

# Direct measurements of flux Richardson number in the nearshore coastal ocean

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## ABSTRACT

We conducted a set of three field experiments using an underwater turbulence tower to address the following questions: (1) “What are the flux Richardson numbers ( $R_f$ ) and vertical diffusivities ( $K_v$ ) in a highly turbulent region with varying stratification?” (2) “Is it valid to extend the Shih et al. [2005] flux Richardson number ( $R_f$ ) parameterization to field situations at higher turbulence

activity numbers  $G = \frac{\epsilon}{\nu N^2}$ ?” The experiments were conducted at three separate field sites in

Monterey Bay, CA; Eilat, Israel; and Mamala Bay, Hawaii using the same experimental platform, instrumentation, and analytical methods. Direct measurements of turbulent buoyancy fluxes and mixing efficiencies, with  $10^2 < G < 10^7$ , confirm the relationship for the flux Richardson number  $R_f$  suggested by Shih et al. [2005]. Additionally, the mixing efficiency  $\Gamma$  is likely to be up to an order of magnitude less than the commonly assumed value of 0.2 over a wide range of turbulence states. This result holds over a range of flow conditions (including presence of internal waves and bores) and environmental conditions (weak and strong stratification) across the three sites.

## 1. INTRODUCTION

Quantifying the vertical turbulent mixing of scalars such as heat, dissolved oxygen, and dissolved inorganic carbon in the ocean, as well as in estuaries and lakes, remains an ongoing challenge [Ivey et al., 2008]. Vertical diffusivity can be directly calculated from measurements of the buoyancy flux if appropriate instrumentation is available. However, due to the difficulty of measuring buoyancy fluxes in the field, the diffusivity is often calculated from parameterizations that involve more easily measured variables (such as the temperature variance), estimates of the dissipation (using for example microstructure profilers, Thorpe-scale density overturns from moored profilers, fine-scale parameterizations, etc.), and an assumed mixing efficiency,  $\Gamma$  [cf. Osborn, 1980; Waterhouse et al., 2014]. Parameterizations for the vertical diffusivity and the mixing efficiency themselves vary widely depending on the level of turbulence and stratification [Dunckley et al., 2012]. Using direct numerical simulations (DNS) of stratified turbulence, Shih et al. [2005] found that  $\Gamma$  expressed in terms of the flux Richardson number,  $R_f$  (see Eq. 5), is a

function of the “turbulent activity number”, which we will refer hereinafter as the Gibson number,  $G = \varepsilon/\nu N^2$  [Gibson, 1980]:

$$R_f = C \left( \frac{\varepsilon}{\nu N^2} \right)^{-0.5} \quad (1)$$

where  $\varepsilon$  is the turbulent dissipation rate,  $\nu$  is the kinematic viscosity,  $N^2 = -\frac{g}{\rho} \frac{\partial \rho}{\partial z}$  is the

buoyancy frequency squared, and  $C = 1.5$  is an empirically derived constant. However, the model fitting was limited to  $G$  less than  $10^3$  due to computational constraints.

While limited to a smaller range of  $G$ , the Shih et al. [2005] parameterization for mixing efficiency is often extrapolated to higher  $G$  without data verification in the higher range. Moreover, controversy exists over the applicability of this DNS parameterization in field situations, where the mixing efficiency is generally taken to be a constant (see, for example, Ivey et al., 2008). Therefore, we seek to address the following questions: What are the flux Richardson numbers ( $R_f$ ) and vertical diffusivities ( $K_v$ ) in a highly turbulent region with varying stratification? Moreover, is it valid to extend the Shih et al. [2005] flux Richardson number ( $R_f$ ) parameterization to field situations at higher turbulence activity numbers? To answer these questions we conducted a set of three field experiments in stratified environments in the near-coastal ocean.

