Transient and Global Instabilities of Internal Solitary Waves

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Abstract

Large amplitude internal solitary waves are common in the atmosphere and ocean and play an important role in mixing and mass transport. While these waves are observed to propagate over large distances, they are susceptible to instabilities that promote wave energy loss, turbulence, and vertical mixing. To gain insight into the mechanisms and bounds on these instabilities, we consider the optimal transient growth of a family of solitary waves in nearly two-layered stratifications obtained from solutions to the Dubreil-Jacotin-Long (DJL) equation. Waves of sufficient amplitude have a zone of self-induced low Richardson number centered on the wave crest and are potentially unstable to Kelvin-Helmholtz instability. Optimal initial disturbances are computed by means of direct-adjoint iteration of the Boussinesq, Navier-Stokes system linearized about the DJL solitary wave solution. The most amplified disturbances resemble wave packets and are initially localized just upstream of the low Richardson number zone. The growth of the disturbance energy is maximized for short time horizons corresponding to travel time across the unstable zone and is shown to increase with the solitary wave phase speed (or amplitude). These optimal transient disturbances are compared to a WKB analysis of spatially growing Kelvin-Helmholtz waves. The comparison demonstrates the connection of the optimal disturbances to the Kelvin-Helmholtz mechanism, but also highlights the significance of initial non-normal growth through the Orr mechanism, which becomes the dominant mechanism for smaller solitary waves. The linear evolution of initial upstream disturbances consisting of a localized packet of short internal waves with carrier frequency equal to that of the optimal growth disturbances is shown to lead to disturbance growth substantially less than the optimal case.

We then consider solitary waves in stratifications that permit trapped cores and show by stability analysis that these waves are subject to a global, or self-sustained, instability that, in contrast to the waves in nearly two-layered systems, does not require continuous upstream noise to feed the instability.

In both cases the nonlinear evolution of the instabilities is explored through numerical solutions of the full Navier-Stokes equations. Implications for breakdown and mixing will be discussed.