Orientation of Vortical Structures in Turbulent Stratified Shear Flow

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Abstract

The results from a series of direct numerical simulations of turbulent stably stratified shear flow are used to determine the orientation of structures present in such flows. The structures are identified using the three-dimensional two-point autocorrelation coefficient of velocity magnitude and vorticity magnitude. Surfaces of a constant autocorrelation coefficient value are observed to resemble an ellipsoid. A least-squares fit of an ellipsoid to the autocorrelation coefficient isosurfaces is performed and the major and minor axes are determined. The inclination angle of the autocorrelation surface (and thus the flow structures) is then determined from the axes. The inclination angle value is fairly unaffected by the choice of autocorrelation coefficient value. The inclination angle is observed to decrease with increasing Richardson number and, hence, directly related to the growth or decay rate of the turbulence in stratified shear flow.

1 Introduction

Homogeneous turbulence in a stably stratified shear flow has been studied extensively in the past due to the wide range of applications in the geophysical environment. The flow considered here has uniform vertical shear with constant rate $S = \partial U/\partial y$ and uniform vertical stratification with constant Brunt-Väisälä frequency $N = \sqrt{-g/\rho_0 \partial \varrho/\partial y}$. In the present study, the Richardson number $Ri = N^2/S^2$ is varied from $Ri = 0$, corresponding to unstratified shear flow, to $Ri = 1$, corresponding to strongly stratified shear flow.

Comprehensive studies of homogeneous turbulent stratified shear flows include the experimental work by Komori et al. (1983), Rohr et al. (1988), Piccirillo and Van Atta (1997), and Keller and Van Atta (2000) as well as the numerical simulations of Gerz et al. (1989), Holt et al. (1992), Jacobitz et al. (1997), and Jacobitz (2002). Turbulent shear flows, including flows with density stratification or system rotation, have identified flow structures inclined in the vertical direction to the downstream direction (e.g. Brethouwer (2005) or Jacobitz et al. (2008)). The three-dimensional two-point autocorrelation coefficient of velocity magnitude and vorticity magnitude is used here to determine the inclination angle of flow structures and to study its dependence on the Richardson number.

2 Numerical Approach

The direct numerical simulations solve the incompressible Navier-Stokes equations in the Boussinesq approximation and an advection-diffusion equation for the fluctuating density. The equations of motion are solved in the Rogallo frame (see Rogallo (1981)) and periodic boundary conditions are applied. The spatial discretization is accomplished by a Fourier collocation method and the solution is advanced in time by a fourth-order Runge-Kutta method. The simulations are performed on a grid with $256 \times 256 \times 256$ points.

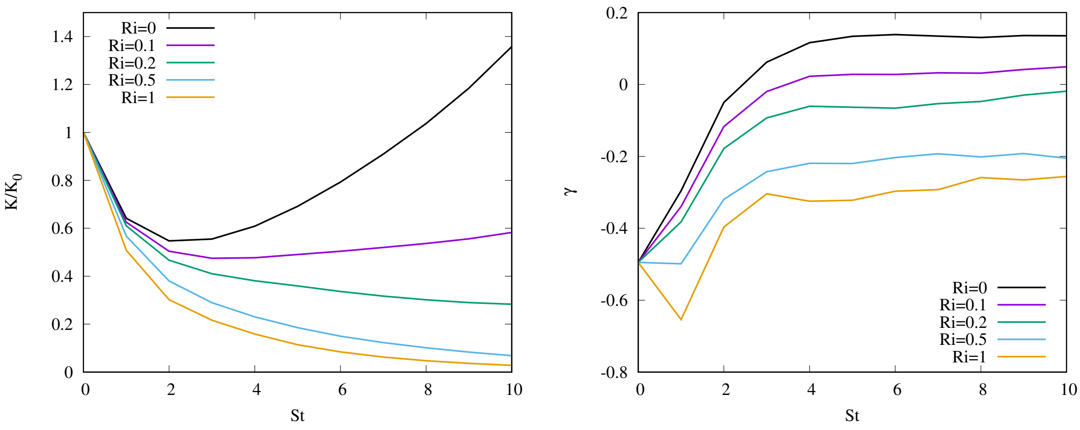

Figure 1: Evolution of the turbulent kinetic energy K (left) and the growth rate γ (right) with non-dimensional time St.

