A Study of a Model for the Generation of Internal Waves by a Moving Body and Its Turbulent Wake

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Abstract

We test a previously proposed model for the generation of internal waves, produced both by the flow over the body and the disturbances produced by its turbulent wake, within the framework of a ray model of the internal waves. This model approximates the stratified flow over the sphere as a combination of a distribution of sources representing the steady internal wave flow, relative to the sphere, and a vertically oscillating sphere, representing the waves generated by the turbulent eddies in the wake. The results of our ray simulations are compared with the few existing laboratory experiments of a towed sphere that oscillates at a fixed frequency in the vertical in a uniformly stratified tank and with some new numerical simulations. We find that the ray calculations using the model source term compare well with the numerical simulations of the laboratory experiments, and these results lead to some insights into how the model can be improved to simulate a fully turbulent wake.

1 Introduction

A body moving through a stratified fluid generates internal waves due to the displacement of fluid by the body as well as due to the disturbance of the fluid by the turbulence in its wake. The purpose of the present study is to test the model proposed by Voisin (1995) and Dupont and Voisin (1996) for the generation of internal waves by the turbulent wake of a sphere in a stratified environment. As observed in experiments Gilreath and Brandt (1985), Hopfinger et al. (1991), Bonneton et al. (1993), Chomaz et al. (1993), the generation mechanism for the internal waves depends mainly on the Froude number $Fr = U/(Na)$, in which U is the speed of the sphere, N is the buoyancy frequency at the depth of the sphere and a is the radius of the sphere. For $0.8 \geq Fr \leq 1.5$, the internal waves are produced primarily by the flow over the sphere and have zero frequency (sometimes called lee waves) in a frame of reference moving with the sphere. For $Fr > 4.5$ and with the Reynolds number, Re, large enough for the wake to be turbulent, the waves are produced primarily by the coherent structures in the wake and have nonzero frequency in the reference frame moving with the sphere (sometimes called random waves). For intermediate Froude numbers, $1.5 < Fr < 4.5$, the wavefield is a combination of the two types of waves. Voisin (1995) proposed to model the generation of the random waves by a vertical oscillation of the sphere at the frequency of the near wake spiral instability, as described by Chomaz et al. (1993).

2 Theory

2.1 Formulation

We consider a stratified fluid in the Boussinesq approximation. As an approximation for the internal waves generated by the body and its turbulent wake, the problem of a source distribution moving horizontally through the fluid while oscillating vertically at frequency σ is solved. The form of the source distribution that describes the motion of a sphere and the value of the frequency of its oscillation that represents the turbulent generated waves will be discussed later. The coordinate system is $\mathbf{r} = (x, y, z)$, with z positive upwards, and is fixed to the mean position of the oscillating source. In this reference frame, the background flow is $\mathbf{U} = (U, 0, 0)$, in which U is the horizontal speed of the sphere. The background buoyancy frequency is $N(z)$.

The internal waves for this system have wavenumber $\mathbf{k} = (k, l, m)$ and *intrinsic* frequency

$$\hat{\omega} = \sigma - kU. \tag{1}$$

The internal-wave dispersion relationship is

$$m = \pm\,(k^2 + l^2)^{1/2}(N^2/\hat{\omega}^2 - 1)^{1/2}. \tag{2}$$

The Fourier transform of the vertical displacement is represented by $\tilde{\zeta}(k, l, z)$. As shown in Broutman and Rottman (2004) this function is the solution of a forced Taylor-Goldstein equation that depends on depth-dependent vertical wavenumber and the source distribution representing the motion of the sphere. Once this function has been obtained, the spatial solution for the wavefield is obtained by inverse Fourier transform. Note that since this is a linear problem, the solutions corresponding to different values of σ can be superimposed. In particular, $\sigma = 0$ represents the waves generated by the motion of the body alone, whereas the waves generated by the turbulence may be represented by a distribution of nonzero values of σ. The function $\tilde{\zeta}(k, l, z)$ cannot be determined analytically except for a few simple buoyancy frequency profiles. We use ray theory to approximate this function for more general cases of buoyancy profiles.

2.2 The Source Distribution

The present work considers internal waves generated by the vertically oscillating sphere of radius a in a uniform flow of speed U. In the limit of large Froude number, $Fr = U/Na$, as shown by Gorodtsov and Teodorovich (1982), Voisin (1995) and Dupont and Voisin (1996), the flow associated with this motion can be represented by the following source distribution function

$$q(\mathbf{r}, t) = -\frac{3}{2}\left[Ue^{-i\sigma_0 t}\frac{x}{a} + h\sigma e^{-i\sigma_1 t}\frac{z}{a}\right]\delta(r - a), \tag{3}$$

where δ is the Dirac delta function. The frequency of the translating portion of the source term is $\sigma_0 = 0$. Following the guidelines of Dupont and Voisin (1996), the dominant internal waves generated by the eddies in the turbulent wake are simulated by choosing a value of 0.2 for the Strouhal number $St = a\sigma_1/\pi U$ and a source frequency of $\sigma_1 =$

