

Internal Hydraulic Jumps with Upstream Shear and Topography

Kelly Ogden^a and Karl Helfrich^b

^aMassachusetts Institute of Technology and Woods Hole Oceanographic Institution,

^bWoods Hole Oceanographic Institution

kaogden@mit.edu

Abstract

The structure and mixing of internal hydraulic jumps with large upstream shear in approximately two-layered flows is investigated using theory and numerical simulations. Two-layered theories and numerical simulations for internal hydraulic jumps with upstream shear were investigated by Ogden and Helfrich (2016). As the upstream shear increases, physically consistent solutions for these theories cease to exist, and when the shear is large enough, all theoretical solutions are lost. However, as shown by Wilkinson and Wood (1971) for one-and-a-half-layer flows, jumps still form. Modifying the two-layer theory to include entrainment allows solutions at higher shear. This is consistent with numerical simulations, which produce high shear jumps that exhibit significantly more entrainment than low shear jumps. Furthermore, the downstream structure of the flow, accounted for with shape functions, which account for continuous velocity profiles, has an important effect on the jump properties. Jumps with a slow upper (inactive) layer exhibit a velocity minimum downstream of the jump, resulting in a sub-critical downstream state, while flows with the same upstream vertical shear and a larger barotropic velocity remain super-critical downstream of the jump. The super- to sub-critical jumps are structurally similar to those described by Wood and Simpson (1984), with an entrainment region and a roller region, as seen in figure 1a. However, super- to super-critical jumps, shown in figure 1b, only exhibit an entrainment region without the subsequent roller region. The entrainment can be predicted through a modification of the approach of Holland et al. (2002) by allowing the energy dissipated in the jump to depend on the square to the upstream shear. The resulting theory can be matched reasonably well with the numerical simulations. However, the results are very sensitive to how the downstream vertical profiles of velocity and density are incorporated into the layered model, highlighting the limitations of the two-layer approximation when the shear is large.

Additionally, the structures of lower shear jumps with topography are investigated numerically. In these flows, the upstream shear is generated by the topography, as occurs in tidally driven channel flows such as Knight Inlet or Hood Canal. Various jump structures can occur, depending on the flow rate, density profile, and topography.

References

- Holland, D., Rosales, R., Stefanica, D., and Tabak, E. (2002). Internal hydraulic jumps and mixing in two-layer flows. *Journal of Fluid Mechanics*, 470:63–83.
- Ogden, K. A. and Helfrich, K. R. (2016). Internal hydraulic jumps in two-layer flows with upstream shear. *Journal of Fluid Mechanics*, 789:64–92.
- Wilkinson, D. and Wood, I. (1971). A rapidly varying flow phenomenon a in two-layer flow. *Journal of Fluids Mechanics*, 47(2):241–256.

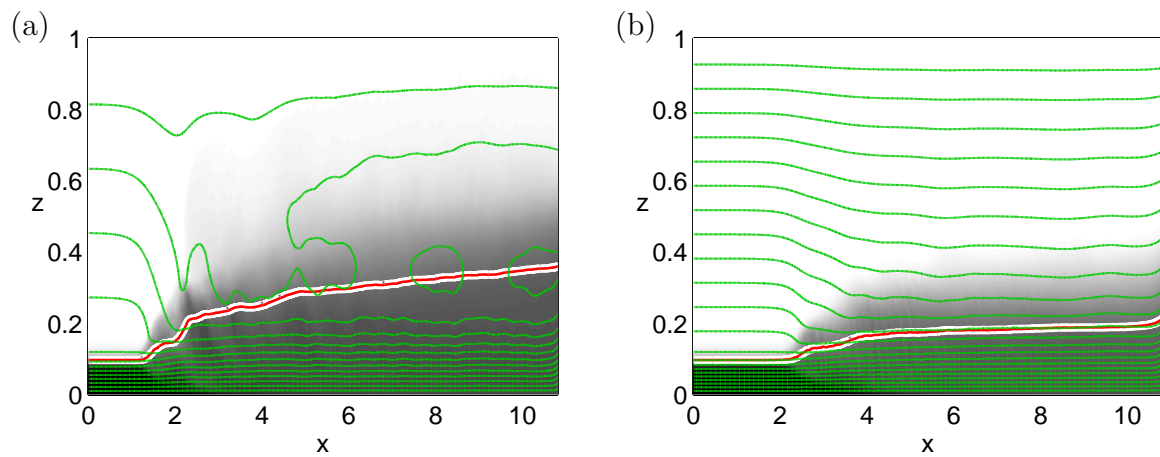


Figure 1: Density field, interface, and streamlines of (a) super- to sub- and (b) super- to super-critical transitions. The super- to sub-critical transition shows closed streamlines, which represent the roller region. Both show streamlines crossing the interface, which indicates the entrainment region.

Wood, I. and Simpson, J. (1984). Jumps in layered miscible fluids. *Journal of Fluids Mechanics*, 140:329–342.