The initial conditions are taken from a simulation of decaying isotropic turbulence without density fluctuations with an initial Taylor-microscale Reynolds number $Re_\lambda = 56$ and an initial shear number $SK/\epsilon = 2$. The Richardson number is varied from $Ri = 0$ to $Ri = 1$. Details on the numerical method can be found in Jacobitz et al. (1997) and the simulation results have previously been used to study acceleration statistics in turbulent stratified shear flows by Jacobitz et al. (2015).

3 Flow Evolution

The evolution of the flow in non-dimensional time St is shown in figure 1. The turbulent kinetic energy $K = 1/2(u^2 + v^2 + w^2)$ (figure 1, left) initially decays due to the isotropic initial conditions. Eventually, the turbulent kinetic energy grows approximately exponentially for small values of the Richardson number Ri and K decays for large values of Ri. The evolution of K is found to change from growth to decay at a critical Richardson number of about $Ri_{cr} \approx 0.15$.

The transport equation of the turbulent kinetic energy K can be written in the following non-dimensional form:

$$\gamma = \frac{1}{SK}\frac{dK}{dt} = \frac{P}{SK} - \frac{B}{SK} - \frac{\epsilon}{SK} \tag{1}$$

Here, γ is the exponential growth rate of K, $P/(SK)$ the normalized production rate with $P = -S\overline{uv}$, $B/(SK)$ the normalized buoyancy flux with $B = g/\rho_0\overline{v\rho}$, and $\epsilon/(SK)$ the normalized dissipation rate.

The evolution of the growth rate γ (figure 1, right) indicates that γ eventually reaches an approximately constant value, indicating an exponential evolution of the flow. The growth rate γ is positive for cases with growing turbulent kinetic energy K and negative for decaying cases. The reduction in γ with increasing Richardson number Ri is mainly due to a decreased normalized production rate $P/(SK)$.

Figure 2 shows the magnitude of velocity $q = \sqrt{2K}$ (top) and the magnitude of vorticity ω (bottom) for two cases with Richardson numbers $Ri = 0.1$ (left) and $Ri = 1$ (right) in the plane of shear (x-y-plane) at non-dimensional time $St = 10$ using the same color map to indicate the decay of the strongly stratified case. Turbulent structures inclined in the vertical direction to the downstream directions are visible. The structures have

