Mixing Efficiency in a Lock Exchange Experiment

Diane Micard¹, Yvan Dossmann², and Louis Gostiaux¹

LMFA, UMR CNRS 5509, Ecole Centrale de Lyon, France
Laboratoire de Physique, UMR 5672, Ecole Normale Supérieure de Lyon, France diane.micard@doctorant.ec-lyon.fr

Abstract

We report on measurements of mixing efficiency in a lock-exchange laboratory experiment. Two reservoirs, separated by a small waterproof gate, are filled with salt water with different densities, and the gate is suddenly opened. All the energy injected into mixing derives from available potential energy. Mixing efficiency is defined as the ratio of the increase of background potential energy to the decrease of available potential energy after opening the gate. The background potential energy is the potential energy computed from cross-sorting density profiles over the two reservoirs. We were thus able to observed that mixing efficiency increases with the hydraulic Reynolds number which confirms the tendency observed by Prastowo et al [4]. Moreover, our method allows to compute the mixing efficiency in the general case where the initial state is composed of two different stratifications. We are also able to discretize the mixing process into elementary mixing events.

Introduction

Abyssal mixing is one of remaining uncertainties for whom wants to understand and predict heat and salt fluxes in the deep ocean. The water mass exchange between the deepest parts of ocean basins is sometimes hydraulically controlled, for instance when the flow is constrained by the geometry of the connections (fracture zones, sills, channels) and is driven by the density difference between water masses. Temperature and salinity fluxes occurring in these regions highly depends on turbulent mixing and entrainment at the fluid interface. As for global oceanic models, an important parameter needed to close the energy budget of the system is mixing efficiency η . The commonly used value is 0.17, obtained from laboratory experiments in the early 80's [5].

Mixing efficiency measures the ratio the turbulent kinetic energy irreversibly transfered into potential energy. Direct measurement of total turbulent kinetic energy is very difficult in a real system, but it can be directly derived from the available potential energy in a lock-exchange experiment. This system is thus an elegant configuration to study mixing efficiency.

Our set-up consists in a rectangular tank separated in two reservoirs by a waterproof wall, in the middle of which a gate has been installed. Inspired by Winters *et al* we propose to cross-sort the density profiles of the two reservoirs in order to define the reference (or background) potential energy [9]. From this quantity, the amount of irreversible mixing is evaluated, as well as mixing efficiency. This follows the work done by Prastowo *et al* [4] and extends it to the general case of lock-exchange between two arbitrarily stratified reservoirs.

1 Set-up and description

Our experimental set-up consists in a glass tank 76 cm wide, 271 cm long and 48 cm deep. The tank is divided in two equal parts by a waterproof gate (see figure 1).

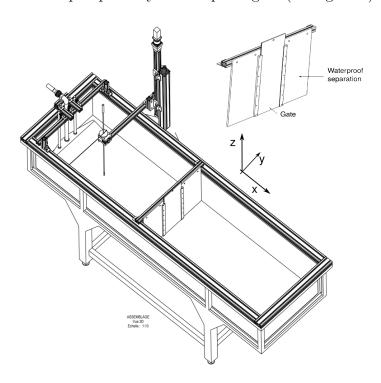


Figure 1: **Experimental set-up**. A glass tank of 271cm long and 76cm wide is divided in two reservoirs by a waterproof gate of 10cm wide. The fluid height in both reservoir is 25cm. The gate can be opened to connect the two reservoirs.

A MSCTI conductivity and temperature probe has been settled on a motorized linear unit which drives the sensor from the surface to the bottom of the tank with a constant speed ([2]). Data acquisition is performed with a Labjack U3 DAQ, and probe displacement is controlled via MODBUS protocol. A set of Python scripts achieves acquisition, control and processing, and density profiles can be obtained [7].

Before the experiment, the gate is closed. Each reservoir i is filled with a fluid of homogeneous density ρ_i . When opened, the gate draws a 10cm aperture in the middle of the tank, and forces a constricted exchange flow between the two reservoirs. Two opposite gravity currents are released at the free surface and at the rigid bottom. After a duration δt the gate is closed again, and density profiles are taken once the fluid is at rest. This operation is called a mixing event.

