

Temperature front formation in stably stratified turbulence

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Abstract

Temperature fronts and associated cliff-ramp structures in stably stratified turbulence are investigated using DNS of the Navier-Stokes equations under the Boussinesq approximation. Vertical profiles of temperature fluctuations show sawtooth wave structures which result in a skewed PDF of the derivative of temperature fluctuation in the z -direction. The mechanisms for generating and maintaining these structures are considered.

1 Introduction

In the atmosphere and oceans, flows are often stably stratified, and ongoing research seeks to understand the dynamics of stratification in geophysical turbulence. An important feature of stably stratified turbulence is the significant influence of internal gravity waves which makes stably stratified turbulence unique compared to homogeneous isotropic turbulence. In this paper, we investigate the genesis of temperature fronts –a crucial subject both practically and fundamentally– in stably stratified turbulence using Direct Numerical Simulations (DNS) with 1024^3 grid points. The simulations are done by solving the following non-dimensionalized 3D momentum and temperature fluctuation equations under the Boussinesq approximation pseudo-spectrally,

$$(\partial_t - \nu \nabla^2) \mathbf{u} = -\mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + \theta \hat{\mathbf{z}} + \mathbf{f} \quad (1)$$

$$(\partial_t - \kappa \nabla^2) \theta = -N^2 w - \mathbf{u} \cdot \nabla \theta \quad (2)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (3)$$

where $\mathbf{u} = (u, v, w)$ are velocity components, θ is the temperature fluctuation about the linear (stable) mean temperature profile $d\bar{T}/dz \equiv -N^2$, and N is the Brunt-Väisälä frequency, $\sqrt{g\alpha(\partial\bar{T}/\partial z)/T_0}$. \mathbf{f} is a stochastic forcing applied to the large scales of the horizontal velocity. In wavenumber space, we set $\tilde{\mathbf{f}}(\mathbf{k}) = (\tilde{f}_x(k_x, k_y, 0), \tilde{f}_y(k_x, k_y, 0), 0)$ so that the forcing excites pure 2D velocity modes uniform in the z -direction and isotropic in (x, y) . For the analysis of velocity field, we use the so-called Craya-Herring decomposition which is based on the following orthonormal coordinates.

$$\mathbf{e}_1(\mathbf{k}) = \frac{\mathbf{k} \times \hat{\mathbf{z}}}{\|\mathbf{k} \times \hat{\mathbf{z}}\|} = \frac{1}{\sqrt{k_x^2 + k_y^2}} \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}, \quad (4)$$

$$\mathbf{e}_2(\mathbf{k}) = \frac{\mathbf{k} \times \mathbf{k} \times \hat{\mathbf{z}}}{\|\mathbf{k} \times \mathbf{k} \times \hat{\mathbf{z}}\|} = \frac{1}{\sqrt{k_x^2 + k_y^2 + k_z^2} \sqrt{k_x^2 + k_y^2}} \begin{pmatrix} k_z k_x \\ k_z k_y \\ -(k_x^2 + k_y^2) \end{pmatrix}, \quad (5)$$

$$\mathbf{e}_3(\mathbf{k}) = \frac{\mathbf{k}}{\|\mathbf{k}\|} = \frac{1}{\sqrt{k_x^2 + k_y^2 + k_z^2}} \begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix}, \quad (6)$$

where $\hat{\mathbf{z}} = (0, 0, 1)$ is the unit vector in the z -direction. The incompressibility condition 3 assures that the Fourier transform the velocity vector $\tilde{\mathbf{u}}(\mathbf{k})$ is perpendicular to the wave vector \mathbf{k}/\mathbf{e}_3 , and thus $\tilde{\mathbf{u}}$ can be spanned by \mathbf{e}_1 and \mathbf{e}_2 as

$$\tilde{\mathbf{u}}(\mathbf{k}) = \phi_1 \mathbf{e}_1(\mathbf{k}) + \phi_2 \mathbf{e}_2(\mathbf{k}). \quad (7)$$

