1DV model of wind-generated internal seiches

Rob Uittenbogaard[a] and Maryam Rezvani[b]

[a]Deltares, Delft, The Netherlands. Rob.uittenbogaard@deltares.nl

[b]PhD alumni, Texas A&M, College Station, USA

Abstract

We propose the addition of a simple equation for the baroclinic pressure gradient to existing 1DV models. This extension allows for the simulation of internal-seiche modes for given longitudinal wave length as demonstrated by a linear stratified case as well as by comparing to observations in the 47m deep thermally-stratified Lake Hallwil. This extension opens another avenue for exploring the energy transfer from wind to internal-seiche modes and subsequently to turbulence and mixing in lakes, notably in benthic boundary layers.

1 Introduction

In the benthic boundary layer of deep and thermally-stratified lakes wind-generated basin-scale internal waves and seiches are the major source of turbulence production (Lorke *et al.*, 2002). These waves contribute also to internal shearing and turbulence production and affect the evolution of cyanobacteria (Cuypers *et al.*, 2011).

At present, 3D numerical models are capable of simulating basin-scale internal waves yet at a notable computational effort for simulating periods of months and with numerical damping by the grid resolution (Hodges *et al.*, 2000) still limited for accurately solving the turbulence model equations (Dijkstra *et al.*, 2016).

Contrary, 1DV models are popular fast performance tools for mixing, thermocline formation and heat exchange in lakes (Perroud *et al.*, 2009), (Read *et al.*, 2014) as part of climate and meteorology models, and for modelling vertical mobility of cyanobacteria (Aparicio Medrano *et al.*, 2013). In these 1DV models the turbulence production by internal seiches is either not simulated and neglected or it is included as some empirical fraction of the power of the wind transferred to the lake (Goudsmit *et al.*, 2002). Hence, we desire to extend 1DV models for simulating the most prominent features of internal seiches. This paper proposes a methodology to be adapted per 1DV solution procedure.

2 The 1DV equations extended for internal seiches

2.1 Determining energy transfer from wind stress to basin-scale internal waves

The principle of including internal seiches in 1DV models is presented for a single horizontal direction but has been extended to the two horizontal directions of orthogonal internal-seiche modes, including the Coriolis force.

Neglecting advection, the 1DV equation for horizontal velocity component u in x-direction, time t, vertical coordinate z, water level ζ , Reynolds stress τ_{xz} , gravitational acceleration g, density ρ, linearized state equation and vertical mass flux F_ρ reads:

$$\frac{\partial u}{\partial t} = -g\frac{\partial \zeta}{\partial x} - P_\rho + \frac{\partial \tau_{xz}}{\partial z} \quad ; \quad P_{\rho,u} = \frac{g}{\rho_0}\frac{\partial}{\partial x}\int_z^{\bar{\zeta}} \rho(z')\,dz' \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \bar{\rho} u}{\partial x} + \frac{\partial \bar{\rho} w}{\partial z} = \frac{\partial F_\rho}{\partial z} \quad ; \quad w = \int_z^{\bar{\zeta}} \frac{\partial u(z')}{\partial x}\,dz' \tag{2}$$

The overbar on ρ and ζ in (1) and (2) defines variables independent of or averaged over time and x, thereby neglecting barotropic-baroclinic coupling and non-linear interactions in mass conservation. From (1) and (2) follows the hydrostatic version of the Taylor-Goldstein equation with inhomogeneous part the derivatives of the shear stress τ_{xz} as well as vertical mass flux F_ρ:

$$\frac{\partial^4 w}{\partial t^2 \partial z^2} - \left(\frac{g}{\rho_0}\frac{d\bar{\rho}}{dz}\right)\frac{\partial^2 w}{\partial x^2} = -\frac{\partial^4 \tau_{xz}}{\partial z^2 \partial x \partial t} - \frac{g}{\rho_0}\frac{\partial^3 F_{\rho,u}}{\partial x^2 \partial z} \tag{3}$$

The RHS of (3) represent important mechanisms of internal-seiche generation but it appears unwieldy complicated to derive the energy transfer from e.g. wind-shear stress variations to the seiche modes through the vertical exchange of horizontal momentum by turbulence subjected to stratification. With the following extension we believe to facilitate further analysis on energy transfer to internal seiche modes.