From density profiles $\{\rho_1(z), \rho_2(z)\}$, one can compute the associated potential energy E:

$$E = g \int_{V_1} \rho_1 z dV_1 + g \int_{V_2} \rho_2 z dV_2 \tag{1}$$

where g is the gravity acceleration and z is the vertical coordinate, orientated positive upward. Density profiles obtained before (resp. after) one mixing event represent the initial state (resp. final state) with potential energy E_I (resp. E_F).

In a first set of experiments, the gate is opened during $\delta t = 30$ s and closed. By repeating this operation, a collection of mixing events is obtained. As the initial state is no longer homogeneous, the computation of mixing efficiency requires a way of evaluating available potential energy that differs from Prastowo *et al.*

In a second set of experiments, the gate remains opened and density profiles are taken when the fluid is at rest. It corresponds to $\delta t = \infty$.

2 Background Potential Energy

In order to compute the average mixing efficiency associated to a mixing event, we first need to extract from the density profiles the amount of turbulent kinetic energy involved.

Kinetic energy directly derives from potential energy as the fluid is at rest at the beginning of the experiment. A relevant quantity to quantify this energy is the available potential energy (APE) defined as:

$$APE = g \int_{V_1} (\rho_1 - \rho^*) z dV_1 + g \int_{V_2} (\rho_2 - \rho^*) z dV_2$$
 (2)

$$= E - g \int_{V} \rho^* z dV \tag{3}$$

$$=E-E^{S} \tag{4}$$

where ρ^* is the unique density profile obtained by sorting adiabatically the density field, as defined by Salehipour and Peltier [6]. In our case, the sorted state is obtained by cross-sorting the fluid particles over the two reservoirs:

$$\{\rho_1(z), \rho_2(z)\} \xrightarrow{Cross} \rho^*(z)$$
 (5)

We use the potential energy E^S of the cross-sorted state as the background potential energy (see figure 2).

As the initial available potential energy is not completely used during a mixing event (the gate is closed before), the amount of energy which is available for mixing $E_a = APE_I - APE_F$ is the variation of APE between the initial and final state.

The effect of irreversible mixing is to increase the background potential energy : $E_{mix} = E_F^S - E_I^S$.

$$\eta = \frac{E_{mix}}{E_a} = \frac{E_F^S - E_I^S}{(E_I - E_I^S) - (E_F - E_F^S)} \tag{6}$$

This definition matches the one of Winters [8], but differs from the one of Prastowo et al.

3 Results

3.1 Results for opening the gate during a finite time

For the first set of experiments, the density difference between the two reservoirs is $\Delta \rho = 6.1 \text{ kg.m}^{-3}$ for a water height of 25.4cm. The gate is opened for duration $\delta t = 30 \text{s}$ and then closed. All measurements are performed after a time long enough (half an hour) to make sure that the fluid is at rest¹. Once the measurement are done, the gate is reopened

¹Diffusive effects are negligible given our timescale.

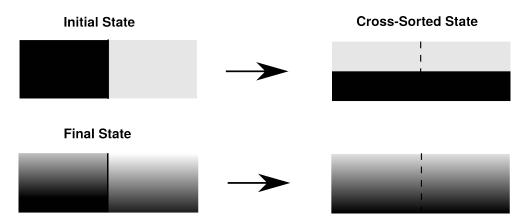


Figure 2: Illustration of the cross-sorting process for initial state and final state.

three times for $\delta t = 30$ s. Finally the gate is left opened at the end of the experiment. A succession of mixing events are thus obtained. Density profiles and cross-sorted density profiles are presented respectively in figure 3 a) and b).

In figure 3 a), the profiles taken in the initial state are the two straight lines. Then, as mixing occurs, profiles from the two reservoirs tend to collapse until the final state is reached (same stratification in both sides of the gate). This stable final state is also the less energetic state as regard to total potential energy. The associated cross-sorted profiles are plotted in figure 3 b). Evolution of those profiles highlight the turbulent diffusion.

