Scattering and Trapping of Obliquely Incident, Low-Mode Internal Tides off a Continental Shelf and Slope

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Abstract

We demonstrate – using linear scattering theory and an idealized, stratified, rotating ocean domain, with a flat bottom deep ocean and a straight continental slope and shelf – that a high-mode (mode 3) Poincare wave incident on the coastal margin can excite large amplitude leaky coastal trapped waves (CTWs) or leaky edge waves along the coastal margin dependent on the frequency and alongshore wavenumber of the incident wave. The large response only occurs when low modes (modes one or two) are evanescent in the deep ocean. This may be a means by which large amplitude, alongshore propagating internal tides can be excited at the coastal margin. The fact that the leaky CTW and edge wave response is sensitive to alongshore wavenumber suggests that their excitation in real oceans will be temporally and spatial intermittent due to variations in incidence angles of deep ocean internal tides and spatial and temporal variations in stratification.

1 Introduction

This work addresses the scattering of obliquely incident internal tides off of a coastal margin and the amplitude of the response on the continental shelf and slope to the incident wave. The specific questions we ask are:

- Under what conditions is the amplitude of the shelf or slope response large, compared to the amplitude of the incident internal tide?
- Can scattering lead to partial trapping of internal tide energy on the shelf and slope, in the form of 'leaky' edge waves or coastal trapped waves?

2 Barotropic Case

To motivate the problem, we first consider the barotropic case. Huthnance (1974) described linear barotropic waves trapped to the coast for a semi-infinite rotating ocean, bounded on one side by a straight continental shelf and slope. The discreet trapped modes (Fig. 1) consist of: *i.* an infinite set of continental shelf waves (Robinson, 1964), *ii.* a Kelvin wave mode, and *iii.* an infinite set of refractively-trapped edge waves (Ursell, 1952). Within the Poincare continuum (shaded region of Fig. 1), Poincare waves can propagate freely in the deep ocean and trapped waves can not exist, as energy isolated to the coast will decay as Poincare waves radiate energy to the deep ocean. However, the amplitude of the response on the shelf to an obliquely incident Poincare wave is dependent on incidence angle, with a large response (relative to the amplitude of the incident wave) occurring along ridges that roughly track the dispersion curves of the edge waves (Fig. 1). Chapman (1982) referred to these ridges of large response as nearly trapped or 'leaky' edge waves.

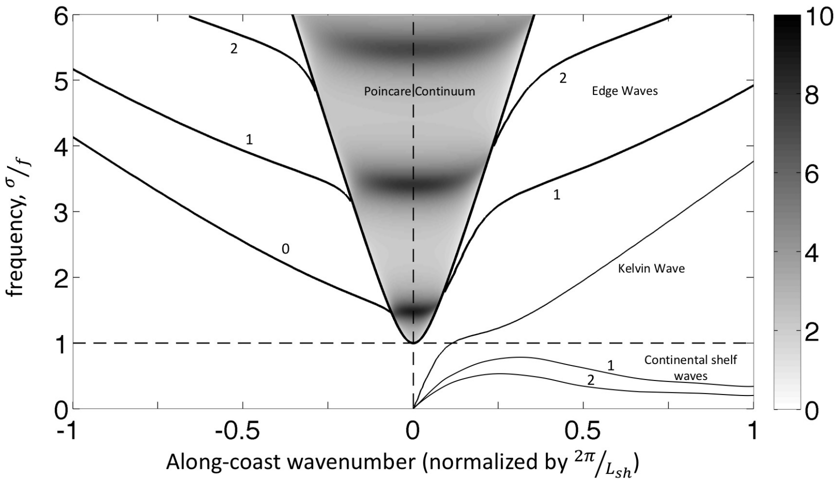

Figure 1: Dispersion relation for barotropic waves on an f-plane, trapped to a straight continental shelf and slope (Huthnance, 1974). Frequency is normalized by the Coriolis frequency, f, and the along-coast wave number, l, is normalized by $2\pi/L_{sh}$, where L_{sh} is the width of the continental shelf. Three classes of discrete trapped modes exist: 1. an infinite set of continental shelf waves at sub-inertial frequencies, 2. a Kelvin wave, and 3. an infinite set of refractively trapped edge waves at super-inertial frequencies. The shaded region indicates the Poincare continuum, where free Poincare waves can exist in the deep ocean and trapping along the coast can not occur, as energy isolated to the coast will decay by the radiation of Poincare waves to the deep ocean. Poincare waves incident on the coast will reflect back to the deep ocean. The shading indicates the amplitude of the response on the shelf relative to the amplitude of the incident Poincare wave. A large shelf response occurs along ridges that track the edge wave dispersion curves. Chapman (1982) referred to these ridges of large response as nearly trapped edge waves.