The experiments were conducted at three separate field sites in Monterey Bay, CA; Eilat, Israel; and Mamala Bay, Hawaii using the same experimental platform, instrumentation, and analytical methods. The basis of the approach was to use fast temperature and conductivity sensors coupled with vertical velocity measurements all mounted on an underwater instrument tower. The coupled instruments were directly connected to power and data lines extending to shore-based laboratories. These measurements allow direct calculation of the flux Richardson number ( $R_f$ ; see Eq. 3 below), mixing efficiency ( $\Gamma$ ; see Eq. 5 below) and the vertical diffusivity ( $K_v$ ; see Eq. 4 below), thereby enabling the examination of the Shih et al. [2005] parameterization at higher turbulence activity numbers, as well as the investigation of the often-used assumption of a constant mixing efficiency.

## 2. BACKGROUND AND METHODS

The flux Richardson number ( $R_f$ ) was originally defined by Osborn [1980] as the proportion of TKE generated by the shear production term ( $P$ ) that gets transferred to potential energy through buoyancy flux term ( $B$ ),

$$R_f = \frac{B}{P} \quad (2)$$

Ivey and Imberger [1991] generalized the definition of  $R_f$  to the following

$$R_f = \frac{B}{B + \varepsilon} \quad (3)$$

They point out that  $B + \varepsilon = m$ , where  $m$  represents the total mechanical energy available to maintain turbulent motions. The standard method of estimating the vertical turbulent diffusivity of density ( $K_\rho$ ) is to apply the steady-state formulation described by Osborn [1980],

$$K_\rho = \Gamma \frac{\varepsilon}{N^2} \quad (4)$$

where  $\Gamma$  is the mixing efficiency, which is linked to  $R_f$  (Eq. 3) through the following relationship:

$$\Gamma = \frac{R_f}{1 - R_f} \quad (5)$$

Calculations of the flux Richardson number have been made through theoretical estimates [Ellison 1957; Osborn, 1980], numerical modeling [Holt et al., 1992, Shih et al., 2005] and laboratory studies [Itsweire et al., 1986; Rohr et al., 1988; among others]. The theoretical maximum has been confirmed by multiple studies [Ellison, 1957; Britter, 1974] which indicate a maximum value in the range of  $R_f = 0.15-0.2$ . Moreover, the standard approach for estimating diapycnal mixing in the field is to use a constant flux Richardson number ( $R_f = 0.17, \Gamma = 0.2$ ).

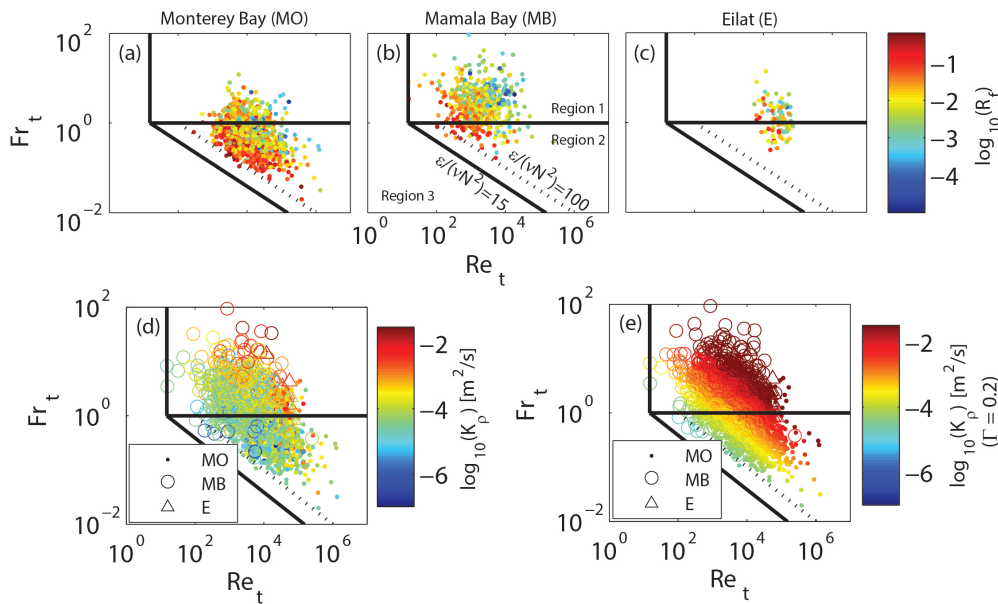
The common element to the measurements we describe below is an underwater turbulent flux tower, deployed in 15, 10, and 23 m of water in Monterey Bay, Eilat, and Mamala Bay, respectively (Walter et al. [2014] - Monterey Bay, Dunckley [2012] - Eilat, and Squibb [2014] - Mamala Bay). On the tower we attached (typically 6) Nortek Acoustic Doppler Velocimeters (ADVs) at various heights above the bed (e.g., 0.3, 1, 2, 4, 6, and 8 meters above the bed for the Monterey Bay and Mamala Bay setup) fixed to arms that extended about 1 meter out from the tower. The ADVs were also outfitted with a Precision Measurement Engineering, Inc. (PME) fast-response thermistor (FP07) and conductivity (foiled electrode ceramic conductivity sensor) sensor (fast CT). The fast CTs were placed roughly 1 cm away from the sampling volume of the ADV so that collocated, synchronized measurements of velocity and density were recorded.

