Shear instabilities in a tilting tube

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Abstract

Shear instabilities were investigated in an exchange flow in a tilting channel. The channel is connected to a freshwater reservoir at one end and a salt water reservoir at the other end. When the channel is initially horizontal, a steady two-layer flow occurs that supports symmetric Holmboe instabilities. When the channel is tilted, as in closed tilting channel experiments, shear increases and Kelvin-Helmholtz instabilities are generated. The KH instabilities break down the sharp interface between the two-layers resulting in a broad region of mixing. Subsequent levelling of the tube results in a three-layer exchange flow. The upper and lower layers are nearly pure fresh and salt water respectively. Separating these two layers is a relatively thin layer of homogeneous mixed fluid. Within this layer a third mode of instability forms that resembles a Taylor-Caulfield instability. The wave characteristics and appearance of these instabilities are compared with the results of linear stability analysis and direct numerical simulations.

1 Introduction

Stratified shear flows in the laboratory can exhibit a variety of wave-like features. In experiments with relatively strong shear, spiralling Kelvin Helmholtz (KH) instabilities occur. These instabilities are stationary and cause intense mixing and broadening of the region of shear. In experiments with relatively weak shear Holmboe instabilities can form. These instabilities form as cusps that propagate. Tilting tube experiments, exchange flow experiments and splitter plate experiments can exhibit either KH or Holmboe instabilities or a complex mix of both KH and Holmboe instabilities. The experiments described here generate initially symmetric Holmboe instabilities, then KH instabilities, and then, during a period of three layer exchange, an instability that resembles the instability predicted by Taylor (1931) and observed by Caulfield et al. (1995). This third mode of instability, referred to here as the Taylor-Caulfield instability, relies on the interaction of two density interfaces (see figure 2).

2 Experimental setup

The shear instabilities are studied in an exchange flow facility with an open ended tilting channel (2.3 m long, 0.23 m wide by 0.2 m deep, figure 1). One end of the channel is open to a fresh water reservoir and the other to a salt water reservoir (1.2 ppt NaCl). Each reservoir holds approximately 6m^3 of water. The channel is tilted by lifting the end of the channel open to the salt water reservoir.

Laser induced fluorescence (LIF) was used to visualize the density interface by illuminating Rhodamine 6G dye mixed into the freshwater with a continuous 2 watt solid state(532 nm) laser. The laser beam was passed through a Powell lens to generate an upward radiating light sheet along the centre of the tilting channel. Images were collected using a digital camera; a sample image is shown in figure 1b. The interfaces were identified by locating the maximum vertical gradient in light intensity.