3 'Leaky' Baroclinic Coastal Trapped Waves and Edge Waves

Our interest here is to understand the response of the continental shelf and slope to obliquely incident Poincare internal waves in a rotating stratified ocean. We consider a rotating ocean with constant stratification and a semi-infinite geometry with a straight coast, a step-like shelf break, and continental shelf (Fig. 2). For this domain, an infinite set of baroclinic trapped waves exist only at sub-inertial frequencies (Huthnance, 1978), and are referred to as coastal trapped waves (CTWs). The cross-shore structure of CTWs is on dependent shelf geometry and stratification. For example, for large Burger number, $S = N_o H_o / f L_{sh}$ – N_o is a typical buoyancy frequency, H_o is the depth of the deep ocean, f is the Coriolis parameter, and L_{sh} is the width of the shelf – the CTW has the structure of an internal Kelvin mode. For weak stratification or short alongshore wavelengths, the CTW behaves as a bottom-trapped wave (Rhines, 1970). For the domain considered here, the pressure field for the mode-one CTW is horizontally-segregated, with a zero crossing roughly at the shelf break (Figs. 3 and 4), and does not change sign in the vertical direction on the shelf and in the deep ocean adjacent to the shelf.

At superinertial frequencies, complete trapping at the coast can not occur, because an infinite set of baroclinic Poincare wave modes can be excited in the deep ocean and variability at the coast will radiate to the deep ocean by these wave modes (Dale and Sherwin, 1996; Dale et al., 1996). However, Dale et al. (1996) showed that CTW dispersion curves extend into super-inertial frequencies. Such waves travel along the coast in the cyclonic direction, but decay over time as energy is radiated to the deep ocean by Poincare waves. In addition, Chapman (1982) demonstrated that the response on the shelf to

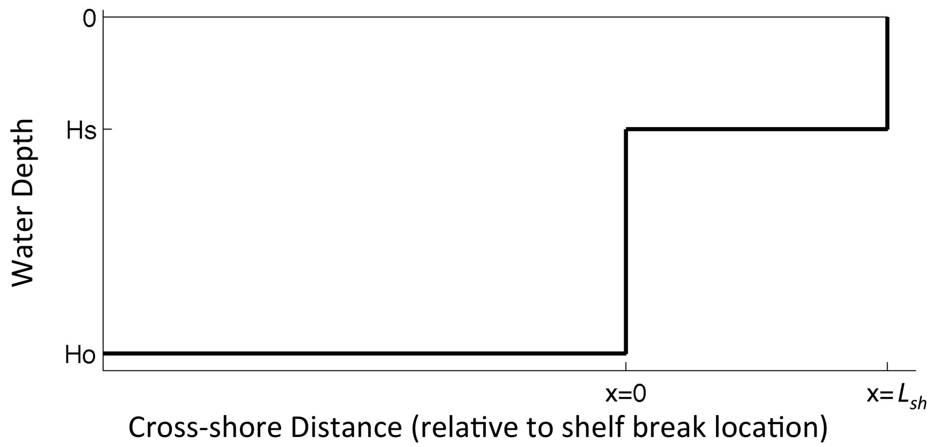

Figure 2: Semi-infinite idealized rotating oceanic domain with a straight coast, flat continental shelf and abrupt shelf break. For this study, the deep ocean water depth, H_o, is 4000 m; $H_s = 0.4H_o$; $L = 100$ km. The fluid has constant stratification ($N = 3.9 \times 10^{-3}\,\mathrm{s}^{-1}$); the Coriolis parameter, f, is $1 \times 10^{-4}\,\mathrm{s}^{-1}$, and the mode-one deformation radius in the deep ocean is 50 km.

incident internal waves can be large under certain incidence conditions and refers to this large response as nearly trapped internal edge waves.

