Porous media plumes: transient filling box solutions

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Abstract

We examine the transient flow of a negatively buoyant, laminar plume in an emptying filling box filled with porous medium. As the system reaches steady state, the box is partitioned into two uniform layers of different densities. However, the approach towards steady state is characterized by a contaminated layer that is continuously stratified. Whereas the presence of a continuous stratification in the contaminated layer for finite time poses nontrivial analytical challenges, we show that it is possible to derive bounds on the range of possible solutions, The validity of this approach is confirmed by drawing a comparison with the turbulent free plume case. A separate component of our study considers time-variable forcing where the laminar plume source strength changes with time. We focus particular attention on cases where the source buoyancy or volume fluxes are either abruptly altered or else turned on and off with non-dimensional half-period $\Delta\tau$.

1 Introduction

Hundreds of studies have been conducted to study high Reynolds number turbulent convection from a discrete source since the seminal work of Morton et al. (1956) who described a turbulent plume in an infinite stratified or unstratified ambient by the means of equations for conservation of mass, momentum and buoyancy, supplemented by a parameterization that describes turbulent entrainment. Baines and Turner (1969) adopted the Morton et al. (1956) equations in order to study a free plume in a confined box. The "filling box" model has since then been used to study numerous environmental and industrial problems, for instance, volcanic eruptions (Woods, 2010), filling of a room with smoke during fires (Kaye and Hunt, 2007), and building ventilation (Nabi and Flynn, 2013).

Linden et al. (1990) first used the term "emptying filling box" in their study of turbulent plumes in a ventilated box. An emptying filling box is connected to an infinite external ambient via upper and lower openings (or fissures). Due to differences in the internal and external hydrostatic pressure distributions, a ventilation flow naturally arises whereby, in the case of a negatively buoyant plume, discharged plume fluid exits the box through the lower fissure(s) and is, in turn, replaced by external ambient fluid, which flows through the upper fissure(s). Emptying filling box models have been broadly adapted to describe various ventilation problems, for example, natural ventilation of one-zone buildings (Linden, 1999), transient ventilation dynamics (Kaye and Hunt, 2004), and ventilation between two chambers (Flynn and Caulfield, 2006).

While recent theoretical and experimental studies have focused on filling box (Sahu and Flynn, 2015) and emptying filling box (Roes et al., 2014) models for porous media plumes, the study of plumes and ventilated filling boxes containing porous media remains in its infancy. This problem is of particular interest in the fields of industrial and environmental fluid mechanics, e.g. in the context of (i) dissolution of non-aqueous phase liquids or geologically-sequestered CO_2 into potable groundwater, (ii) leakage of contaminants from waste piles or composting facilities, and (iii) enhanced oil recovery technologies such as cyclic steam stimulation and steam-assisted gravity drainage – see Roes et al. (2014) and the references therein.

In this study, we investigate time-dependent flows in an emptying filling box filled with porous media. Here, the box is comprised of two layers, an upper uncontaminated layer containing ambient fluid and a lower layer containing discharged plume fluid. The latter layer is continuously stratified, but approaches a uniform density in the long time limit. Although the density profile in the contaminated layer for the case of a free turbulent plume can be obtained by adapting the numerical technique of Germeles (1975), a model for the density profile of the contaminated layer for the case of porous media plumes has not yet been developed. In this paper, we show that even in the absence of such a model, bounds on the range of possible solutions can be obtained. Our approach is substantiated by drawing comparisons with the analogue turbulent free plume flow.

2 Theory

We begin investigating the motion of a negatively buoyant plume in ventilated boxes filled with and devoid of porous medium by applying the conservation of volume and buoyancy. Consistent with our previous discussion, the transient dynamics due to turbulent free plumes are well-established, and the associated equations and some of the results presented below are shown only for the sake of comparison with the porous media plumes.