Turbulence quantities were processed using ten-minute data intervals (50% overlap between adjacent windows), following standard practices [e.g., Soulsby, 1980; Davis and Monismith, 2011; Walter et al., 2011; Walter et al., 2014]. Prior to calculating turbulent statistics, a number of quality control measures were implemented filtering the data using the phase-space method described in Goring and Nikora [2002]. Surface wave effects were removed either using the adaptive filtering technique described in Feddersen and Williams [2007] or the spectral “phase” decomposition method of Bricker and Monismith [2007]. Dissipation of TKE was calculated using the Feddersen et al. [2007] inertial subrange fit method that accounts for the advection of turbulent eddies past the sensor due to surface waves. Additional data conditioning techniques were also similar to those of Gerbi et al. [2008] and Bluteau et al. [2011].

### 3. RESULTS AND CONCLUSIONS

During the tower deployments, all three sites experienced transient stratification and mixing events associated with shoaling internal tides (bores). In order to characterize the stratified turbulence, we examined the turbulent Reynolds number ( $Re_t$ ) and turbulent Froude number ( $Fr_t$ ) parameter space [e.g., Ivey and Imberger, 1991; Davis and Monismith, 2011; Dunckley et al.,

2012; Walter et al., 2014]. The stratified turbulence at each of the three sites occupies a slightly different portion of the  $Re_t$ - $Fr_t$  parameter space (Fig. 1a-c). The data from Monterey Bay (Fig. 1a) are predominantly in Region 2 (buoyancy-controlled regime; see Figure 1b), while the data for Mamala Bay (Fig. 1b) are mostly in Region 1 (buoyancy-affected regime). Interestingly, the data from Eilat (Fig. 1c) are centered on the boundary between the two regions. In all cases observations where the combined action of buoyancy and viscosity significantly suppress turbulent motions (Region 3; buoyancy dominated regime) were largely absent.