2.2 1DV differential equation for the baroclinic pressure gradient

Therefore we looked for, and found the following simple procedure for simulating internal seiche motions in a 1DV model concept. The hydrostatic baroclinic pressure force $P_{\rho,u}$ in (1) is due to the horizontal gradient in x-direction of the mass stored in the water column above level z. This gradient changes in time by vertical mass exchange through mixing and by vertical and horizontal advection. By (2) the latter can be expressed into the horizontal velocity component yielding a closed set of integro-differential equations. Our addition to (1) is the following differential equation for baroclinic pressure force $P_{\rho,u}$, derived from (2):

$$\frac{\rho_0}{g}\frac{\partial P_{\rho,u}}{\partial t} = \frac{\partial^2}{\partial x^2}\int_z^{\zeta}\left[\bar{\rho}(z)-\bar{\rho}(z')\right]u(z')\,dz' + \frac{\partial F_\rho(x,z,t)}{\partial x} - \frac{\partial F_\rho(x,\zeta,t)}{\partial x} \tag{4}$$

We assumed small-amplitude internal seiches decoupled from the barotropic mode, see (Horn & Imberger, 2001) for the applicability of these assumptions. The last two terms in (4) allow insight in the forcing mechanism by internal vertical mixing events represented by $F_\rho(x,z,t)$, such as bubble plumes (Rezvani, 2016), as well as differential surface heating by $F_\rho(x,\zeta,t)$. However, because of the omitted horizontal resolution in a 1DV model we discard the vertical mass fluxes F_ρ . Yet, for internal seiches of horizontal integer mode *n* over basin scale L_x we apply the decomposition $u=\tilde{u}(z,t)\cos(\pi n\,x/L_x)$ with the lake's centre at x=0 and $\bar{\rho}(z)$ the horizontally-averaged density:

$$\frac{\partial P_{\rho,u}}{\partial t} = -\frac{g}{\rho_0}\left(\frac{\pi n}{L_x}\right)^2\int_z^{\zeta}\left[\bar{\rho}(z)-\bar{\rho}(z')\right]\tilde{u}(z',t)\,dz' \tag{5}$$

Hence, the momentum equation (1) and the rate of change in baroclinic pressure gradient (4) are simplified and solve the vertical mode $\tilde{u}(z,t)$ for given horizontal wave length or horizontal mode of the internal seiche.

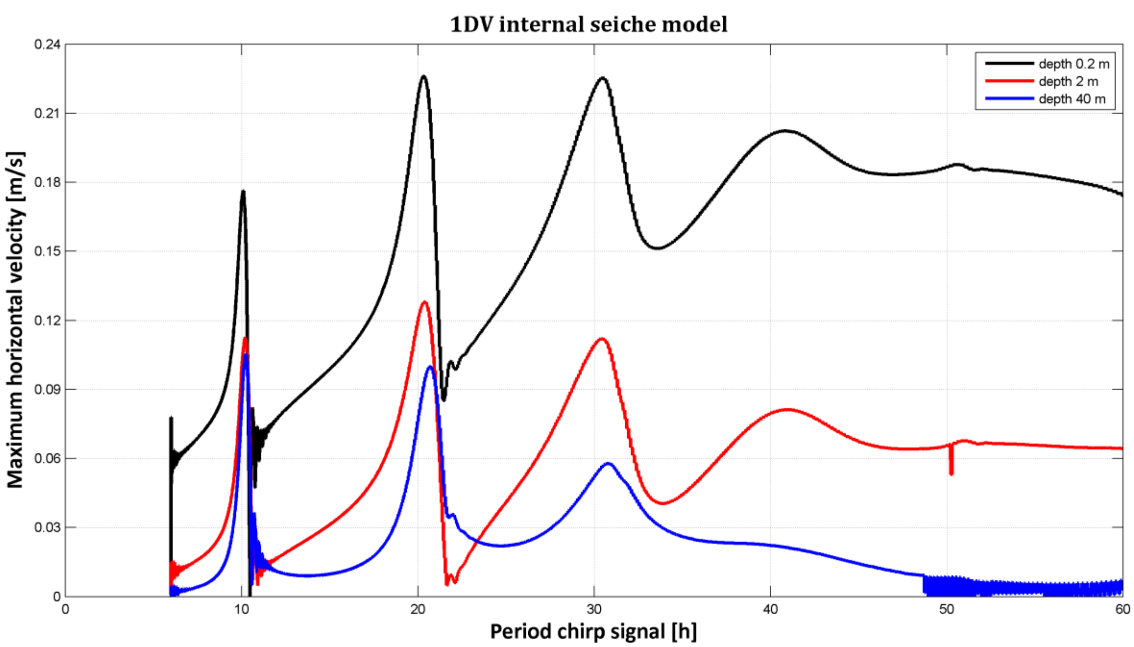
2.3 Model extensions and numerical procedure