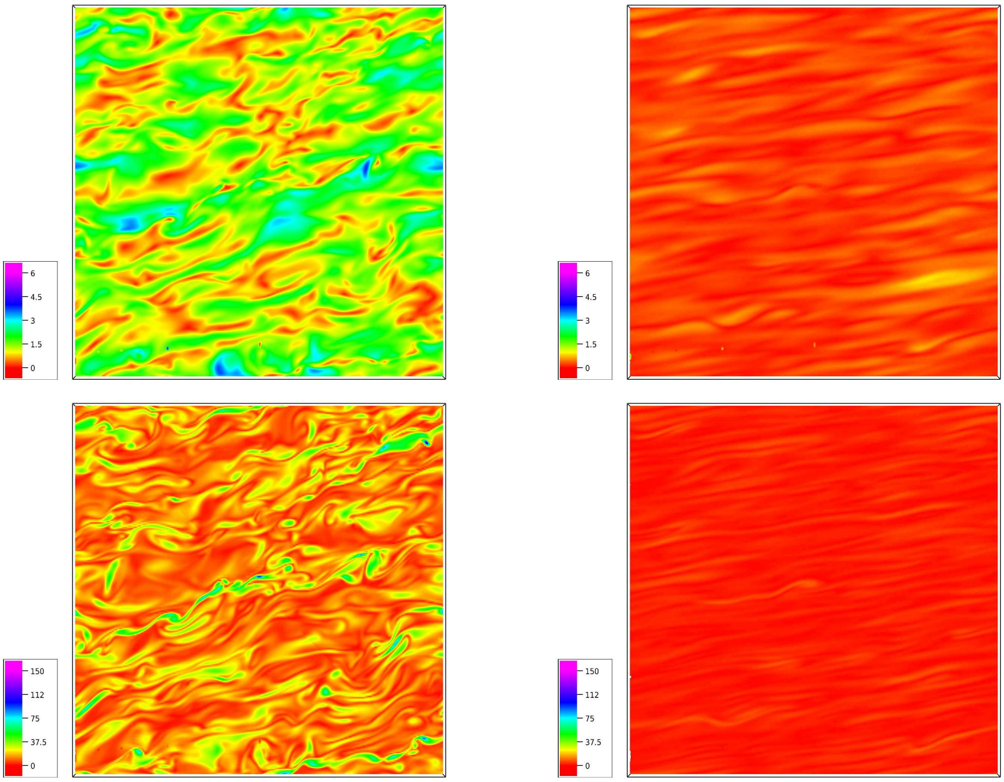

Figure 2: Velocity magnitude (top) and vorticity magnitude (bottom) in the plane of shear for Richardson numbers $Ri = 0.1$ (left) and $Ri = 1$ (right) at non-dimensional time $St = 10$.

the largest inclination angle for unstratified shear flow. With increasing stratification, vertical velocity fluctuations are suppressed and the flow structures are less inclined to the downstream direction.

4 Structure Orientation

In order to quantify the structure orientation, the three-dimensional two-point autocorrelation coefficient Co of velocity magnitude q and vorticity magnitude ω is considered:

$$Co_q(\mathbf{r}) = \frac{\overline{q(\mathbf{x})q(\mathbf{x}+\mathbf{r})}}{q^2} \qquad \text{and} \qquad Co_\omega(\mathbf{r}) = \frac{\overline{\omega(\mathbf{x})\omega(\mathbf{x}+\mathbf{r})}}{\omega^2} \tag{2}$$

Figure 3 shows isosurfaces of constant two-point autocorrelation coefficient $Co = 0.3$ of velocity magnitude q (top) and vorticity magnitude ω (bottom) for two cases with Richardson numbers $Ri = 0.1$ (left) and $Ri = 1$ (right) at non-dimensional time $St = 10$. Also shown in the figure are ellipsoid least-squares fits to the isosurfaces. The fitted ellipsoids allow for the computation of the major and minor axes as well as the inclination angle α of the ellipsoid fit to the downstream direction. The orientation of the two-point autocorrelation coefficients isosurface closely matches the inclination angle of the original structures visible in figure 2 and, hence, the ellipsoid fit allows for its determination.

Figure 4 shows the dependence of the angle α on the two-point autocorrelation coefficient Co of velocity magnitude q (top) and vorticity magnitude ω (bottom) used in its determination for two cases with Richardson numbers $Ri = 0.1$ (left) and $Ri = 1$ (right) at non-dimensional time $St = 10$. For all Ri, the inclination angle value does not change