$N\pi StFr$. The vertical displacement amplitude of the oscillation of the sphere is h, and its vertical velocity is $h\sigma_1 e^{-i\sigma_1 t}$.The amplitude of the vertical oscillation of the source must be a function of Fr, and more weakly a function of Re, but at this stage its value is chosen to match the amplitude of the observed waves. The three-dimensional Fourier transform of q is

$$Q(\mathbf{k},t) = \frac{3}{4} i\pi^{-2} a^3 \left[U e^{-i\sigma_0 t} k + h\sigma e^{-i\sigma_1 t} m \right] \frac{j_1(Ka)}{Ka}, \tag{4}$$

where $K = |\mathbf{k}|$, and $j_1(z) = (\sin z)/z^2 - (\cos z)/z$ is the spherical Bessel function of order unity.

The non-oscillating portion of the source distribution is an accurate representation of non-stratified flow over a sphere, but has been shown (and will also be shown here) that it also is a very good representation of the flow over sphere even for moderate Froude numbers. The oscillating portion of the source distribution has been used to model internal wave generation by turbulent eddies in a uniformly stratified flow in the wake of an obstacle by Dupont and Voisin (1996) based on some experimental work by Gilreath and Brandt (1985); Bonneton et al. (1993); Chomaz et al. (1993).

3 Experiments

The only directly applicable set of experiments we are aware of are those reported in Dupont and Voisin (1996). These experiments were of a towed sphere in a uniformly stratified fluid with the sphere oscillating vertically at a fixed frequency. The experiments were conducted in a tank 0.5 m wide, 0.5 m deep and 4 m long. A sphere of radius 0.012 m was towed horizontally through uniformly stratified fluid while it was oscillated vertically at a number of frequencies. The Reynolds number was low, about 100, the vertical amplitudes of the oscillations smaller than the sphere diameter, and the vertical speed of the sphere was small compared with its horizontal speed.

4 Numerical Simulations

Another avenue for getting the necessary measurements is to do high resolution direct numerical simulations, DNS, or implicit large eddy simulations, iLES, of stratified flow over a sphere. Here we use the numerical code NFA (Numerical Flow Analysis), Rottman et al. (2010). NFA solves the incompressible Navier-Stokes equations on a Cartesian-grid using immersed boundary methods to treat the free surface and solid boundary. The solid boundary is treated using a cut-cell and finite-volume method to solve the pressure field and a ghost-cell immersed boundary method to implement the velocity boundary conditions. In the ghost-cell immersed boundary method we enforce a no-slip boundary condition by setting ghost-cells within the body such that the average between an image point (in the fluid) and the ghost point (in the solid body) along the normal direction is equal to the body velocity. The image point in the fluid is determined by interpolating from neighboring values, using a second order Taylor series expansion, yielding a second order technique. In addition, for the species concentration we use a no-flux boundary condition. The advective terms are treated using a slope-limited, third-order QUICK Leonard (1997) scheme, which functions as an implicit subgrid-scale model.

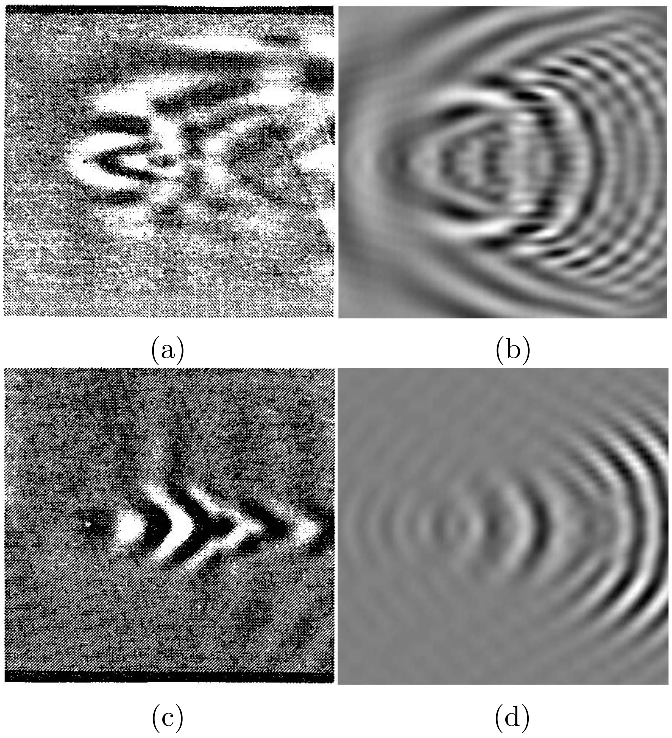

Figure 1: Comparison of experimental results (left) and ray calculations (right) for the vertical displacement field for the uniform stratification cases with $\sigma_1/N = 0.8$ (a,b) and $\sigma_1/N = 1.2$ (c,d). Figures a and c reproduced with permission from Dupont, P. "Ondes Internes Engendrees par une Source Oscillanted en Mouvement", PhD Dissertation, Université Joseph Fourier, 1995 (in French).