Applying conservation of volume and buoyancy to the lower contaminated layer leads to

$$\frac{dV}{dt} = Q_p - Q_{out}, \tag{1}$$
$$\frac{dVg'}{dt} = F_0 - Q_{out}g'(x = H, t), \tag{2}$$

where V is the volume of the contaminated layer. In the case of a box devoid of porous medium, $V = Sh$ where S and h denote the cross-sectional area of the box (independent of height) and the depth of the lower contaminated layer, respectively. If the box is filled with porous medium, $V = \phi Sh$ in which ϕ is the porosity of the box. Q_p is the volume flux of the plume at the interface, Q_{out} is the volume flow rate out of the box, F_0 is the source buoyancy flux, and $g'(x = H, t)$ is the reduced gravity of the contaminated layer at the bottom of the box. Expressions for Q_p and Q_{out} are as follows (Kaye and Hunt, 2004; Roes et al., 2014):

$$\text{free turbulent plumes: } Q_p = C_j F_0^{1/3}(x + x_0)^j, \qquad Q_{out} = A^\star\sqrt{I}, \tag{3}$$

$$\text{point-source porous media plume: } Q_p = 8\pi D\phi(x + x_0), \qquad Q_{out} = \frac{Ak_f I}{\nu b}, \tag{4}$$

$$\text{line-source porous media plume: } Q_p = \left(\frac{36D\phi F_0 k(x + x_0)\Lambda^2}{\nu}\right)^{1/3}, Q_{out} = \frac{Ak_f I}{\nu b}. \tag{5}$$

In (3) j depends on the plume geometry, i.e. $j = 5/3$ and $j = 1$ correspond to point- and line-source plumes, respectively. Furthermore, the constant C_j is based on the entrainment coefficient and source geometry, x is the vertical coordinate measured relative to the source, x_0 is the virtual origin correction for non-ideal plumes having a finite source volume flux, $A^\star$ is the weighted area of the lower and upper openings, $I = \int_{H-h}^{H+b} g'dx$

is the integrated buoyancy in the lower layer, and b is the depth of lower fissure(s). In (4) and (5), D is the solute dispersion coefficient, A and k_F denote, respectively, the cross-sectional area and permeability of the lower fissure(s), ν is the kinematic viscosity, k denotes the permeability of the box, and Λ is the depth of the line-source into the page.

We define the average reduced gravity of the contaminated layer as $g' = \frac{I}{h+b}$ and introduce the following dimensionless parameters

$$\xi = \frac{h}{H} \quad \text{and} \quad \left| \begin{array}{ll} \delta = g' \frac{C_j H^j}{F_0^{2/3}} & \text{(free turbulent plumes)} \\ \delta = g' \frac{8\pi D \phi H}{F_0} & \text{(point-source porous media plume)} \\ \delta = g' \left(\frac{36 D \phi k H \Lambda^2}{F_0^2 \nu} \right)^{1/3} & \text{(line-source porous media plume)} \end{array} \right. \tag{6}$$

For an emptying filling box problem, two time-scales are typically considered, namely the draining (T_d) and filling (T_f) time-scales, which are defined as

$$\begin{array}{lll} \text{free turbulent plumes:} & T_d = \dfrac{S C_j^{1/3} H^{(j+1)/2}}{A^\star F_0^{1/3}}, & T_f = \dfrac{S}{C_j F_0^{1/3} H^{(j-1)}}, \\ \text{point-source porous media plume:} & T_d = \dfrac{S}{A} \dfrac{b\nu}{k_f} \dfrac{8\pi D \phi^2 H}{F_0}, & T_f = \dfrac{S}{8\pi D}, \\ \text{line-source porous media plume:} & T_d = \dfrac{S}{A} \left(\dfrac{36 D \phi^4 k H \Lambda^2}{F_0^2 \nu} \right)^{1/3}, & T_f = \left(\dfrac{S^3 \phi^2 H^2 \nu}{36 D F_0 k \Lambda^2} \right)^{1/3}. \end{array}$$

Now, we can introduce a dimensionless time $t = \sqrt{T_d T_f}\tau$ to rewrite (1) and (2) for a free turbulent plume as

$$\frac{d\xi}{d\tau} = \sqrt{\mu} \left(1 - \xi + \frac{x_0}{H} \right)^j - \frac{1}{\sqrt{\mu}} \sqrt{\delta \left(\xi + \frac{b}{H} \right)}, \tag{7}$$

$$\frac{d\delta}{d\tau} = \sqrt{\mu} \frac{1 - \delta \left(1 - \xi + \frac{b}{H} \right)^j}{\xi} + \frac{1}{\sqrt{\mu}} \sqrt{\frac{\delta}{\xi}} \left[\delta - \delta(x = H, \tau) \right], \tag{8}$$