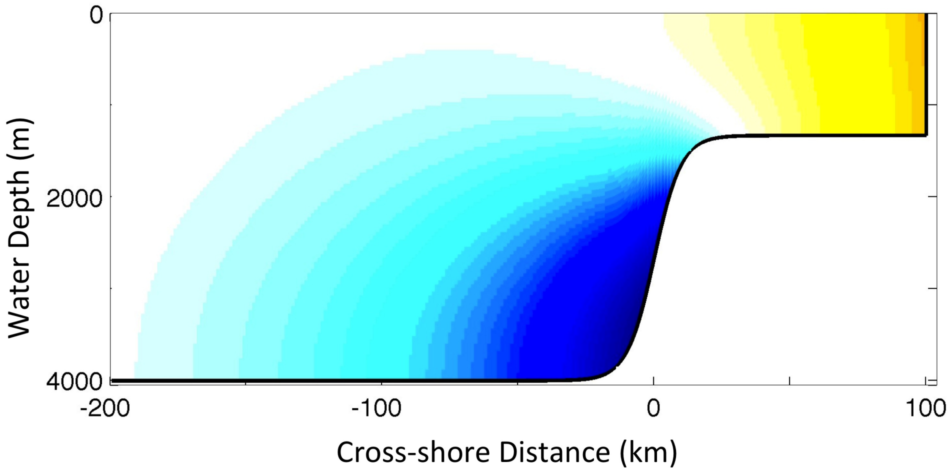

Figure 3: Pressure field for a mode-one, sub-inertial, baroclinic coastal trapped wave for the domain described in Fig. 2 ($\sigma/f = 0.825$ and along-shore wavenumber $l = 1.32 \times 10^{-5}\,\mathrm{m}^{-1}$). The location for this wave on the mode-one dispersion curve is indicated by the black circle in Fig. 4.

We follow the methods Chapman (1982) and Dale et al. (1996) to study the coastal response of the domain described in Fig. 2 to incident Poincare waves. Dispersion diagrams are shown in Figs. 4 and 5. For a particular wave mode, n, the wave can only propagate freely within the deep ocean within the Poincare continuum for that mode. Thus, in (σ,l) space – where σ is the wave frequency and l is the alongshore wavenumber – incident free mode-n waves only exist in the region bounded by the mode-n dispersion curve. Outside of that curve, mode-n energy is evanescent.

When a mode-one wave is incident on the coast, most of the energy is reflected back to

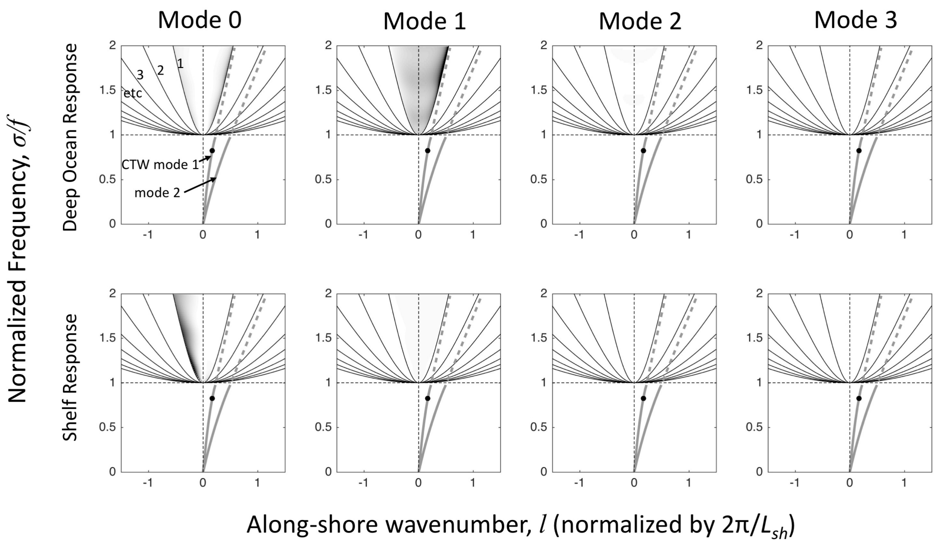

Figure 4: Response of the continental shelf to an incident mode-one internal Poincare wave, in the ocean/shelf domain described in Fig. 2, as a function of frequency, σ,and along-shore wavenumber, l, of the incident wave. Calculation follows the method described in (Chapman, 1982). Shading in upper panels indicate the amplitude-squared of modes 0-3 in the deep ocean. Mode 0 is always evanescent, with a decay length scale of l^{-1}. Higher modes are reflected Poincare waves within the Poincare continuum for that wave mode. Outside the Poincare continuum, the mode is evanescent and decays offshore. Shading in lower four panels indicate amplitude-squared of modes 0-3 response on the continental shelf. Thick lines above $\sigma/f = 1$ are dispersion curves for mode 1-3 internal waves in the deep ocean, thus showing the bound of the Poincare continuum for each mode. Thick, gray lines below $\sigma/f = 1$ indicate dispersion curves for CTW modes one and two. The black circle indicates the frequency and alongshore wavenumber of the CTW shown in Fig. 3.

the deep ocean as a mode-one wave (Fig. 4). The shelf response is very weak, with only a mode-0 response at negative alongshore wavenumbers (5^{th} panel of Fig. 4).

When a mode-three wave is incident on the coast, the scattering into reflected wave modes is dependent on frequency and alongshore wavenumber (panels 1 to 4 in Fig. 5). For waves propagating in the cyclonic direction along the coast (positive l; with the coast to the right in the northern hemisphere), a very strong mode-one or mode-two response occurs in the deep ocean when that mode is evanescent (Panels two and three in Fig. 5). These regions of large response track the extensions of CTW dispersion curves into superintertial frequencies (thick gray dashed lines in the figure), suggesting that the incident Poincare wave is exciting a leaky mode-one or mode-two CTW along these dispersion curve extensions.