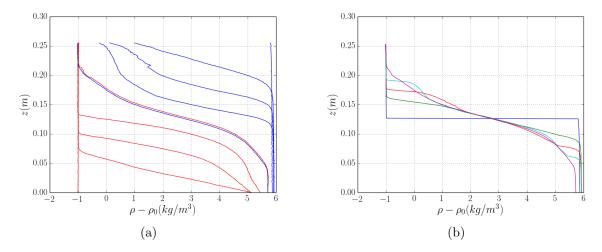


Figure 3: a) Evolution of density profiles through mixing events: red curves represents the density profiles taken in the fresh water reservoir and blue one in the salted reservoir. b) Cross-sorted profiles. Evolution along mixing events goes from blue, green, red, light blue to deep purple.

From those profiles one compute the potential energy and the background potential energy figure 4 a). At each mixing event, the potential energy is released and decreases due to reorganization of fluid particles (fluid mass exchange between the two reservoirs). The background potential energy increases with the number of mixing events.

Cumulative mixing efficiency, is computed for each intermediate state and for the final state (E_I and E_I^S are fixed as respectively the potential energy of the first profile and the first cross-sorted profile). Results are plotted in figure 4 b), and shows that cumulative mixing efficiency decreases through mixing events. The final mixing efficiency reached is

 $\eta = 0.145.$

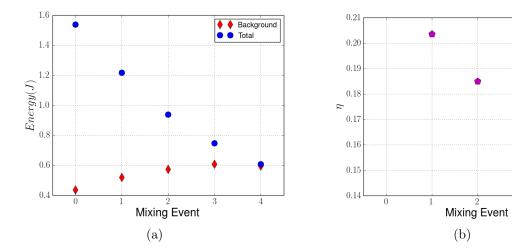


Figure 4: a) Potential Energies as a function of the number of mixing event (opening of the gate). Red diamonds represent the background potential energy, and blue dots the total potential energy. b) Cumulative Mixing efficiency as a function of the number of mixing events. For all the data points, the initial state is the same: the first state before opening the gate.

3.2 Results for opening the gate for an infinite amount of time

We will now compare mixing efficiency for different values of density difference, all other parameters being constant. In this set of experiment the gate is opened once and remains open for the entire experiment. After measurements, the tank is refilled with a different density difference $(\Delta \rho)$ and the experiment is repeated. Figure 5 a) is an example of density profiles obtained in this experiment. The straight lines are the profiles taken during the initial state where the density is constant in each reservoir. The two others witness reorganization of the fluid and mixing. The cross-sorted profiles over the two reservoirs computed for initial and final state are represented in figure 5 b).

The resulting mixing efficiency is given in figure 6 a) for different $\Delta \rho$. In this set of experiments, mixing efficiency is increasing with the initial density difference between the two reservoirs. One can also notice that the mixing efficiency seems to reach an asymptotic value of around 0.17.

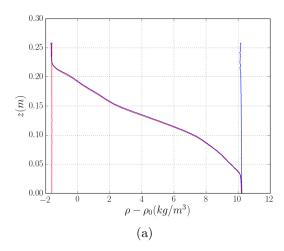
In order to relate $\Delta \rho$ to the fluid velocity, we use the analysis done by Hogg *et al*[3] for the dynamic of an hydraulic flow in the same kind of experiment, we define our Reynolds number as:

$$Re_{H} = \frac{UH}{\nu} = \frac{\frac{1}{2}\sqrt{Hg'}H}{\nu} \tag{7}$$

where where $g' = \frac{\Delta \rho}{\rho_0}$ is the reduced gravity, H is the water height (we assume each fluid layer to be of the same thickness $\frac{H}{2}$), and ν this the cinematic viscosity.

Figure 6 b) shows mixing efficiency as a function as this Reynolds number.

Figure 6 b) shows mixing efficiency as a function as this Reynolds number. We observe here the same tendency as seen by Prastowo $et\ al\ [4]$: mixing efficiency increases with the Reynolds number until it reaches its constant value. Our efficiency is higher than the one computed by Prastowo $et\ al$ as our definitions differ. The mixing efficiency estimated



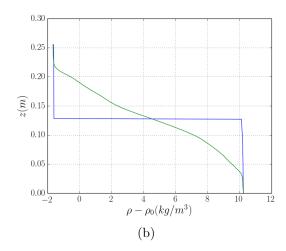
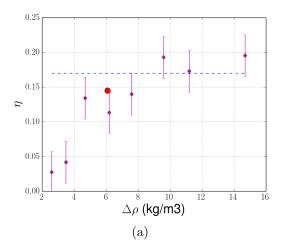


Figure 5: **Initial and final density profiles.** a) Profiles taken in both side of the separation. The straight curves are the density profiles in the initial state. The red curves are taken in the reservoir which initially contained the lightest fluid and the blue ones are taken in the other reservoir. Here $\rho_0 = 1000 \text{kg/m}^3$. b) **Cross-sorted profiles** associated with the density profiles. The blue one represents the sorted initial state and the green one represents the final sorted state. From those profiles background potential energy is computed.