in which the parameter μ is the ratio of the draining time-scale to the filling time-scale. Meanwhile, (1) and (2) for a porous media plume can be rewritten as

$$\frac{d\xi}{d\tau} = \sqrt{\mu} \left(1 - \xi + \frac{x_0}{H} \right)^k - \frac{1}{\sqrt{\mu}} \delta \left(\xi + \frac{b}{H} \right), \tag{9}$$

$$\frac{d\delta}{d\tau} = \sqrt{\mu} \frac{1 - \delta \left(1 - \xi + \frac{b}{H} \right)^k}{\xi} + \frac{1}{\sqrt{\mu}} \frac{\delta \left[\delta - \delta(x = H, \tau) \right]}{\xi}, \tag{10}$$

where $k = 1$ and $k = 1/3$ correspond to point- and line-source porous media plumes, respectively.

In order to derive bounds on the range of solutions for the height and reduced gravity of the contaminated layer, we consider two limiting cases. In the former, the lower layer is, in spite of the possible presence of porous media, assumed to be well-mixed so that the latter terms from the right-hand sides of (8) and (10) disappear. In the latter case (hereafter referred to as the approximated stratified model), the density of the lower layer at the bottom of the box is assumed to equal the plume density at the level of the interface whether or not this interface is stationary. In order to validate our approach, we apply the Germeles (1975) numerical technique to the free turbulent problem to confirm that Germeles (1975) "exact" filling box solution is bounded by the above limiting cases, at least for not small τ. A comparison of model output is presented in figure 1, which shows the exact filling box, well mixed, and approximated stratified solutions.

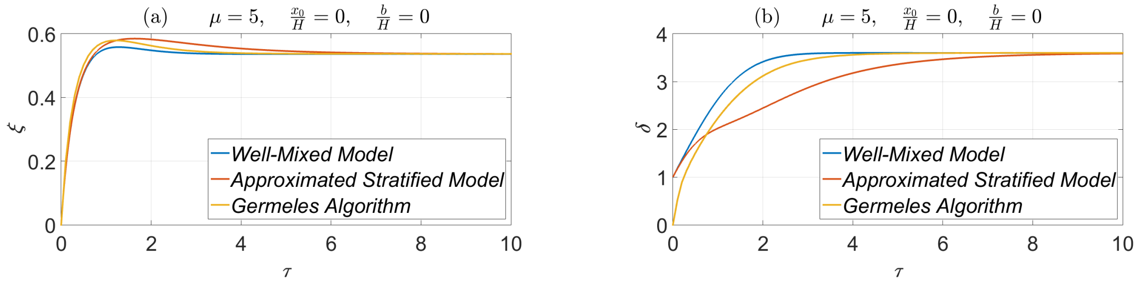

Figure 1: [color] The evolution of the contaminated layer non-dimensional height (panel a) and reduced gravity (panel b) calculated from Germeles (1975), the well-mixed model given by (7) and (8) with uniform δ, and the approximated stratified model given by (7) and (8) with $\delta(x = H, \tau)$ specified by the plume density at the interface height. Here we assume an ideal point-source free turbulent plume with $\frac{x_0}{H} = 0$; moreover, $\frac{b}{H} = 0$.

Some time after "turning on" the source, figure 1 indicates that the Germeles (1975) solution becomes bracketed by those of the well-mixed and approximated stratified models. The initial discrepancy is due to the fact that in our numerical solutions of (7) and (8) the non-dimensional reduced gravity of the contaminated layer can never start from a value less than unity, i.e. in our limiting cases the initial conditions are given by

$$\text{free turbulent plumes: } \xi = 0, \quad \text{and} \quad \delta = \frac{1}{\left(1 + \frac{x_0}{H}\right)^j} \quad \text{at} \quad \tau = 0, \tag{11}$$

whereas the initial conditions for the Germeles (1975) algorithm are $\xi = 0$ and $\delta = 0$ at $\tau = 0$. Hence, it takes some finite, but generally small, time for the system to evolve to the point that the Germeles (1975) solution becomes bracketed.

