclet number as

$$Pe = \frac{U\lambda}{\kappa}\,. \tag{5}$$

The largest closed isopycnal is obtained from the numerical model and the area is computed allowing for the calculation of the concentration value inside the core $C = N_p^i/A$, where N_p^i is the number of particles inside the core at any given time t, and the area of the core A is defined by the largest closed isopycnal. Because we are advecting the steady solution of the wave, we assume A to be constant with time. The actual concentration value inside the core is normalized by the concentration value inside the core if all the particles were trapped $C_f = N_p/A$, where N_p is the total number of particles. Hence, to help quantify the rate of entrainment and detraiment, we define the core concentration ratio

$$\frac{C}{C_f} = \frac{N_p^i}{N_p}\,. \tag{6}$$

3 Results and discussion

3.1 Entrainment

To study particle entrainment into the trapped core, we introduce a particle cloud ahead of the wave. Several values of the Péclet number are considered by varying the vertical turbulent diffusivity κ, see (5). The evolution of a particle cloud for time $0 \leq t \leq 3T$ is shown in Figure 1, which also shows the horizontal distribution of particles after twenty periods ($20T$) have elapsed. For this case we employ a vertical diffusivity $\kappa = 0.005$ m ^{2} s^{-1} based

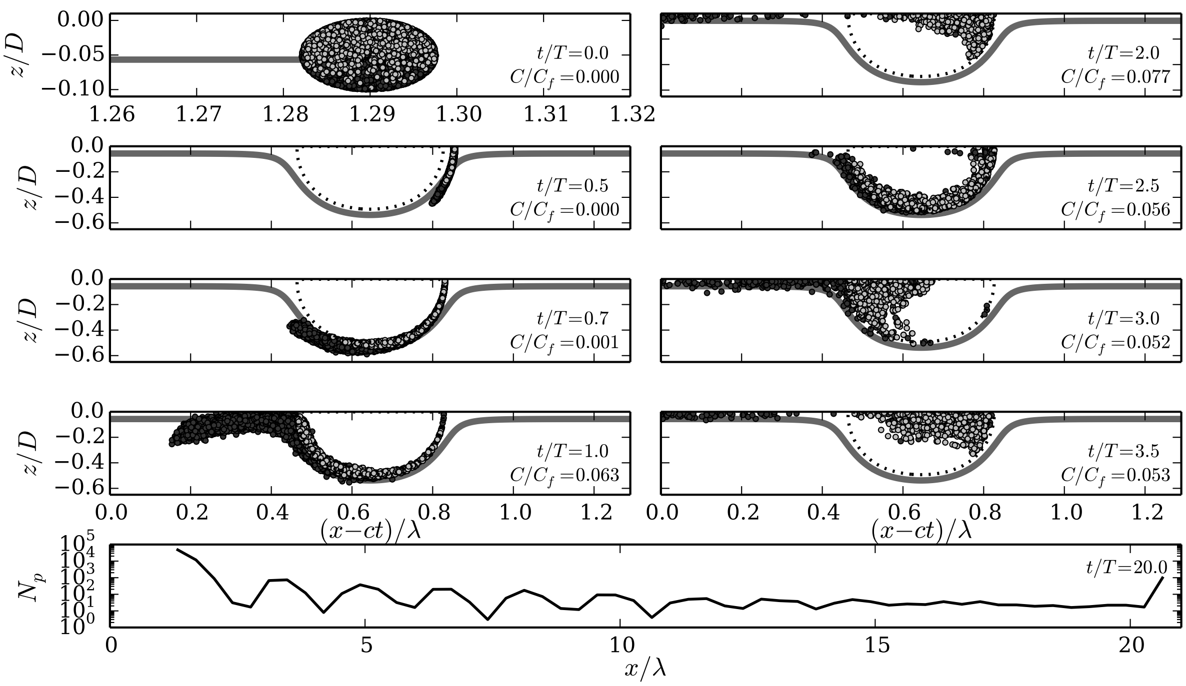

Figure 1: Evolution of a particle cloud entraining/detraining into/from a solitary wave with a trapped core. A zoomed-in view of the initial particle distribution is shown in the first panel. The particles that remain inside the core by time $t/T = 20$ are depicted by light circles and the rest are shown by dark circles. The solid gray line depicts the isopycnal where $\rho = (\rho_1 + \rho_1)/2$ and the dotted line represents the largest closed isopycnal. Wave propagation is from left to right. Note that, for clarity, not all particles are included. The bottom panel represents the final particle distribution at time $t/T = 20$.

on field measurements of large-amplitude internal solitary waves propagating towards the shore over the Oregon continental shelf, yielding $Pe = 6.0 \times 10^4$ (Moum et al., 2003). We initialize $N_p = 65,536$ particles uniformly distributed inside a circle with a diameter $d = h_1$ and vertical center of mass at $z = -h_1$. Since half of the particles are initially located above $z = -h_1$, those particles have a higher chance of entraining and remaining inside the core than particles that are initialized below $z = -h_1$.