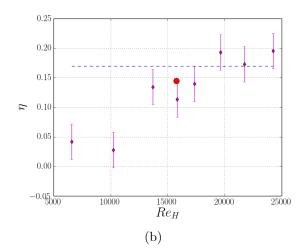


Figure 6: **Mixing Efficiency.** The red dot is the cumulative potential energy computed from the final state of the previous experiment where mixing was discretize in events of duration δt . The purple diamonds represent the mixing efficiency computed after opening the gate without closing it back. The dash blue line represent the commonly used value for η . Uncertainties are estimated in the worst case scenario where a 0.5 millimeter of heavy fluid is not detected at the free surface. **a)**: Mixing efficiency as a function of the initial density difference between the reservoirs. Reynolds number is estimated under the assumption of a two layer flow hydraulically driven. **b)**: Mixing efficiency as a function of the Reynolds number.

from the final stage of the stepwise experiment described in section 3.1 is marked as a red dot on figure 6 a) and b). It stands in the confidence interval measured in the single opening experiment.

4 Conclusion

We showed that in a lock-exchange experiment, when the gate is opened for an "infinite" amount of time, mixing efficiency increases with the initial density difference between the two fluids and with the associated hydraulic Reynolds number until a plateau is reached. In the case of repeated opening and closing down of the gate, the same value of η is attained, but the early mixing events show higher values of mixing efficiency. This rises the question of the definition of mixing efficiency in unstationary regimes. In the oceanic context, the intensity of exchange flows can for instance be modulated by the tide [1]. We plan to use this set-up to study mixing efficiency in other configurations: intrusions, convecting flows, and mixing in continuously stratified environments.

Acknowledgements

This research is funded by the Agence Nationale de la Recherche, ANR 13-JS09-0004-01 (STRATIMIX). We are grateful to Bruno Poirel and Alexandre Azouzi for the technical support, and to Kraig Winters for his comments and discussions during his stay in 2016 as invited professor from the Ecole Centrale de Lyon.

References

- [1] Hans Haren, Louis Gostiaux, Eugene Morozov, and Roman Tarakanov. Extremely long kelvin-helmholtz billow trains in the romanche fracture zone. *Geophysical Research Letters*, 41(23):8445–8451, 2014.
- [2] Michael Jay Head. The use of miniature four-electrode conductivity probes for high resolution measurement of turbulent density or temperature variations in salt-stratified water flows. University of California San Diego, 1983.
- [3] Andrew M Hogg, Gregory N Ivey, and Kraig B Winters. Hydraulics and mixing in controlled exchange flows. *Journal of Geophysical Research: Oceans*, 106(C1):959–972, 2001.
- [4] Tjipto Prastowo, Ross W. Griffiths, Graham O. Hugues, and Andrew McC. Hogg. Mixing efficiency in controlled exchange flows. *J. Fluid Mech.*, 600:235–244, 2008.
- [5] James Rohr and Charles Van Atta. Mixing efficiency in stably stratified growing turbulence. *Journal of Geophysical Research: Oceans*, 92(C5):5481–5488, 1987.
- [6] Hesam Salehipour and W. R. Peltier. Diapycnal diffusivity, turbulent prandtl number and mixing efficiency in boussinesq stratified turbulence. *Journal of Fluid Mechanics*, 775:464–500, 7 2015.
- [7] John M Stewart. Python for scientists. Cambridge University Press, 2014.
- [8] Kraig B Winters and Roy Barkan. Available potential energy density for boussinesq fluid flow. *Journal of Fluid Mechanics*, 714:476–488, 2013.
- [9] Kraig B Winters, Peter N Lombard, James J Riley, and Eric A D'Asaro. Available potential energy and mixing in density-stratified fluids. *Journal of Fluid Mechanics*, 289:115–128, 1995.