On the shelf, significant mode-one energy is excited (panel 6 of Fig. 5 and Fig. 6). Within the Poincare continuum of the deep ocean mode-one wave in (σ,l) space, the mode-one amplitude on the shelf is comparable to the amplitude of the incident wave – that is, $\mathcal{O}(1)$ (Fig. 6). The response amplitude is slightly amplified in the mode-two Poincare continuum where mode-one is evanescent. When both mode-one and mode-two are evanescent, ridges of high mode-one response occur along ridges corresponding to leaky edge waves. These waves can propagate in both the cyclonic and anticyclonic directions – that is, ridges of large response occur for both positive and negative l.

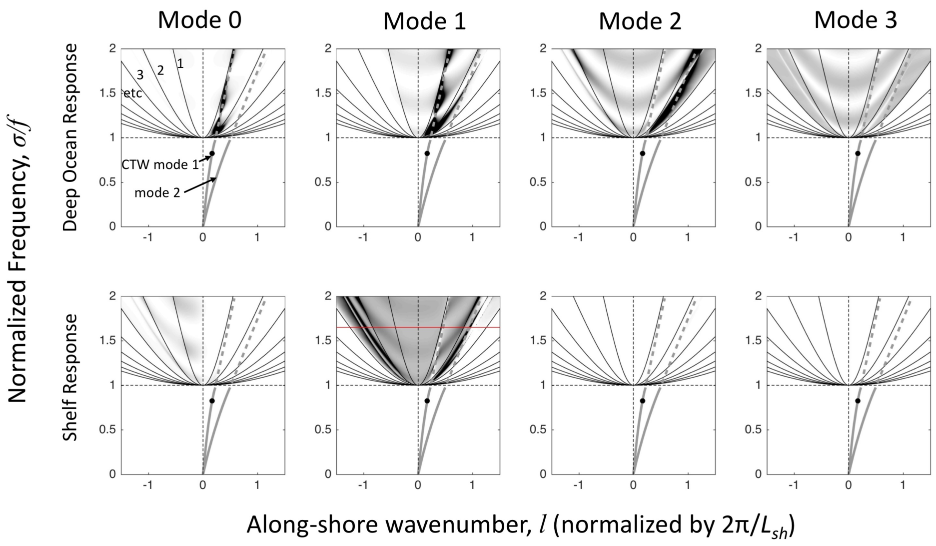

Figure 5: Same as Fig. 4, except the incident Poincare wave is mode-three. The horizontal red line indicates the frequency ($\sigma/f = 1.65$) at which the mode-one response on the shelf is plotted as a function of alongshore wavenumber in Fig. 6.

4 Summary

Excitation of leaky edge waves and leaky CTWs may be a means by which large amplitude, alongshore propagating internal tides can be excited at the coastal margin. The fact that the response of these nearly trapped waves is sensitive to alongshore wavenumber suggests that their excitation in real oceans will be temporally and spatial intermittent due to variations in incidence angles of deep ocean internal tides and spatial and temporal variations in stratification.

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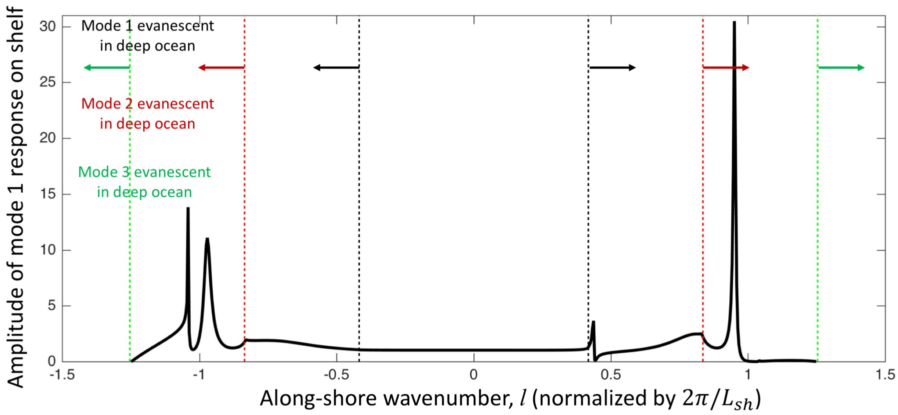

Figure 6: Amplitude of the mode-one response on the continental shelf to an incident mode-three Poincare wave as a function of alongshore wave number, l. The normalized frequency of the wave is $\sigma/f = 1.65$, and is indicated by the horizontal red line in Fig. 5. Colored horizontal lines indicate the bounds of the deep ocean Poincare continuum for modes one to three. Large leaky edge wave responses occur when modes one and two are evanescent in the deep ocean.

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