It should be re-emphasized that figure 1 presents results for a relatively classical problem, namely the emptying filling box problem in a box devoid of porous medium, rather the more novel problem of a laminar, porous media plume in a ventilated box where, of course, no analogue of Germeles (1975) is available. However, whereas the physics describing plume development are fundamentally different between free turbulent plumes and their porous media analogues, many dynamical features are similar as are the forms of (7-8) and (9-10).So although an exact solution is unavailable in the porous media case, we expect that such a solution would be is bounded by the appropriate limiting cases of (9) and (10) and that these bounds would become increasingly sharp in the limit of large μ.

3 Initial Transient

We expect that from figure 1 that we can reasonably bracket solutions using equations like (9) and (10). We now proceed to solve these equations in the relevant limits so that we may infer the true behavior of porous media plumes. For the case where the initial interior density is the ambient density, we take $\tau = 0$ as the moment when the plume first touches the bottom of the box. Therefore, similar to (11), the initial conditions read

$$\text{porous media plumes: } \xi = 0, \quad \text{and} \quad \delta = \frac{1}{\left(1 + \frac{x_0}{H}\right)^k} \quad \text{at} \quad \tau = 0. \tag{12}$$

Figure 2 shows the evolution of ξ and δ for ideal line-source free turbulent and porous media plumes with $\frac{b}{H} = 0.1$. When μ is large, the filling time-scale is smaller than the draining time-scale; hence, the contaminated layer thickens at a faster rate compared to the draining capacity of the box. Consequently, the contaminated layer height first reaches then exceeds its steady state value. Over time, ξ relaxes to this steady state value as the filling and draining of the contaminated layer become balanced.

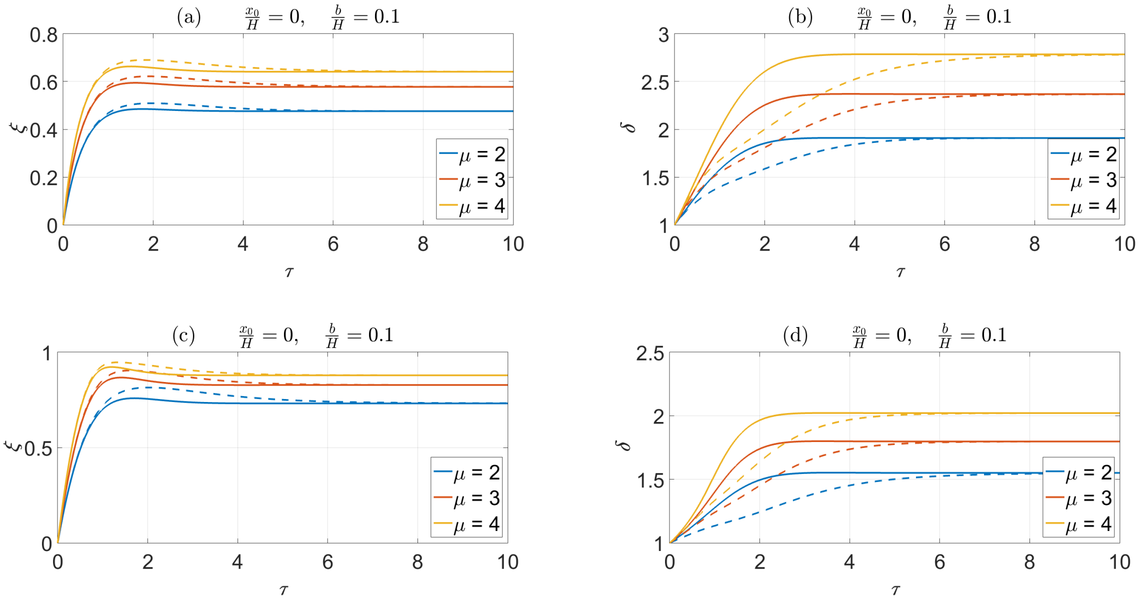

Figure 2: [color] Ideal line-source free turbulent and porous media plumes with $\mu = \{2, 3, 4\}$ and $\frac{b}{H} = 0.1$. (a, b) free turbulent plumes and (c, d) porous media plumes. The solid and dashed curves correspond to the well-mixed and approximated stratified models, respectively.