Here the coefficients ϕ_1 and ϕ_2 can be written in components as

$$\phi_1 = \tilde{\mathbf{u}} \cdot \mathbf{e}_1 = \frac{1}{\sqrt{k_x^2 + k_y^2}} (k_y \tilde{u} - k_x \tilde{v}) = \frac{i}{\sqrt{k_x^2 + k_y^2}} \tilde{\omega}_z, \quad (8)$$

$$\phi_2 = \tilde{\mathbf{u}} \cdot \mathbf{e}_2 = -\frac{\sqrt{k_x^2 + k_y^2 + k_z^2}}{\sqrt{k_x^2 + k_y^2}} \tilde{w}, \quad (9)$$

and we see that they have the meaning of a vortex mode (ϕ_1) and a wave mode (ϕ_2), respectively [1].

2 Numerical results

2.1 Velocity field

As an initial condition, we set all velocity components zero, and adding the above 2D forcing, we excite the velocity without coupling to the temperature field first. After the 2D velocity field is developed, we switch on the coupling of stratification [2].

Figure 1 shows the temporal growth of the energy in ϕ_1 and ϕ_2 for $N^2 = 10$ and forcing wavenumber $k_f = 4$. Stratification is switched on at $t = 10$, and because of the

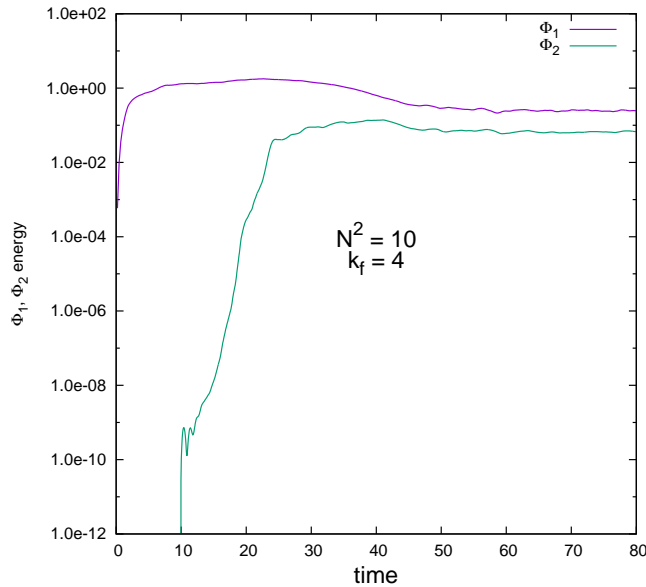


Figure 1: The growth of ϕ_1 and ϕ_2 energy.

interaction with the vortex mode, the wave mode ϕ_2 grows nearly exponentially and becomes comparable with ϕ_1 around $t = 20$. It takes a rather long period for the ϕ_1 and ϕ_2 modes to become stationary around $t = 50$. At the early stage, the ϕ_1 energy continues to increase until the ϕ_2 energy grows sufficiently. By looking at the energy spectrum, this increase of the ϕ_1 energy is confirmed as the inverse cascade of energy towards the largest scale and the characteristic slope of $-5/3$ is observed. It is interesting to notice that the

ϕ_1 energy begins to decrease after the ϕ_2 energy becomes close to the value of ϕ_1 . Around this time we observe that the $-5/3$ spectrum disappears and the energy spectrum becomes almost flat.

2.2 Generation of temperature fronts

Figure 2 shows a gray-scale contour plot of temperature fluctuations at slice $y = \pi$ (or $y = 512$ in grid number coordinates) at $t = 64.7$. We observe many (almost vertically periodic) wavy structures with black (negative) and white (positive) layers stacked together with clear boundaries implying a sharp temperature fronts. Similar temperature front structures are observed in simulations of homogeneous shear flows [3],[4] as well as in large-eddy simulations of the stably stratified atmospheric boundary layer [5]. Also, temperature fronts are ubiquitous features in passive scalar turbulence [6]. The fronts are tilted at angles that depend on the ratio of horizontal to vertical gradients of temperature; Chung and Matheou [4] show that for fixed shear the fronts become nearly horizontally when the mean temperature gradient is very large. Cliff-ramp structures are one of the