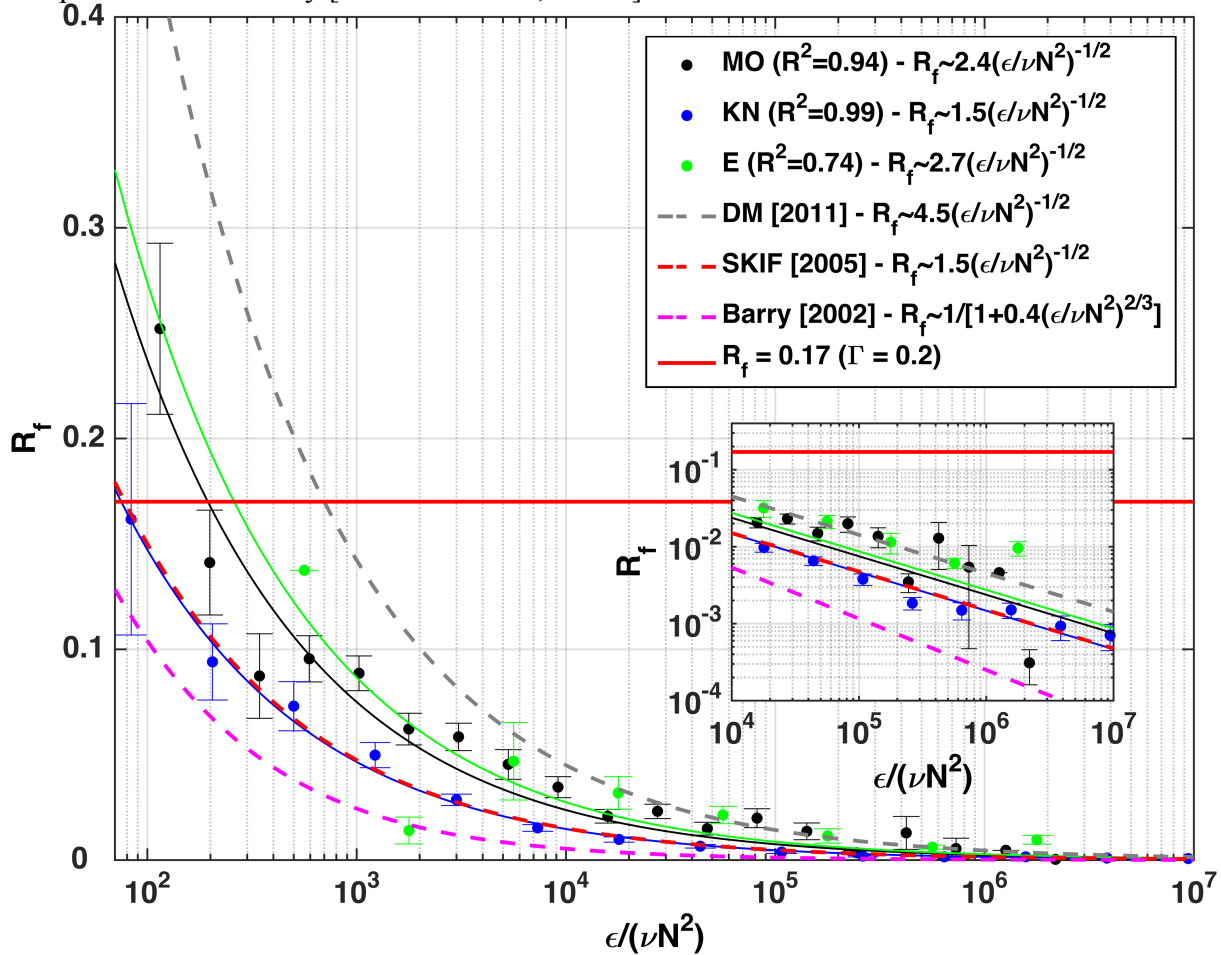


**Figure 1:** Characterization of turbulence at all three sites using  $Fr_t$ - $Re_t$  phase diagram of Ivey and Imberger (1991). Panels (a)-(c) are color coded to show the flux Richardson number of the turbulence as a function of  $Fr_t$  and  $Re_t$ . Panel (d) shows the turbulent diffusivity calculated from direct measurements of the mixing efficiency for all three sites. Panel (e) shows the turbulent diffusivity calculated from Equation (6) assuming a mixing efficiency  $\Gamma = 0.2$ .

Measurements at all three field sites reveal the highly variable nature of  $R_f$ , with most of the observations falling below the commonly assumed “critical” value of 0.17 ( $\Gamma = 0.2$ ). However, all three sites indicate that for increasing  $Re_t$  and  $Fr_t$ , or equivalently an increase in  $G$  (diagonal lines in Fig. 1), the flux Richardson number, and hence mixing efficiency, decreases. Vertical turbulent diffusivity was estimated at all three field sites using two methods: the diffusivity calculated using direct observations of the mixing efficiency (Fig. 1d) and the diffusivity estimated using a constant mixing efficiency ( $R_f = 0.17$ ,  $\Gamma = 0.2$ ) and Equation 4 (Fig. 1e). Diffusivities calculated using a constant mixing efficiency were consistently larger (up to several orders of magnitude) than those estimated using a variable mixing efficiency.