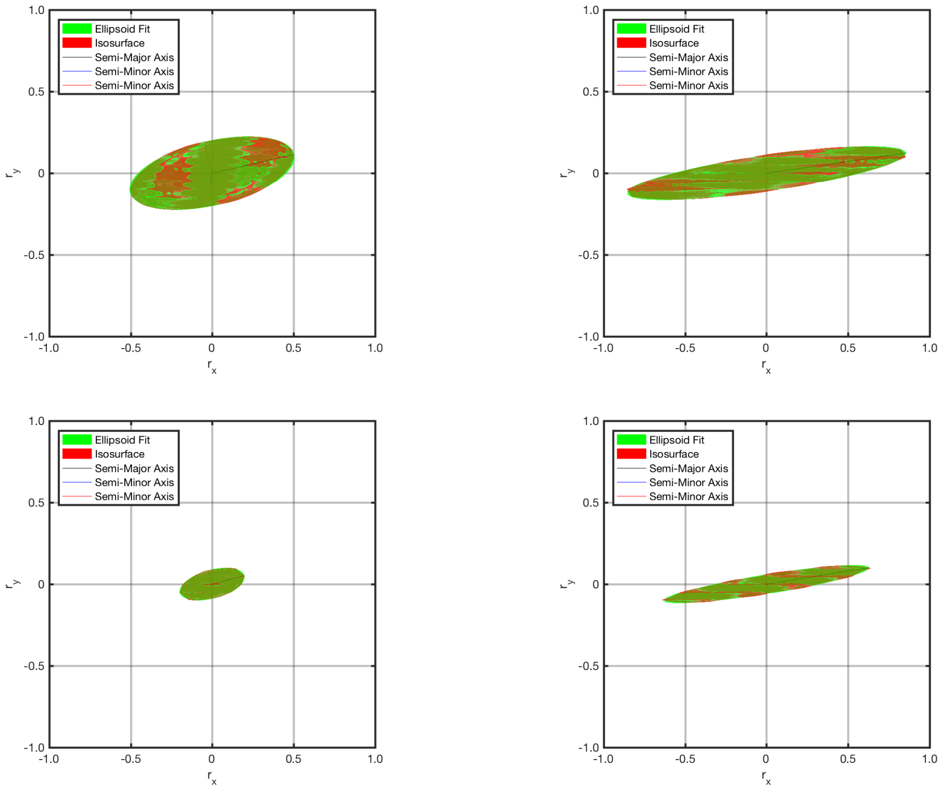

Figure 3: Iso-surface of the three-dimensional two-point autocorrelation of velocity magnitude (top) and vorticity magnitude (bottom) for a correlation coefficient of 0.3 in the plane of shear for Richardson numbers $Ri = 0.1$ (left) and $Ri = 1$ (right) at non-dimensional time $St = 10$.

substantially for an interval of autocorrelation coefficients ranging from about $Co = 0.2$ to $Co = 0.6$. The figure also indicates the number of isovalues used in the least-squares fit of the ellipsoid. For large autocorrelation coefficient values, only few points are used to fit the ellipsoid and its orientation is not always well-defined. For small values, the isosurface does not resemble an ellipsoid shape.

5 Inclination Angles

Figure 5 shows the evolution of the inclination angle α determined from two-point autocorrelation coefficients of velocity magnitude (left) and vorticity magnitude (right) with non-dimensional time St. For the isotropic initial condition, the isosurfaces of autocorrelation coefficient form spheres and the inclination angle is not defined. The inclination angles α decrease with increasing St and eventually reach approximately constant values for most values of the Richardson number. At a given St, the values of α decrease with increasing stratification.

Figure 6 compares the dependence of the growth rate γ (left) on the Richardson number Ri to that of the inclination angle α (right) at non-dimensional time $St = 10$. Both γ and α decrease with increasing stratification. While the angles determined from autocorrelation coefficients of velocity magnitude and vorticity magnitude are similar, the angles obtained from vorticity magnitude are always a little larger than those from velocity magnitude.

The dependence of the inclination angle α on the growth rate γ at non-dimensional time