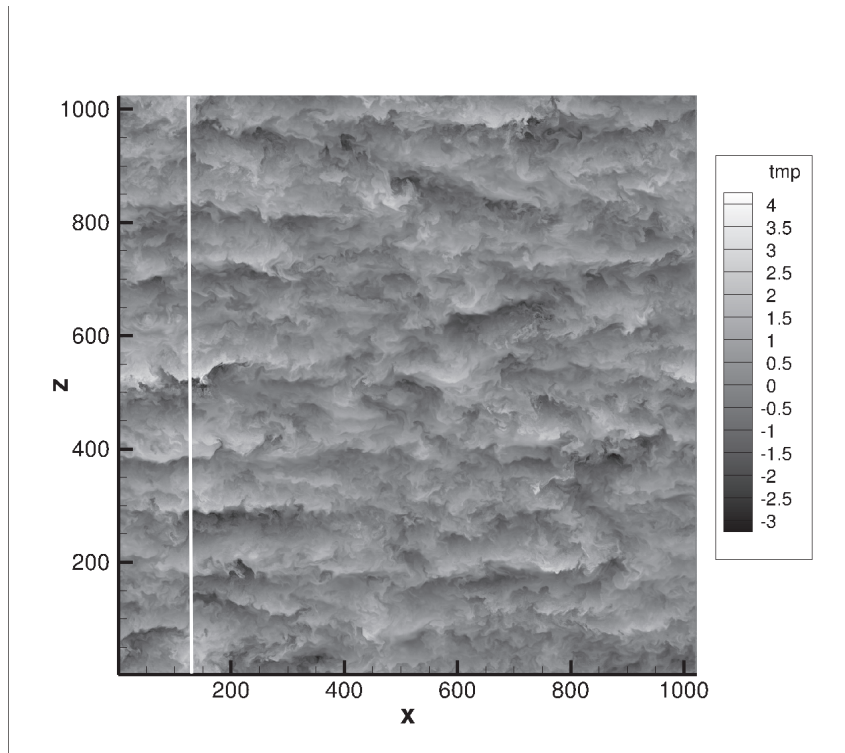


Figure 2: Contour plot of the temperature fluctuations on the slice at $y = \pi$ at $t = 64.7$.

signatures of temperature fronts, and to show this more clearly, a vertical cut is taken at $x = 132$ in the region where the most intense fronts are visible in Figure 2 (see the white line in Figure 2). Temperature fluctuations and the total temperature along this cut are plotted in Figure 3. Figure 3-a) shows numerous sharp jumps in the temperature fluctuations with increasing z . These large jumps result in a staircase pattern for the total temperature shown in Figure 3-b). The sawtooth waves consist of gradual decreasing θ with z with rapid recovery to a positive value as the frontal boundary is crossed. This asymmetry of gradients comes from the structure that warm temperature region lies on top of cool temperature region. The asymmetry of the positive and negative gradients

