The Influence of a Shoaling Internal Gravity Wave on a Dense Gravity Current

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Abstract

Shoaling internal waves may play a major role in diluting the dense gravity currents that occur in cold river inflows to lakes and brine effluent to coastal oceans. We consider a model of a gravity current entering a two-layer ambient with a shoaling internal wave at the pycnocline. Energetic considerations suggest that waves above a critical amplitude can prevent the gravity current from passing through the lower layer of the ambient and instead divert the gravity current into the pycnocline. Internal waves in a receiving water may therefore have a dramatic impact on transport by dense gravity currents in practical applications in the environment.

1 Background

In many geophysical settings, inclined gravity currents fall through stratified ambients that contain internal gravity wave motion. Examples include effluent from seawater desalination facilities discharging into the coastal oceans where internal waves shoal on the continental slope, and dense river inflows entering lakes where winds drive internal seiches. When the internal wave field has enough energy relative to the gravity current, dilution and transport of the gravity current fluid can be strongly influenced by the internal wave field (Cenedese and Adduce, 2010).

Prior research on the interaction of gravity currents and internal waves has focussed on how gravity currents can cause internal waves in stratified ambients (Monaghan et al., 1999; Maurer and Linden, 2014; Ungarish and Huppert, 2002). Very little work has been done on how incident internal waves alter gravity currents. Whilst Baines' experimental observations of gravity currents in linearly stratified ambients interestingly suggested that internal waves influenced the entrainment and detrainment exchanges between the gravity current and the ambient, the internal waves were not a focus of his study (Baines, 2001).

Our work is based on the gravity current experiments carried out by Cortés et al. (2014) and on work on shoaling internal waves (Moore et al., 2016; Walter et al., 2012; Aghsaee et al., 2010). The work of Cortés et al. (2014) showed that when a gravity current falls through the pycnocline of a two-layer ambient, the lighter outer layers of the gravity current separate from the underflowing gravity current and become an intrusion at the pycnocline. The authors derived a model for the proportion of the gravity current that continues to fall through the lower ambient layer. This and other studies have only considered the case of a still receiving water. The influence of incident internal waves, which are a common feature practical applications, have been neglected.

Shoaling waves have many different behaviours, depending on the characteristics of the incident waves and the bathymetry. When interfacial gravity waves are incident on a shallow slope, the wave shoals, steepens and then breaks to send a pulse of dense water

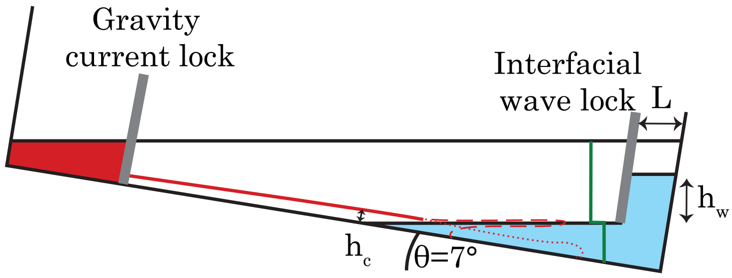

Figure 1: Schematic of gravity current and internal wave system. The two possible paths of the homogeneous gravity current are shown - penetrating the lower layer as an underflow with the dotted line, and forming an interflow at the pycnocline with the dashed line.

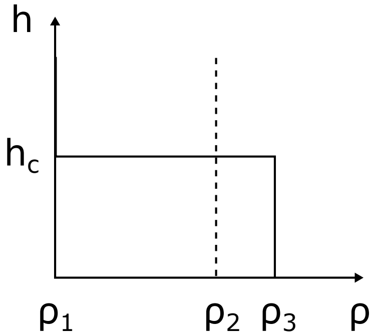

Figure 2: Density profile in the gravity current is shown by the solid line. The density of the lower layer is shown by the dashed line, indicating the threshold for the current to descend as an underflow.

up the slope (Walter et al., 2012). For shallow waves the wave forms a coherent bolus that travels up the slope (Moore et al., 2016). Our study focusses on this wave regime.

In this paper, we address the influence of internal waves on gravity currents. Energetic considerations will be presented that constrain when shoaling internal waves can prevent a gravity current from descending through the lower layer of a receiving two-layer ambient. We then show visualisations to demonstrate the principle used in this analysis.