Figure 2 indicates that the overshoot is, in magnitude, more prevalent in the porous media plume case which suggests a greater mismatch between the time-scales over which the contaminated layer density and depth evolve in time. However, as μ increases, the difference between the relative height of overshoot for the free turbulent vs. porous media plume decreases. Indeed, and for very large values of μ (not shown), the overshoot relative to final interface height is actually larger in case of free turbulent plumes.

4 Transient source turned on and off

Consider a situation where the buoyancy source is sequentially turned on and off with a half-period of $\Delta\tau$. When the buoyancy source is off, contaminated fluid continues to be discharged from the bottom of the box. As a consequence, the lower layer depth decreases monotonically in time until the source is again turned on. If the contaminated layer drains completely, i.e. if $\Delta\tau$ is sufficiently large, the box then consists of uniform ambient fluid just as with the initial condition. For $\Delta\tau$ less than this critical value, the contaminated layer remains of finite thickness; its reduced gravity equals the value at the instant the source was turned off.

Figure 3 shows the evolution of ξ and δ for ideal line-source free turbulent and porous media plumes with $\frac{b}{H} = 0.1$, $\mu = 3$, and $\Delta\tau = 0.5$. If $\Delta\tau$ is smaller than the non-dimensional time required for the system to approach steady state, ξ_{ss} and δ_{ss} will never be realized regardless of the number of cycles. Rather, after a few cycles, the height of the lower layer oscillates between two extrema whose values depend on the magnitude of $\Delta\tau$ in addition to $\frac{x_0}{H}$, $\frac{b}{H}$, and μ.

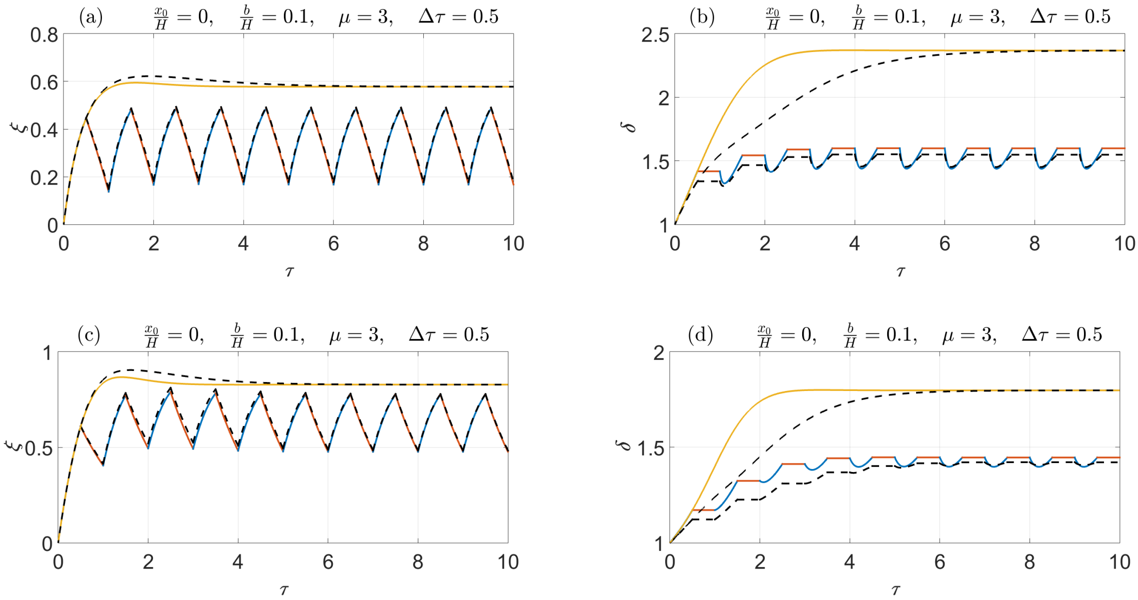

Figure 3: [color] Free turbulent and porous media plume responses to ideal cyclic line-sources with $\frac{b}{H}=0.1$, $\mu=3$ and $\Delta\tau=0.5$ (a, b) free turbulent plume and (c, d) porous media plume. The yellow curves pertain to the well-mixed case where the source remains on indefinitely. The blue and red curves correspond to the period when the source is respectively on and off. Also, the dashed curves show model predictions corresponding to the approximated stratified model.