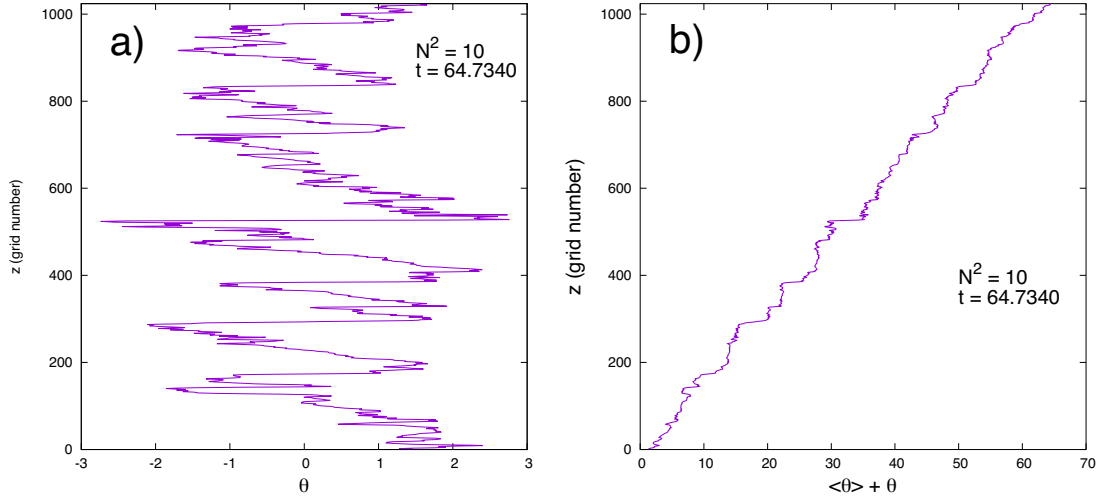


Figure 3: a): Temperature fluctuations along the line at $x = 132$, b): The total temperature.

in the z direction is further shown in the probability density function (PDF) of temperature. Figure 4 shows the PDFs of θ and the three components of its derivatives. We see that the PDF of θ is near Gaussian for small values, while the tails exhibit strong non-Gaussianity for all components of the gradient of θ . Among the three components, $\partial\theta/\partial z$ shows skewness while the horizontal derivatives are not skewed.

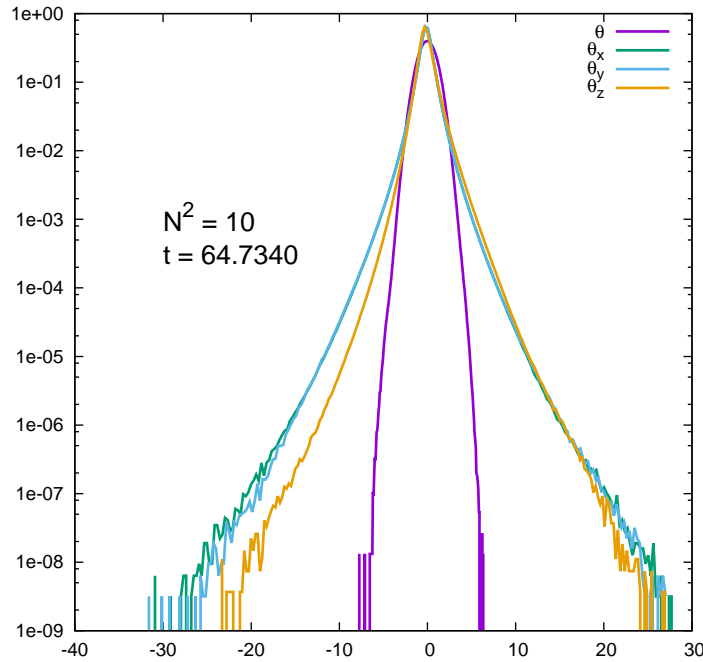


Figure 4: PDFs of the temperature fluctuations and the derivatives.

3 Discussion

We have seen that the strong temperature fronts can be produced in stably stratified turbulence even without external shear flows, and that asymmetry and the skewed statistics

are associated with those fronts. The mechanism of production of those fronts and statistics in particular in pure stratified turbulence is still an open problem, and investigation is ongoing. It may be useful to raise some questions as a summary of the article and a chart of the future study:

- (1) How is the strong temperature front produced in relation of the effects of gravity waves? (How does it grow along the development of ϕ_1 and ϕ_2 in Figure 1 ?)
- (2) What are the relevant flow structures to maintain the strong temperature front in stably stratified turbulence? (Are they (approximate) vortex rings [5], pancake vortices [7] or any kind of colliding waves?)
- (3) How does the intermittency of the derivative of temperature fluctuations develop as stratification varies? (Departure from Gaussian increases/decreases with stratification?)
- (4) What kind of interactions do flow instabilities in stably stratified turbulence have with temperature fronts? zig-zag or Kelvin-Helmholtz (Zigzag instability [8][9] causes the front genesis ? Kelvin-Helmholtz instability enhances/suppresses it ?)

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