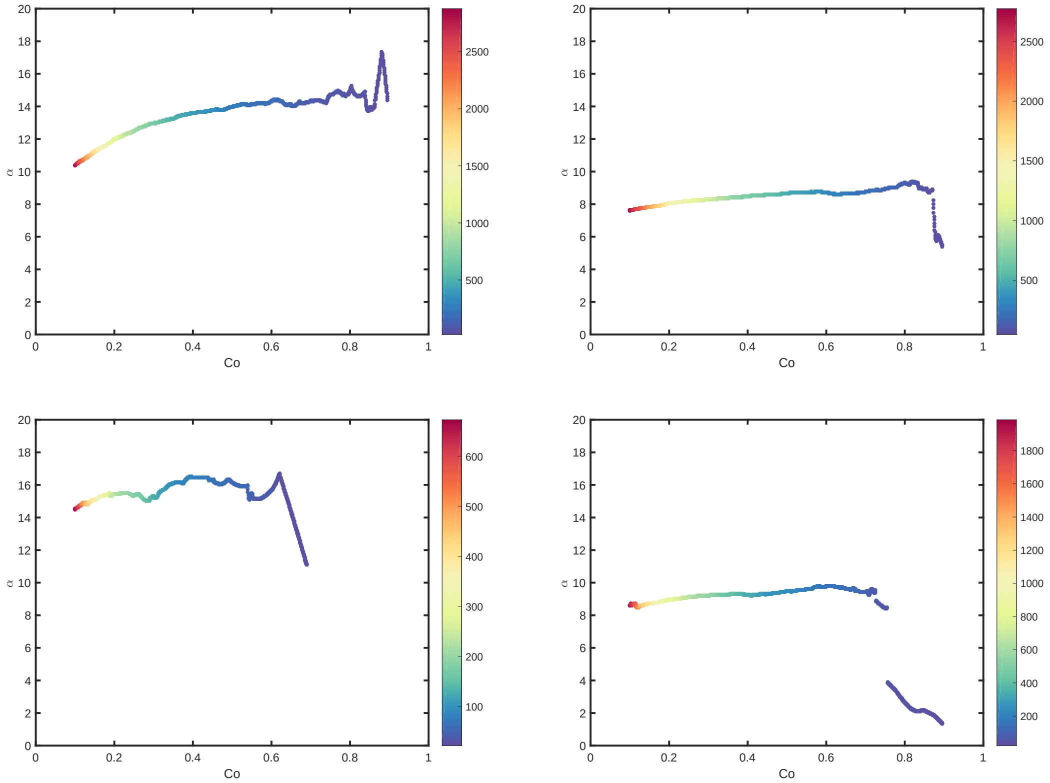

Figure 4: Dependence of the inclination angle α of the three-dimensional two-point autocorrelation of velocity magnitude (top) and vorticity magnitude (bottom) on the value of the correlation coefficient Co chosen for Richardson numbers $Ri = 0.1$ (left) and $Ri = 1$ (right) at non-dimensional time $St = 10$. The color scale indicates the number of data points available to determine the angle.

$St = 10$ is given in figure 7. Again, the angle obtained from vorticity magnitude is a little larger than that from velocity magnitude. In both cases, an approximately linear relationship between the inclination angle and the growth rate is obtained. Hence, it appears that the eventual evolution of the turbulence in stratified shear flow is directly related to the orientation of structures present in the flow.

6 Summary

Using the results of direct numerical simulations of turbulent stratified shear flows, the orientation of structures present in the flows is determined using isosurfaces of three-dimensional two-point autocorrelation coefficients of velocity magnitude and vorticity magnitude. The isosurfaces have an approximately ellipsoid shape and the inclination angle of the structures are determined from the major and minor axes of an ellipsoid least-squares fit to the isosurfaces.

The angles were observed to decrease with increasing Richardson number and an approximately linear relationship between the inclination angle and the growth rate of the turbulent kinetic energy was observed. Therefore, the inclination angle of structures in turbulent stratified shear flow appears to be directly related to the dynamics of the turbulent motion.