Diffusion of particles in the vicinity of the largest closed isopycnal causes a small fraction of particles to be entrained into the core (see time $0 \leq t \leq 1T$ in Figure 1). As shown in Figure 2, the concentration ratio C/C_f is a maximum at time $t = 2T$, when approximately 8% of the particle cloud has been entrained into the core. Although the particles can exit the core at any location along the largest closed isopycnal, a majority of particles detrain at the trailing edge of the core. This is because they have had more time to experience turbulent diffusion and migrate outside the largest closed isopycnal into a region where $F < 1$ for a sufficient amount of time. The majority of particles exit the core above the pycnocline, leading to a horizontal distribution that is confined to the upper layer of the water column.

As depicted for times $2T < t \leq 3.5T$ in Figure 1, it takes approximately $1.5T$ for the entrained particles to perform a full revolution within the core followed by further detraining at the back of the core. As shown by the final distribution of the particles along the wave path in Figure 1, this ejection process is repeated until the core is empty. In the initial stages of the simulation, the particles within the core are not not well mixed, resulting in intermittent particle detrainment over time. However, for $t > 15T$, the particles inside the core have become well mixed so that particle ejection from the core occurs in a continuous fashion resulting in a more uniform distribution as highlighted by the bottom panel of Figure 1.

To study the effect of the Péclet number, we depict the evolution of the concentration ratio C/C_f in Figure 2. The greatest potential for entraining particles into the core is when

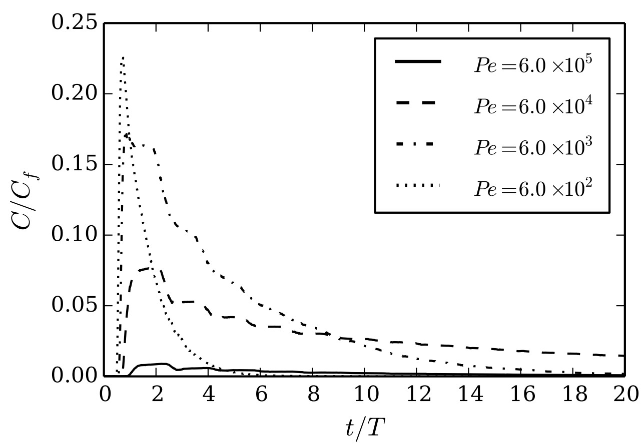

Figure 2: The effect of the Péclet number Pe on the temporal evolution of the concentration ratio C/C_f for particles entraining/detraining into/from the core.

Pe is small (large diffusivity). When $Pe = 6.0 \times 10^2$, approximately 22% of the particles are entrained into the core within one wave period. Conversely, when $Pe = 6.0 \times 10^5$, less than one percent of the particles are entrained. However, a larger diffusivity also results in a much larger rate of detrainment, suggesting that there is an optimum value of Pe for enhanced longitudinal dispersion of material. For example, the core is emptied four times faster when $Pe = 6.0 \times 10^2$ compared to when $Pe = 6.0 \times 10^3$. Due to rapid mixing when κ is large, intermittent ejection of particles does not occur when $Pe = 6.0 \times 10^2$, but instead particles are detrained from the core continuously in time.

3.2 Detrainment

To investigate the detrainment mechanism in more detail we consider a case with the same wave and stratification parameters as in the previous section. However, this time we initialize the particles ($N_p = 131,072$) with a uniform distribution within the largest closed isopycnal (i.e. $C/C_f = 1$ at $t = 0$). The evolution of the entrained material is depicted in Figure 3, where the last particle to exit the core is depicted in a lighter color. Because, in this case, the particles are uniformly initialized inside the core it is well mixed by definition. Hence, the particle ejection process occurs continuously (albeit with an increasingly weaker rate of detrainment), as exemplified by the final horizontal distribution