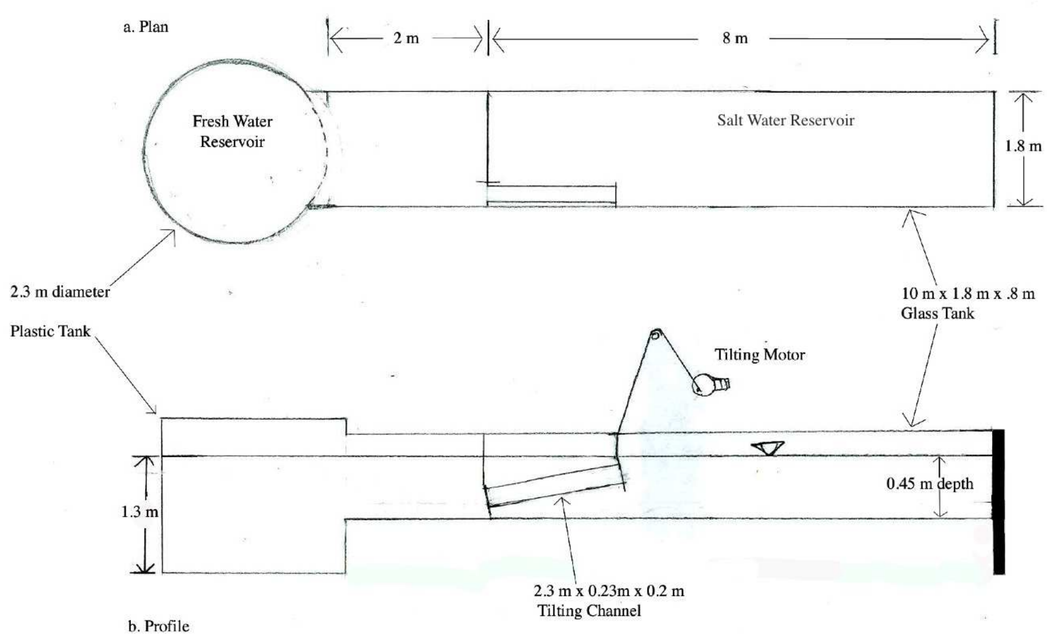

Figure 1: (a) Plan and (b) profile of experimental setup.

A Dantec particle image velocimetry (PIV) system was used to measure the velocity of pliolite VT-L particles (Goodyear Chemical Co.). The particles were pulverized and sieved to diameters less than 0.24 mm. Pairs of images ($\Delta t = 0.04$s) were collected every 3 seconds. A 3-step adaptive correlation algorithm was used to calculate velocities. The Dantec system was also used to determine density by quantitative measurement of dye fluorescence.

The first experiment (LIF) was performed over the entire viewable region of the tank (-1m$< x <$ 1m). This experiment provided a description of the evolution of the instabilities and their wave characteristics. In the second experiment both PIV was performed

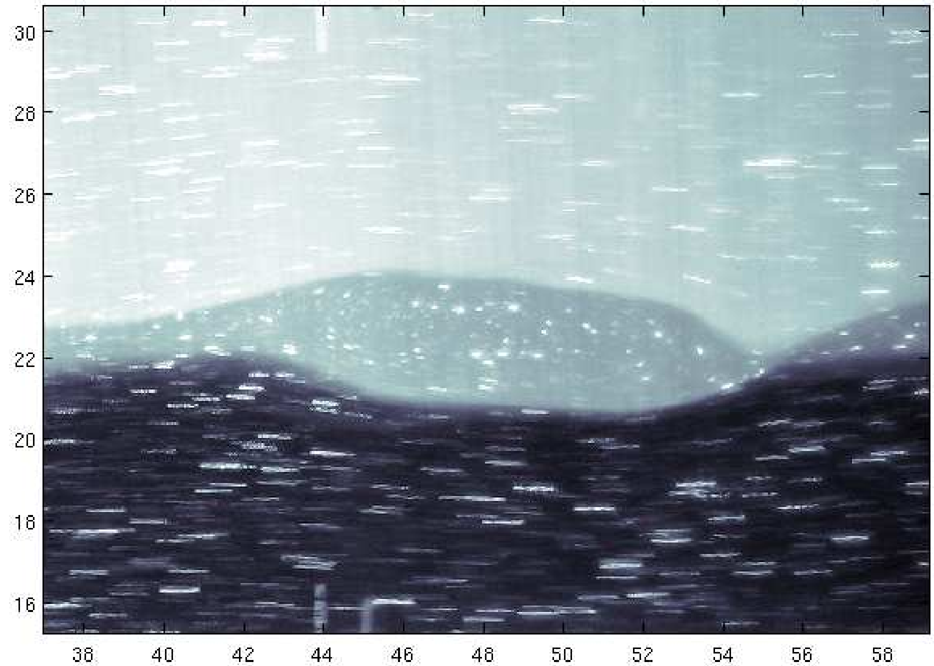

Figure 2: Particle streak image taken at approximately t = 800 seconds. The dyed upper layer is fresh and moving to the right and the clear (dark) lower layer is saline and moving to the left. The mixed fluid (gray) is nearly stationary. The maximum thickness of the middle layer in this image is approximately 3 cm. To generate particle streaks the shutter speed of the camera was set to 0.5 seconds.

simultaneously to get the evolution of the velocity profile for the stability analysis. Particle streak images (figure 2) were collected in a third experiment to examine the structure of individual instabilities. Finally, fluid samples were collected with a syringe and analyzed in a densitometer to verify $\Delta\rho$ between the layers.