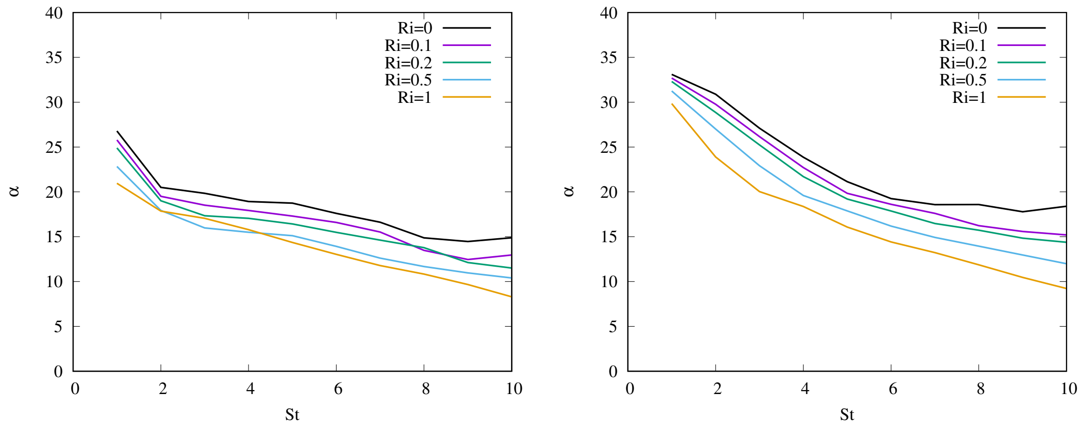

Figure 5: Evolution of the inclination angle α of velocity magnitude (left) and vorticity magnitude (right) with non-dimensional time St.

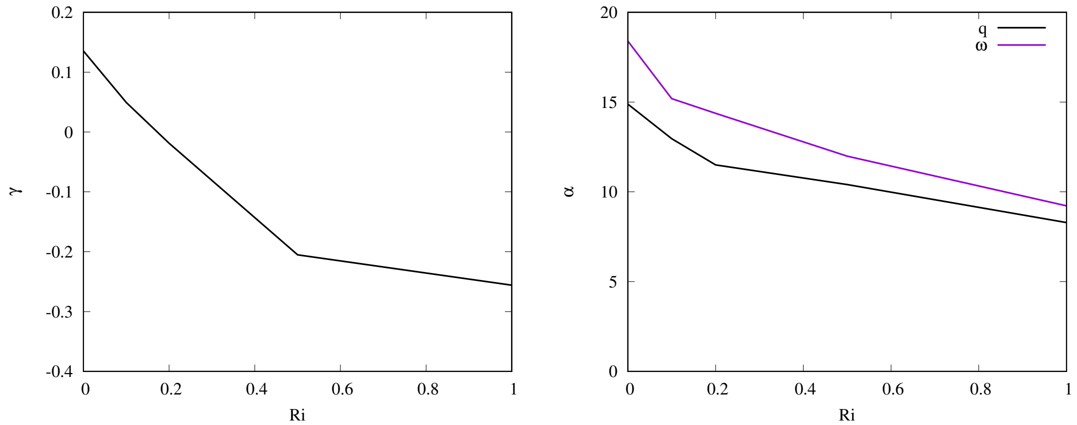

Figure 6: Dependence of the growth rate γ (left) and the inclination angle α (right) on the Richardson number Ri at non-dimensional time $St = 10$.

Acknowledgments

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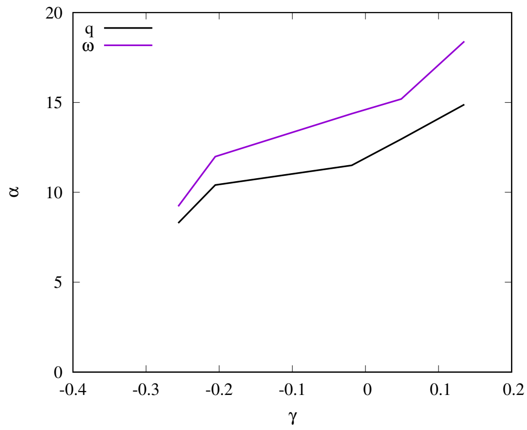

Figure 7: Dependence of the inclination angle α on the growth rate γ at non-dimensional time $St = 10$.

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