Observations from all three sites with  $10^2 < G < 10^7$  confirm Eq. 1, the functional relationship for  $R_f$  suggested by Shih et al. [2005], with values of  $C$  from the three sites as follows: Eilat,  $C = 2.7$  ( $R^2 = 0.82$ ); Mamala Bay,  $C = 1.5$  ( $R^2 = 0.99$ ), and Monterey Bay,  $C = 2.4$  ( $R^2 = 0.94$ ). Similar measurements reported in Davis and Monismith (2011) gave  $C = 4.5$ . At

lower turbulent activity numbers (i.e.,  $G < 10^3$ ),  $R_f$  showed the most variation across the different sites, although many of the observations show overlapping error bars. At higher values of  $G$  ( $G > 10^3$ ), the flux Richardson numbers at the various sites converge and show more uniform agreement. Variation in  $C$  across the different sites is likely attributable to the variability in  $R_f$  at lower  $G$  values, which is the lower portion of the energetic turbulence regime identified by Shih et al. [2005]. The larger coefficient ( $C = 4.5$ ) in the Davis and Monismith [2011] study may be due to the fact that the authors were calculating density fluctuations using measured temperature fluctuations and salinity values calculated using an empirical relationship between temperature and salinity [cf. Walter et al., 2014a].



**Figure 2:** The flux Richardson number as a function of the turbulence activity number showing how the results from the three sites compare with the form of Equation 1 proposed by Shih et al. [2005] (SKIF). The solid dots represent bin-averaged values, while the error bars signify the standard error. Also included are the forms of Equation 1 found by Davis and Monismith [2011] (DM) and a collection of laboratory experiments compiled by Barry et al. [2002]. The red horizontal line denotes the commonly assumed constant mixing efficiency ( $R_f = 0.17, \Gamma = 0.2$ ).

The data show that in the relatively shallow near-coastal environment when stratification is present the value of  $\Gamma$  is likely to be up to an order of magnitude less than the commonly assumed value of 0.2 over a wide range of turbulence states (as given by  $G$ ). This result holds over a range of flow conditions from the weakly stratified reef where internal waves and bores are present (Mamala Bay), to an energetic near-coastal regime where strong internal waves and bores are present (Monterey), to a less energetic, more strongly stratified shallow coral reef environment (Eilat). Given the relatively high correlation coefficients for all three sites ( $R^2$  from 0.82 to 0.99), the relationship originally proposed by Shih et al. [2005] proves to be robust and applicable to oceanic turbulence.

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#### 5. REFERENCES

- Barry, M. E. (2002), Mixing in stratified turbulence, PhD thesis, University of Western Australia, Centre for Water Research.
- Britter, R.E. (1974) An experiment on turbulence in a density stratified fluid. PhD thesis, Monash University, Victoria, Australia.
- Bluteau, C. E., N. L. Jones, G. N. Ivey (2011), Estimating turbulent kinetic energy dissipation using the inertial subrange method in environmental flows, *Limnol. Oceanogr. Methods*, 9, 302-321.
- Bricker, J. D. and S. G. Monismith (2007), Spectral wave-turbulence decomposition, *J. Atmos. Ocean. Technol.*, 24, 1479-1487.
- Davis, K. A. and S. G. Monismith (2011), The modification of bottom boundary layer turbulence and mixing by internal waves shoaling on a barrier reef, *J. Phys. Oceanogr.*, 41, 2223-2241.
- Dunckley, J. F., J. R. Koseff, J. V. Steinbuck, S. G. Monismith, and A. Genin (2012), Comparison of mixing efficiency and vertical diffusivity models from temperature microstructure, *J. Geophys. Res.*, 117, C10008, doi: 10.1029/2012JC007967.
- Dunckley, J. F. (2012), Mixing in Near-Shore Coastal Environments, Ph.D. Dissertation, Department of Civil and Environmental Engineering, Stanford University, Stanford, CA.
- Ellison, T.H. (1957) Turbulent transport of heat and momentum from an infinite rough plane., *Journal of Fluid Mechanics*. 2, 456-466
- Feddersen, F. and A. J. Williams (2007), Direct estimation of the Reynolds stress vertical structure in the nearshore, *J. Atmos. Ocean. Technol.*, 24, 102-116.
- Feddersen, F., J. H. Trowbridge, and A. J. Williams (2007), Vertical structure of dissipation in the nearshore, *J. Phys. Oceanogr.*, 37, 1764-1777.