Assuming orthogonal decomposition of the seiche motions, i.e. avoiding mixed spatial derivatives in (4) and (5), the equations for horizontal momentum components and baroclinic pressure gradients are decomposed along the orthogonal seiche directions (x,y) with velocity components (u,v). Thus the v-momentum equation is driven by a baroclinic pressure gradient $P_{\rho,\mathrm{v}}$ similar to (5) but for basin scale L_y in y-direction and horizontal mode *m*. The vertical exchange of horizontal momentum and temperature is modelled by the k-ε turbulence model driven by the vertical gradients of (u,v) and partial-slip bed-shear stress boundary conditions. The water-level slope $\partial\zeta/\partial x$ is adjusted to maintain a given depth-averaged and seiche-mode averaged velocity e.g. zero for lakes. The vertical profile of the basin-wide or background temperature is modelled by the standard vertical diffusion equation, depending on the eddy-diffusivity but without vertical advection by seiche motions. The temperature model is driven by a composition of all heat fluxes through the water surface as well as by absorption of solar irradiance, see (Aparicio *et al.*, 2013). The momentum equations as well as all diffusion and sink terms in the temperature and turbulence models are solved Euler implicit on a vertically staggered grid with velocity and temperature in cell centres and fluxes and turbulence properties at cell interfaces (Dijkstra *et al.*, 2016); the baroclinic pressure gradient (5) is integrated Euler explicit.

Under strong stratification with negligible mixing predicted by the k-ε turbulence model, the coupling of (5) to (1) may produce step-wise velocity profiles that disappear if (5) is solved with a marginal diffusion of 1.10^{-6} m² s^{-1}.

2.4 Testing the 1DV internal seiche model

The essence of the model concept is solving (1) and (5) for a single horizontal velocity component. For given horizontal wave length L_x and H1 mode (n=1) we tested the appearance of the vertical seiche modes depending on the frequency of the wind-shear stress. To that purpose (1) is forced by a so-called *"chirp"* of 3 m/s wind amplitude with fixed wind direction but with periodicity increasing from 6 to 60 hours over 720 days. The internal wave length, L_x in (5), and frozen linear-density stratification have been selected such that the periods of the vertical modes of the internal seiche are a multiple of 10 hour. Here we applied a constant eddy-viscosity of 1.10^{-4} m² s^{-1}, Chézy bed friction of 60 m^{½} s^{-1}, 46m water depth, zero depth-averaged velocity, 100 equidistant layers and 5 minute time step. Figure 1 shows that the 1DV internal-seiche model agrees with the period and the corresponding vertical internal-wave modes $\tilde{u}(z,t)$.