2 Theory

We consider a tank of water with a two-layer density stratification, shown schematically in figure 1. The upper layer has density ρ_1 and the lower layer has density ρ_2, separated by a sharp pycnocline. We will first consider the case of a quiescent background ambient, and then later consider interfacial waves at the pycnocline.

The gravity current is assumed for simplicity to be of uniform density. The density profile of the gravity current which contains fluid of density ρ_3 with thickness h_c just before it reaches the pycnocline is shown in figure 2. When $\rho_3 > \rho_2$, the gravity current will penetrate the lower layer and continue as an underflow. When $\rho_3 < \rho_2$, the gravity current will form an interflow at the pycnocline.

The underflowing case can be diverted to become an interflow if the gravity current is diluted with fluid from the upper layer so that it has a diluted density $\rho_d \leq \rho_2$. Accounting for conservation of mass shows that diluting the current with upper ambient fluid to change its density from ρ_3 to ρ_2 would increase the volume of the gravity current by a factor $(\rho_3 - \rho_1)/(\rho_2 - \rho_1) = g'_3/g'_2$. Here, $g'_3 = g(\rho_3 - \rho_1)/\rho_r$ is the reduced gravity of the gravity current in the upper ambient fluid, g'_2 is the reduced gravity of the lower ambient fluid in the upper ambient fluid, and $\rho_r = (\rho_3 + \rho_1)/2$ is a reference density. If the gravity current only adds to its volume vertically, the new gravity current height after dilution is $h_d = h_c g'_3/g'_2$. This assumes that the current increases in volume by getting thicker due to the entrainment, which is reasonable given the aspect ratio of the gravity current. This also assumes that the gravity current is diluted with fluid from the upper ambient fluid. Diluting of the gravity current with fluid from the lower layer cannot make the gravity current lighter than the lower layer. For the gravity current to be diluted to become lighter than the lower layer, the centre of mass of the gravity current must rise by at least

$$h_m = \frac{h_c}{2}\left(\frac{g'_3}{g'_2} - 1\right). \tag{1}$$

For a volume per unit width of the current, V_c, the additional potential energy required is

$$E_d = V_c g'_3 \rho_r \frac{h_c}{2}\left(\frac{g'_3}{g'_2} - 1\right) \tag{2}$$

The energy to dilute the gravity current may be provided by waves on the pycnocline breaking and mixing the current with fluid from the upper layer.

In the laboratory, a wave may conveniently be initiated from rest using a lock arrangement to hold the pycnocline in a Heaviside step function with amplitude h_w and length L, as shown in figure 1. The potential energy initially in the elevated pycnocline is

$$E_w = \rho_r g'_2 h_w^2 L/2. \tag{3}$$

Some of the wave's energy incident on the slope will leave the slope in reflected waves, leaving a proportion Λ_r of the initial energy at the slope. Viscous losses during the mixing process will mean that only a proportion Λ_p of the energy from the wave will be transferred into gravitational potential energy in the current by dilution. Some of the potential energy of the initial pycnocline shape will not be transported by the interfacial wave, adding a further reduction in the energy delivered by the wave Λ_i. We expect the gravity current to be diverted from the underflow to the interflow when

$$E_w \Lambda_i \Lambda_r \Lambda_p = E_w \Lambda > E_d, \tag{4}$$

meaning that when the initial height of the wave lock release is greater than a critical height,

$$h_w^{crit} = \sqrt{\frac{V_c g'_3 h_c}{L \Lambda g'_2}\left(\frac{g'_3}{g'_2} - 1\right)}, \tag{5}$$

the gravity current can be diluted and thus prevented from descending through the lower ambient layer.

3 Visualisations

To demonstrate the principle of this analysis, we carried out visualisations of gravity currents falling through a two-layer ambient environment with and without shoaling internal waves at the pycnocline. The visualisations were carried out in a tank (0.15 by 1.2 by 0.2 m) inclined at 7°. The tank was initially set up with a two-layer stratification with fresh water above salt water of density $\rho_2 = 1045$ kg m^{-3} separated by a sharp pycnocline approximately 1 cm thick. A lock at the top of the slope held salt water of density 1050 kg m^{-3}, which was released to form a gravity current. A lock at the other end of the