5 Transient source turned up or down

Building on the material of the previous section, we now consider a limited increase or decrease in the source buoyancy flux or volume flux. Thus the source buoyancy flux might change from an initial value of F_0 to $F_0+\Delta F$ or the source volume flux might change from an initial value of Q_0 to $Q_0+\Delta Q$. Note that ΔF and ΔQ can be either positive or negative quantities but, by assumption, $F_0+\Delta F>0$ and $Q_0+\Delta Q>0$. Note also that a change in Q is not supposed to result in a change in F, i.e. ΔQ and ΔF are assumed to be uncorrelated. In the figures to follow, changes in the source conditions are represented by two new variables, namely $\chi_1=\frac{F_0+\Delta F}{F_0}$ and $\chi_2=\frac{Q_0+\Delta Q}{Q_0}$.

Figure 4 illustrates the effect of changing the source buoyancy flux of ideal line-source free turbulent and porous media plumes with $\mu=3$ and $\frac{b}{H}=0.1$. In the case of a free turbulent plume, an increase (decrease) in the source buoyancy flux results in a transient increase (decrease) in the depth of the contaminated layer. However, as the outflow from the contaminated layer adjusts, so too does ξ; ultimately the interface height returns to its previous value. Qualitatively different behavior is seen for the case of porous media plumes. Increasing (decreasing) the source buoyancy flux increases (decreases) the plume volume flux. Thus, the lower layer height initially rises (falls). Correspondingly, the outflow volume flux increases (decreases) and ultimately balances the plume volume flux at the ambient interface. Unlike the free turbulent plume case where the steady-state height of the contaminated layer is independent of the source buoyancy flux, the new value for ξ_{ss} is now different from the value observed before the change of F_0.

Figure 5 shows the effect of changing the source volume flux of line-source free turbulent

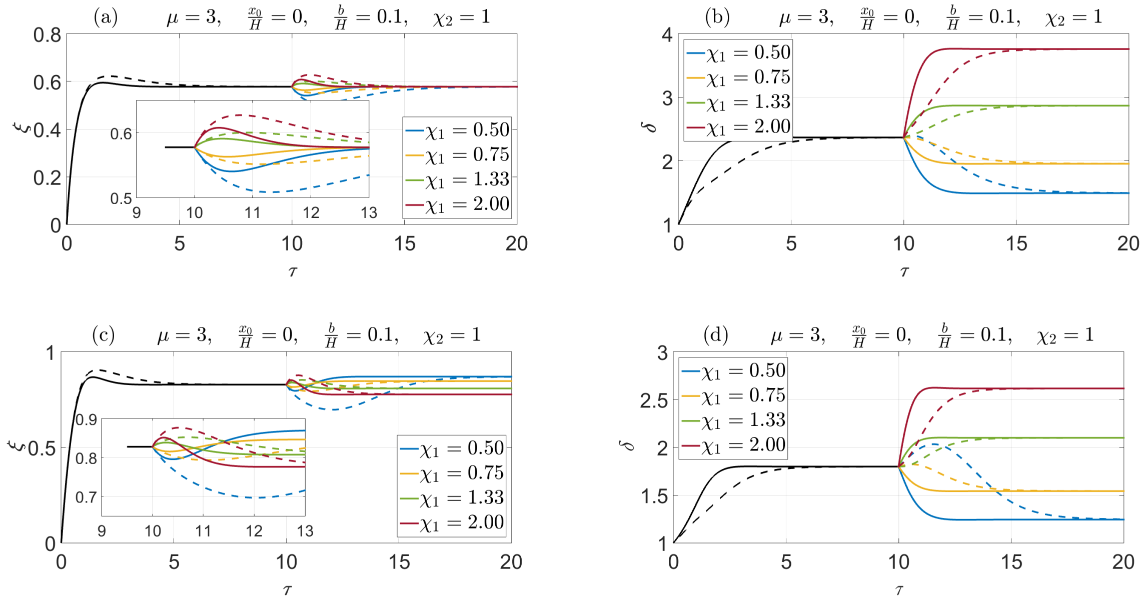

Figure 4: [color] Effects of changing the source buoyancy flux of ideal line-source free turbulent and porous media plumes with $\mu = 3$ and $\frac{b}{H} = 0.1$. (a, b) free turbulent plume and (c, d) porous media plume. The black curves show the initial evolution of the system toward steady state values before changing the source buoyancy flux at $\tau = 10$. Also, the dashed curves show model predictions corresponding to the approximated stratified model.

