Ageostrophic Instability and Mixing in a Dense Overflow

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Abstract

We will discuss a number of aspects of ageostrophic instability in two-layer systems, with specific applications to deep channel flows such as that of the Denmark Strait. The work touches on the extension to two layers of Ripa's Theorem, including a geometric interpretation, linear stability analysis and finite amplitude simulations using a numerical model. Among other things, we attempt to explain the 2–4 day oscillations detected in the approach flow to the Denmark Strait sill.

1. Introduction

The Griffiths-Killworth-Stern instability (Griffiths, et. al. 1982) acts in a 1.5-layer layer system in which the thickness of the active layer vanishes at the edges. The instability, which was explored in the context of an eddy or gyre, can be attributed to resonance between different edge modes arising on the frontal boundaries (Paldor, 1983). Pratt et al. (2008) hereafter PHL, and Simeonov and Stern (2008) examined a similar linear instability in a parallel flow confined to a parabolic channel, where the topography potentially has a stabilizing influence. The growing modes are particularly intriguing in that they have zero wave energy and therefore do not formally extract energy from the background flow, a possibility first point out by Hayashi and Young (1987). Although such instabilities alter the mean flow, they cannot easily be classified as barotropic or baroclinic.

The parabolic-channel configuration is often used to explore the rotating hydraulics of deep overflows (Borenäs and Lundberg, 1986 and 1988 and Pratt and Whitehead, 2006). PHL also considered the finite amplitude growth of the unstable edge waves, using a numerical model to track growth and equilibration. Some success in predicting the equilibrated states was enabled by reference to Ripa's Theorem (Ripa, 1983), a sufficient condition for ageostrophic stability that involves conditions on both the potential vorticity gradient and the layer Froude number, thus combining elements of classical quasigeographic instability theory with hydraulics. To make a connection with deep overflows PHL also considered channels that begin in a deep, broad basin and narrow to a sill section where the active layer spills (Figure 1).

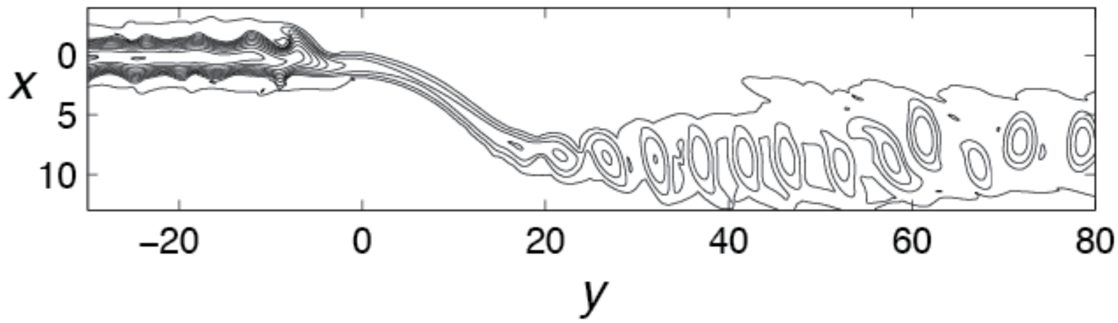

Figure 1: From numerical simulation by Pratt et al. (2008) showing finite amplitude state achieved as the result of unstable growth of instabilities in a single-layer, ageostrophic flow in a deep channel with a parabolic cross section. The contours represent the elevation of the interface. The mean flow is from left to right and the channel contains a sill at y=0. To the right of the sill, the dense layer spills and becomes banked on the right side (facing downstream) of the channel.

2. Results

Among the climate-relevant deep ocean overflows, that of the Denmark Strait is particularly complex because of its extreme variability on short time scales, even well upstream of the sill (Harden, et al. 2016). It is further complicated by a strong barotropic component, introduced by the influence of the East Greenland Current. This factor makes it difficult to apply the 1.5-layer results mentioned above. For one thing, it is nearly impossible to configure such a model so that it looks anything like the mean flow in the Denmark Strait, as documented by Harden et al. (2016). For this reason we have extended our analysis to include two-active layers, still in a channel with a parabolic cross-section. This extension is nontrivial since the central part of the channel is covered by both layers, while the outside flange regions are covered only by the upper layer. In addition, the upper layer may now contain a potential vorticity gradient, a factor that enlarges the zoological range of possible unstable modes. Ripa's theorem can be extended, but the required bounds are more difficult to derive.

Our presentation will include a discussion of the linear stability problem, growth rates, conditions for the dominance of zero-energy waves, and the derivation, implications and geometric interpretation of Ripa's Theorem in two layers. We will also show numerical simulations (an example of which appears in Figure 2) in both strait channels and channels with sills and width contractions. We will discuss the possible use of Ripa's Theorem to predict equilibrated states. We will also compare findings with recent observations of time-variability (Harden et al., 2016) in the Denmark Strait.

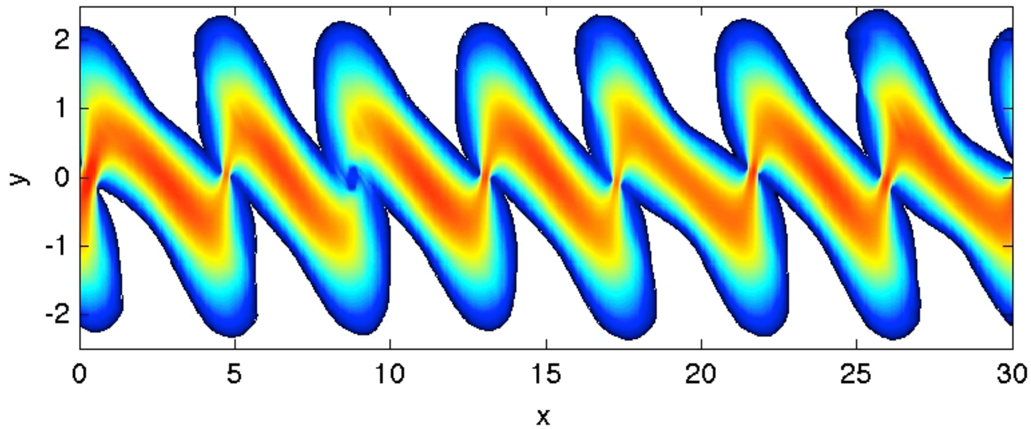

Figure 2: Plan view showing lower layer thickness during the development of an instability in a parabolic channel. The channel is aligned in the x-direction.

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