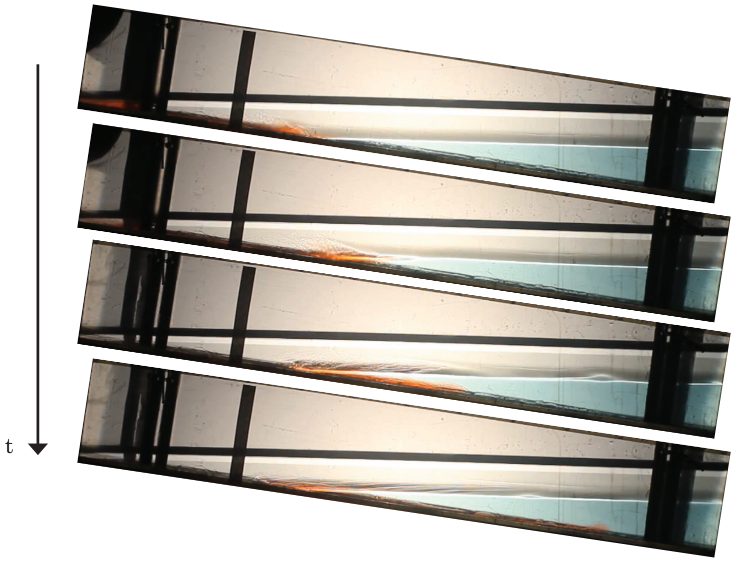

Figure 3: Time series of the case with no internal wave.

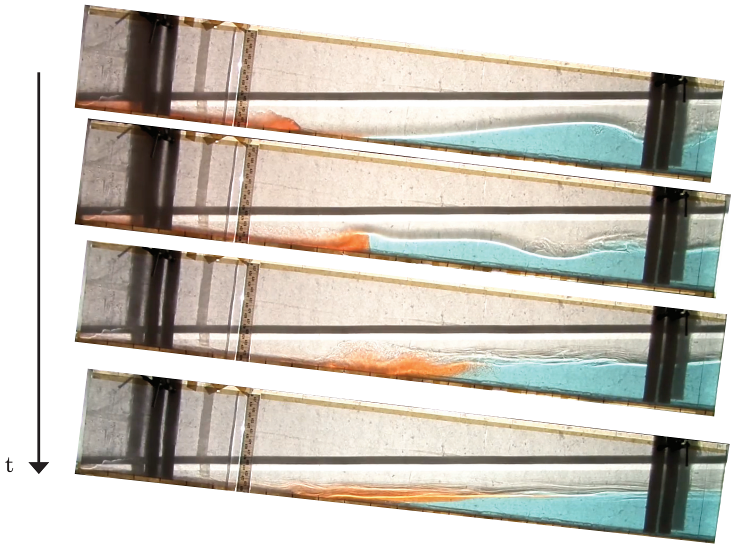

Figure 4: Time series of the case with an incident internal wave.

tank held a raised portion of the pycnocline, which was released to form an internal wave. Releasing the two locks simultaneously led the gravity current to reach the pycnocline as the interfacial wave was breaking.

A gravity current release with no internal wave is shown in figure 3. In this case, part of the gravity current fluid, which was dyed red, descended through the lower ambient layer and part of the gravity current fluid remained at the pycnocline. This can be seen in the final frame of figure 3.

A gravity current released with an internal wave incident on the current is shown in figure 4. In this case, the wave caused the gravity current to mix more with the upper layer than in the no wave case. This increased mixing meant that none of the red dyed gravity current fluid was dense enough to descend through the lower ambient layer. Instead, all the gravity current fluid remained at the pycnocline. This case is a demonstration of an incident wave with amplitude larger than the critical value h_w^{crit}.

We are now continuing this work by comparing final density profiles in cases with different amplitude wave releases. This will show quantitatively the influence of the internal wave on how much of the gravity current falls as an underflow beneath the lower layer. We will also examine the density field at the collision of the internal wave bolus and the gravity current head in more detail, where we believe different regimes of interaction can occur.

4 Conclusions

Gravity currents falling through two-layer ambients will descend through the lower layer if the gravity current fluid remains denser than the lower layer. Energetic considerations show that internal waves with amplitude greater than a critical amplitude h_w^{crit} can dilute the gravity current enough to prevent it from descending through the lower layer. In future work we will determine how the wave characteristics, such as wave steepness and phase relative to the gravity current, influence the prefactor Λ, which describes the proportion of the incident energy that raises the potential energy of the gravity current.

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