and porous media plumes with $\mu = 2$, $\frac{x_0}{H} = 0.1$, and $\frac{b}{H} = 0.1$. In both cases, the lower layer reduced gravity decreases (increases) for an increase (decrease) in the source volume flux. As a result, the outflow volume flux becomes smaller (larger) and the lower layer height rises (falls) to compensate. It should be emphasized that, for both cases of free turbulent and laminar porous media plumes, the steady state height of the contaminated layer is dependent on the source volume flux. Therefore changing the source volume flux results in a permanent change in the steady state values of ξ (and, of course, δ).

6 Conclusions

The principal contribution of this study is to outline a mathematical methodology by which the emptying filling box behavior of a control volume filled with porous media may be approximated. Specific reference is made to two limiting cases, namely one where the contaminated layer is assumed to be uniform and another where the density of the discharged fluid is supposed to equal the plume density at the level of the interface. The predictions associated with these models typically represent upper and lower bounds; the bounds become increasingly sharp in the limit of large μ where μ is defined by the ratio of the draining time-scale to the filling time-scale. Moreover, as is shown in various figures and also by (7-8) and (9-10), which , through judicious non-dimensionalization, have a comparable form, the dynamical evolution of these ostensibly very distinct emptying filling box flows are confirmed to be similar.

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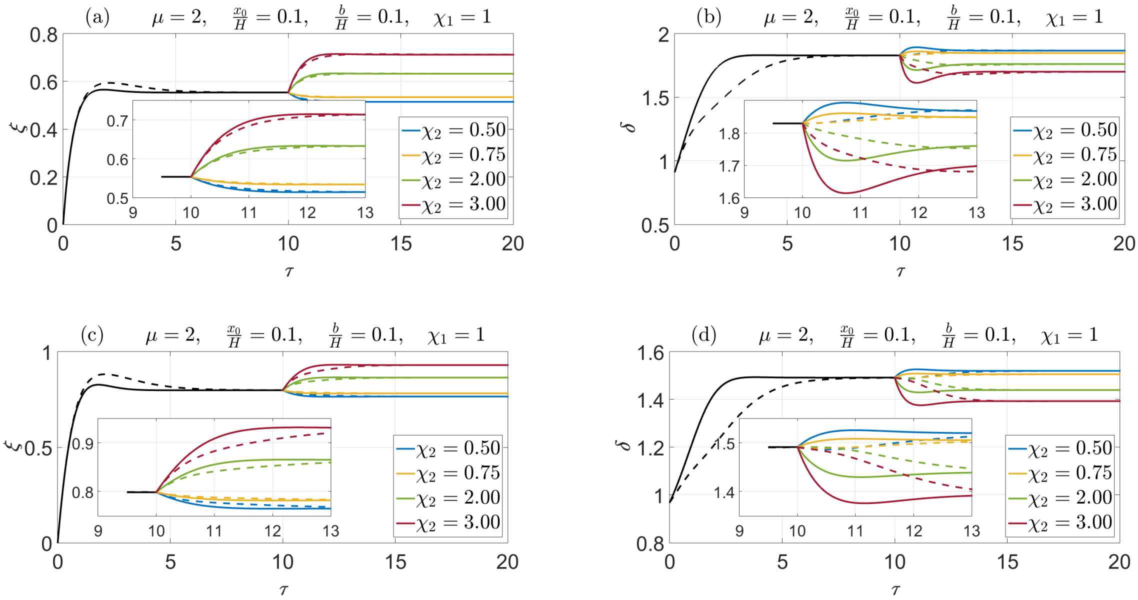

Figure 5: [color] Effects of changing the source volume flux of line-source free turbulent and porous media plumes with $\mu = 2$, $\frac{b}{H} = 0.1$n and initial $\frac{x_0}{H} = 0.1$. (a, b) free turbulent plume and (c, d) porous media plume. The black curves show the initial evolution of the system toward steady state values before changing the source buoyancy flux at $\tau = 10$. Also, the dashed curves show model predictions corresponding to the approximated stratified model.

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