3 Evolution of the mean flow

The experiments are initialised by removing a gate separating the two reservoirs. When the gate is removed gravity currents propagate through the channel and cause intense mixing. After approximately 8 minutes, the flow becomes steady and consists of two homogeneous layers. The two layers are separated by a sharp density interface centred within a relatively broad shear layer. The sheared interface supports two oppositely propagating symmetric Holmboe instabilities.

The channel is then slowly tilted over 30 seconds from horizontal to a slope of 1/50 (rise/run). The maximum velocity in each layer increases from 1.9 cm/s to 2.7 cm/s. As in the tilting tube experiments of Thorpe (1971), the additional shear associated with tilting generates KH instabilities that grow and break down the sharp density interface. The resulting mixed fluid forms a layer of intermediate density at mid-depth (time 350 to 550 s) that is approximately 3 cm in thickness.

The channel is then tilted back to the horizontal position (time 550 to 600 s). The flow decelerates, mixing diminishes, and strain is able to thin the layer of intermediate density. During this stage there are rightward propagating instabilities on the upper density interface and leftward propagating instabilities on the lower density interface that both resemble the Holmboe instability. There is also a stationary or nearly-stationary mode that resembles the instability (Figure 1) initially predicted by Taylor (1931) and investigated further by Caulfield (1995). This mode persists until strain completely advects the mixed fluid of the intermediate layer out of the channel.

4 Linear stability analysis and Direct Numerical simulations

A linear stability analysis was conducted on the stratified shear flow observed in the channel during the period after the tube was brought back to a horizontal position. The linear stability analysis (figure 3a) suggests the flow is susceptible to both Taylor-Caulfield and Holmboe instabilities. The stability properties shown in figure 3 were calculated assuming a tanh-form for the velocity profile, and a double tanh form for the density field. The growth rate curve shows two peaks at wavelengths of approximately 10 and 18 cm. The shorter wavelength instabilities are of the Holmboe type and have a phase speed relative to the mean flow, whereas the TC instabilities are stationary and have longer wavelengths.

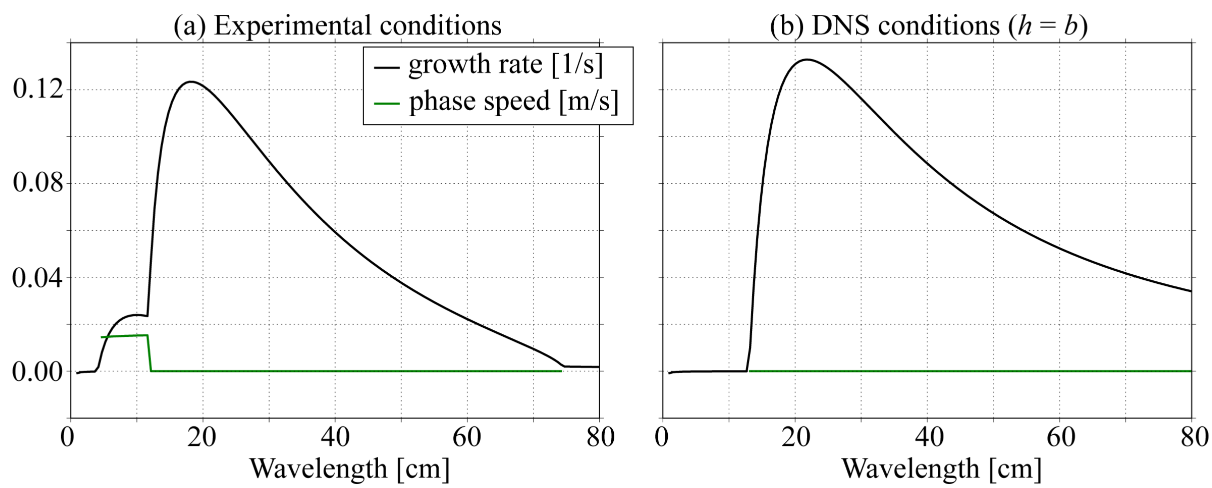

Figure 3: Stability properties of the experimental conditions (a) during the time of TC instabilities, and for the conditions simulated by the DNS (b).