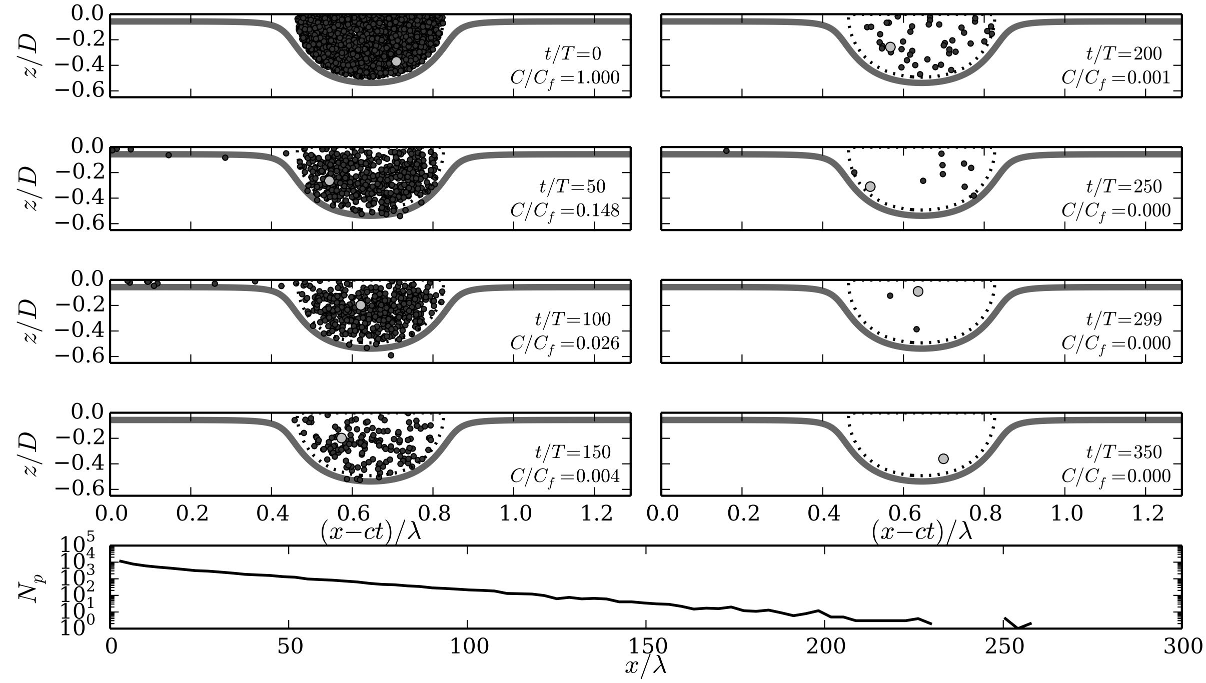

Figure 3: Evolution of a particle cloud detraining from a solitary wave with a trapped core. The last particle to exit the core is depicted by a light circle and the rest are shown by dark circles. The solid gray line depicts the isopycnal where $\rho = (\rho_1 + \rho_1)/2$ and the dotted line represents the largest closed isopycnal. Wave propagation is from left to right. Note that, for clarity, not all particles are included. The bottom panel represents the final particle distribution at time $t/T = 350$.

of the particle cloud in the bottom panel of Figure 3. The final particle distribution shows that the bulk of detrainment occurs in the first twenty periods ($20T$).

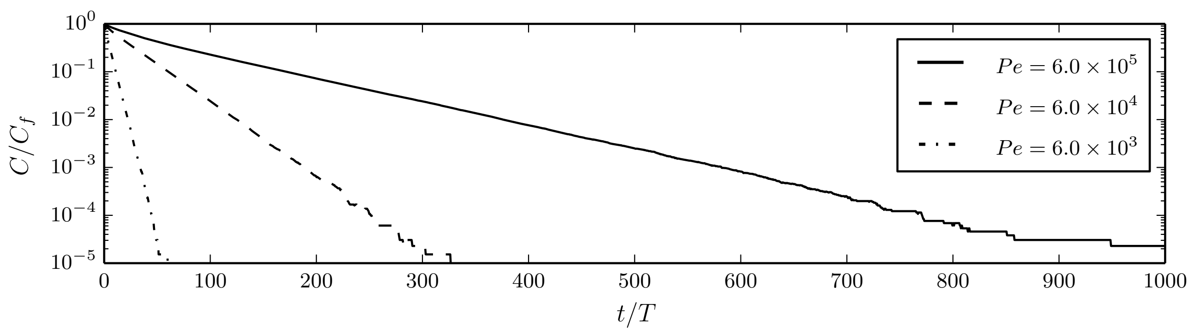

Figure 4: The effect of the Péclet number Pe on the temporal evolution of the concentration ratio C/C_f for particles detraining from the core.

As depicted in Figure 4, the detrainment rate of is exponential with an e-folding time scale T_e that depends on the value of the turbulent diffusivity, such that we expect

$$C/C_f = e^{-t/T_e} \, . \tag{7}$$

Namely, a larger value of turbulent diffusivity results in faster ejection rate from the core. As highlighted in Figure 4, the e-folding time scale is $T_e/T = [4, 24, 62]$ for $Pe = [6.0 \times 10^3, 6.0 \times 10^4, 6.0 \times 10^5]$, respectively. Therefore, it takes a total time $t/T = [40, 227, 695]$ for the concentration in the core to become less than one percent ($C/C_f < 1\%$) when $Pe = [6.0 \times 10^3, 6.0 \times 10^4, 6.0 \times 10^5]$.

4 Conclusion

In this paper we investigated the potential for large amplitude internal solitary waves with trapped cores to transport and disperse particulate matter. Depending on the value of the Péclet number, the fraction of particles that become entrained into the core can be relatively high (up to 22% for very low Péclet number). Based on measured oceanic values of the turbulent diffusivity in internal solitary waves, we expect a wave to entrain between $5 - 15\%$ of material in the vicinity of the pycnocline as it propagates. The detrainment rate is also a function of the Péclet number and decays exponentially, with smaller e-folding time scales for smaller Péclet numbers. For example, for small Péclet number, the ratio of final to initial values of the particle concentration in the core is equal to $1/e$ of its initial value in approximately five wave periods. However, it may take up to sixty wave periods if the Péclet number is large. Therefore, there is an optimum value of the Péclet number that maximizes the number of particles that are trapped in the core. Future work will focus on understanding this optimum Péclet number. Regardless of the Péclet number, however, the results suggest that trapped cores can entrain and distribute mass over distances much larger than solitary waves without trapped cores.

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