- Gerbi, G.P., J.H. Trowbridge, E.A. Terray, A.J. Plueddemann, and T. Kukulka (2008), Observations of Turbulence in the Ocean Surface Boundary Layer: Energetics and Transport, *Journal of Physical Oceanography*, 39,5: 1077-1096.
- Gibson, C. H. (1980), Fossil temperature, salinity and vorticity in the ocean. *Marine Turbulence*, J. C. T. Nihoul, Ed., Elsevier, 221–258.
- Goring, D. G. and V. I. Nikora (2002), Despiking acoustic Doppler velocimeter data, *J. Hyd. Eng.*, 128(1), 117-126.
- Holt, S. E., J. R. Koseff, and J. H. Ferziger (1992), A numerical study of the evolution and structure of homogeneous stably stratified sheared turbulence, *J. Fluid. Mech.*, 237, 499-539.
- Itsweire, E. C., K. N. Helland, and C. W. Van Atta (1986), The evolution of grid-generated turbulence in a stably stratified fluid, *J. Fluid Mech.*, 162, 299-338.
- Ivey, G. N. and J. Imberger (1991), On the nature of turbulence in a stratified fluid. Part I: The energetics of mixing, *J. Phys. Oceanogr.*, 21, 650-658.
- Ivey, G. N., K. B. Winters, and J. R. Koseff (2008), Density stratification, turbulence, but how much mixing?, *Annu. Rev. Fluid. Mech.*, 40, 168-184.
- Osborn, T. R. (1980), Estimates of the local rate of vertical diffusion from dissipation measurements, *J. Phys. Oceanogr.*, 10, 83-89.
- Rohr, J. J., K. N. Helland, and C. W. Van Atta (1988), Growth and decay of turbulence in a stably stratified shear flow., *J. Fluid Mech.*, 195, 77-111.
- Shih, L. H., J. R. Koseff, G. N. Ivey, and J. H. Ferziger (2005), Parameterization of turbulent fluxes and scales using homogeneous sheared stably stratified turbulence simulations, *J. Fluid Mech.*, 525, 193.
- Soulsby, R. L. (1980), Selecting record length and digitization rate for near-bed turbulence measurements, *J. Phys. Oceanogr.*, 10, 208-219.
- Squibb, M. E. (2014), Dynamics of shoaling internal waves in the near-shore, Ph.D. Dissertation, Department of Civil and Environmental Engineering, Stanford University, Stanford, CA.
- Walter, R. K. (2014), Nonlinear internal waves, internal bores, and turbulent mixing in the nearshore coastal environment, Ph.D. Dissertation, Department of Civil and Environmental Engineering, Stanford University, Stanford, CA.
- Walter, R.K, et al. (2014), Stratified Turbulence in the nearshore coastal ocean: Dynamics and evolution in the presence of internal bores, *J. Geophys. Res. Oceans*, 119, doi:10.1002/2014JC010396.
- Walter, R. K., N. J. Nidziko, and S. G. Monismith (2011), Similarity scaling of turbulence spectra and cospectra in a shallow tidal flow, *J. Geophys. Res.*, 116, C10019, doi:10.1029/2011JC007144.
- Waterhouse, A. F., et al. (2014), Global patterns of diapycnal mixing from measurements of the turbulent dissipation rate, *J. Phys. Oceanogr.*, 44, 1854-1872.