In order to isolate the Holmboe and the TC instabilities from each other in a series of direct numerical simulations (DNS) we have also carried out a stability analysis with the thickness of the intermediate layer set equal to that of the shear layer thickness. In this case, the stability properties show only the presence of a TC mode (figure 3b). This allows for the examination of the nonlinear form of the TC mode through DNS. The two-dimensional DNS were carried out with the code described in Winters et al. (2004) with initial profiles as in the linear stability analysis. To avoid excessive diffusion of the density field during the growth of the instabilities the diffusion of the background profile of density was removed. The results of the simulations are shown in the density fields plotted in figure 4. An instability at the wavelength of maximum growth predicted by the linear stability analysis develops from an initially random noise perturbation at $t = 0$ s (figure 4a). They appear to have the same qualitative form as those observed in the experiments, despite the presence of Holmboe instabilities simultaneously in the experiments. At later times in the DNS the TC "billows" breakdown into finer scale motions, which is expected to feed a turbulent transition at high Reynold's numbers, and in three dimensions. In addition, both pairing of TC billows and the development of secondary Holmboe modes on the density interfaces appears after the initial TC instabilities become nonlinear and interact with one another.

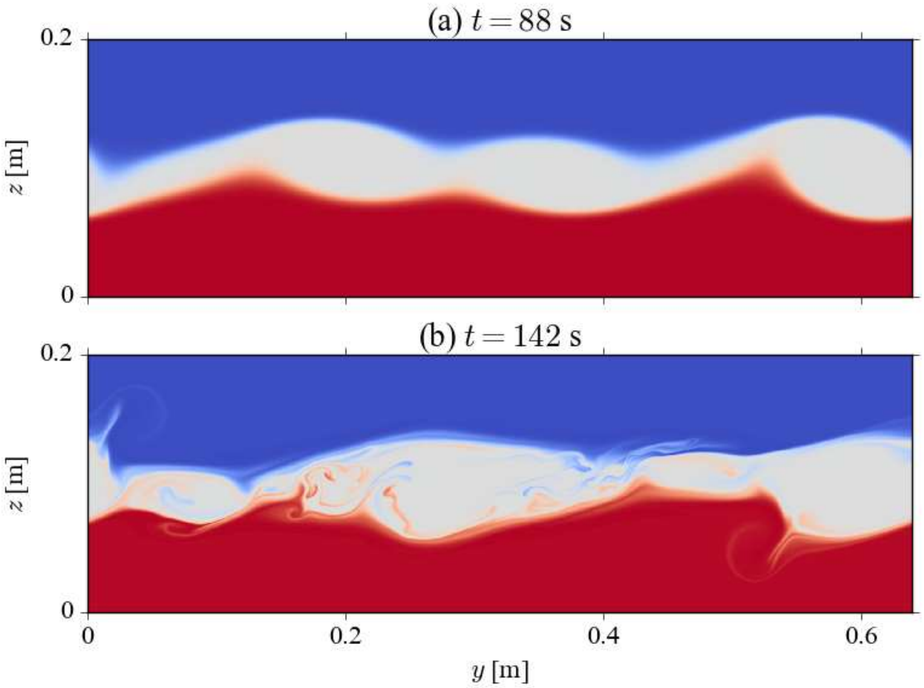

Figure 4: Density field at two different times of the DNS showing the development and breakdown of TC instabilities.

5 Summary

Exchange flow in a tilting channel connecting two reservoirs supports a variety of instabilities. When the channel is horizontal and the flow is able to attain a steady state Holmboe instabilities form. When the channel is tilted, shear increases and the Holmboe instabilities are overwhelmed by KH instabilities. The KH instabilities cause intense mixing and broadening of the shear layer. When the channel is tilted back to horizontal, the Holmboe instabilities return. The presence of a third intermediate layer allows

Taylor-Caulfield instabilities to form. The results of linear stibility analysis and DNS are consistent with the observed TC instabilities.

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