Body geometries are included in the code using a triangulated surface mesh, or through a specification of the distance function, and the algorithms are capable of handling complex surface features. The code is written in FORTRAN 2003 and parrallelized using the Message Passing Interface (MPI-2). A detailed description of the numerical algorithm and its implementation on distributed memory high performance computing (HPC) platforms can be found in Dommermuth et al. (2006), Dommermuth et al. (2007), Dommermuth et al. (2010), O'Shea et al. (2008) and Brucker et al. (2010).

5 Results

5.1 Theory and Experiments

Figure 1 show comparisons of the ray calculations with the experimental measurements of the vertical displacement for the two cases with $\sigma_1 = 0.8$ and 1.2. The experimental results are images from a figure in Dupont (1995) that are difficult to compare quantitatively with the current simulations. However, there is some qualitative agreement between the two images. The simulations appear to capture most of the major features of the flow, but of course the simulations have much higher resolution and therefore more fine scale features than can be seen in the experimental images.These comparisons are encouraging and give us confidence that the source term for an oscillating sphere are accurate.

5.2 Numerical simulation

Here we perform a three-dimensional numerical simulation of a sphere oscillating in linearly stratified environment. In the simulation we use the following dimensionless (scaled by the diameter) domain size: $L_x = 65$, $L_y = 50$ and $L_z = 50$. The grid is clustered around the body and into the wake with the smallest grid spacing being 0.035 dimensionless units and the grid is stretched away from the body. The background flow is slowly ramped-up with a period of 0.5 dimensionless time units to reduce the impact of initial transients. The smallest time step used is $\Delta t = 5 \times 10^{-3}$ dimensionless units and the simulation is run long enough to get a statistically steady solution in the region near the sphere.

In Figure 2 we are showing the species concentration at a point in time for two different oscillation frequencies. In the simulations and experiments a moderate Reynolds number is used so the wake behind the sphere is laminar, extending a finite distance downstream. As the sphere oscillates up and down the wake also oscillates, leading to an interesting wave-like structure that develops downstream. Both simulations compare very well with the experimental results near the sphere. For the moderate frequency case the agreement is not as good far downstream but this may be due to some finite domain effects in the experiment. In the Figure we are also comparing the effect of two different Reynolds numbers Re of 100 and 1000; the former represents the experiments and the later is more appropriate for the Ray theory. In the large Reynolds number case the laminar wake downstream extends further and has more frequency components. Near the sphere the internal gravity waves are similar to the lower Reynolds number case but here we get lee waves, which have a similar structure to the ray theory results.

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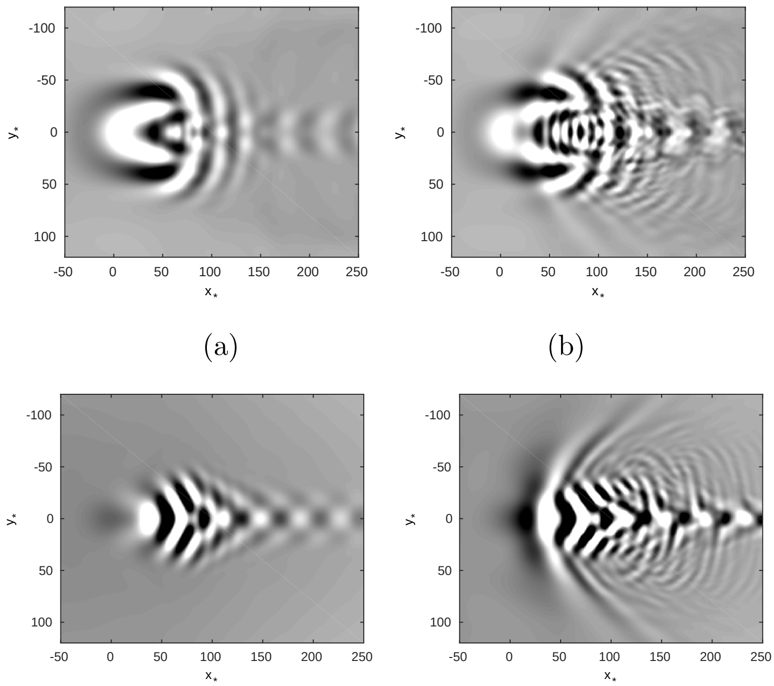

(a) (b) (c) (d)

Figure 2: Numerical simulations, (left) Reynolds number $Re = 100$ and (right) $Re = 1000$, for the vertical displacement field for the cases with $\sigma_1/N = 0.8$ (a,b) and $\sigma_1/N = 1.2$ (c,d). The nondimensional horizontal coordinates, x^* and y^*, are the dimensional coordinates, x and y, scaled by the factor U/N. Note that the domain for these figures is larger than that in Figure 1.

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