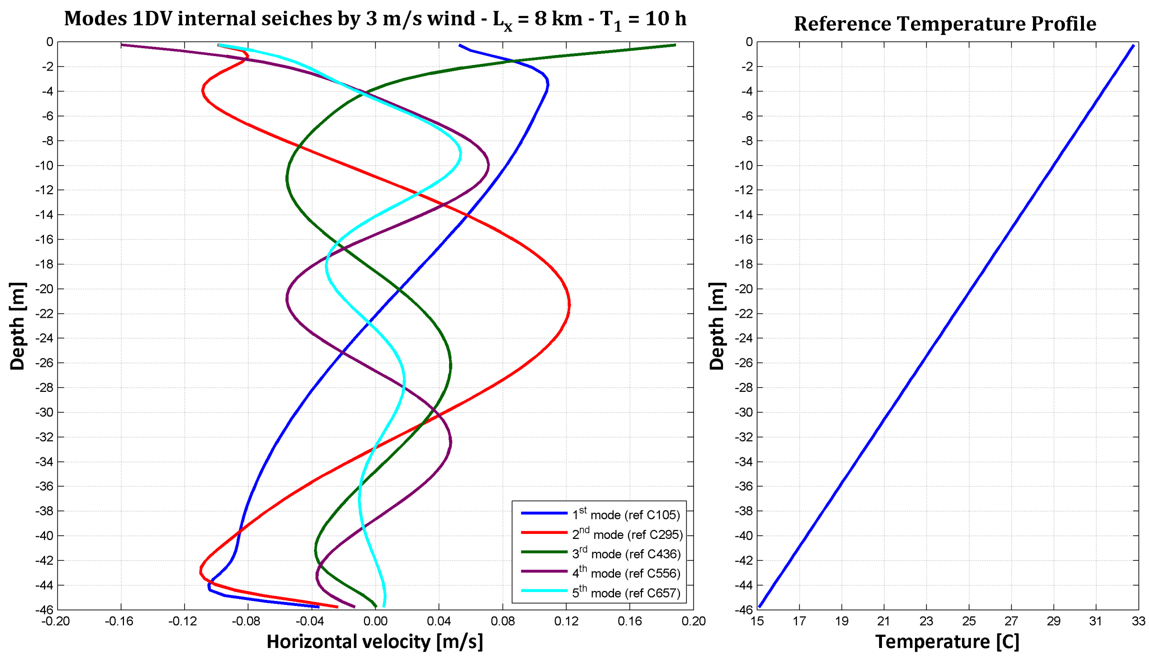


Figure 1: Response of velocity amplitude of the 1DV internal-seiche model (1) and (5) to wind oscillating with period increasing from 6 to 60 hour (top) passing through the first five vertical modes (bottom) in a lake with fixed linear density stratification and with constant eddy viscosity.

3 Comparison to velocity observations in Lake Hallwil

Figure 3 compares the horizontal velocity components observed 1m above the 47 m deep bed in the centre of Lake Hallwil in Switzerland (Figure 2) with those simulated. The surrounding terrain steers the wind in longitudinal direction and the wind exhibits a diurnal oscillating pattern stimulating a basin-long V1H1 seiche mode (McGinnis *et al.*, 2004). Here we applied our two-velocity component 1DV internal-seiche model with orthogonal longitudinal seiche modes set at 7.5 km and 1.25 km, the hypsometry of Lake Hallwil in vertical diffusion terms, with non-linear state equation, Coriolis force, k-ε turbulence model including buoyancy flux, extensive heat flux model with Stanton and Dalton number equal to the wind-drag coefficient according to (Wüest & Lorke, 2003), 100 equidistant layers and 1 minute time step. The 1DV model is driven by locally-observed meteorology and faithfully simulates the water temperature and thermocline evolution over 5 years of monthly observed temperature profiles (Lemonnier *et al.*, 2016).

At the start of the observations a strong wind pulse in basin-length direction generates the observed seiching that gradually dampens; the model response and comparison appear in good agreement.

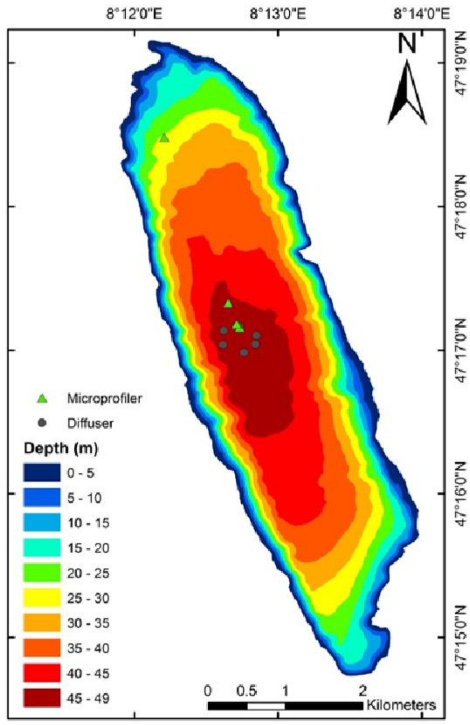


Figure 2: Lake Hallwil (Switzerland), 47m maximum depth, 8 km long and 1.3 km wide. The observations of Figure 3 were made in the lake's centre near "microprofiler".

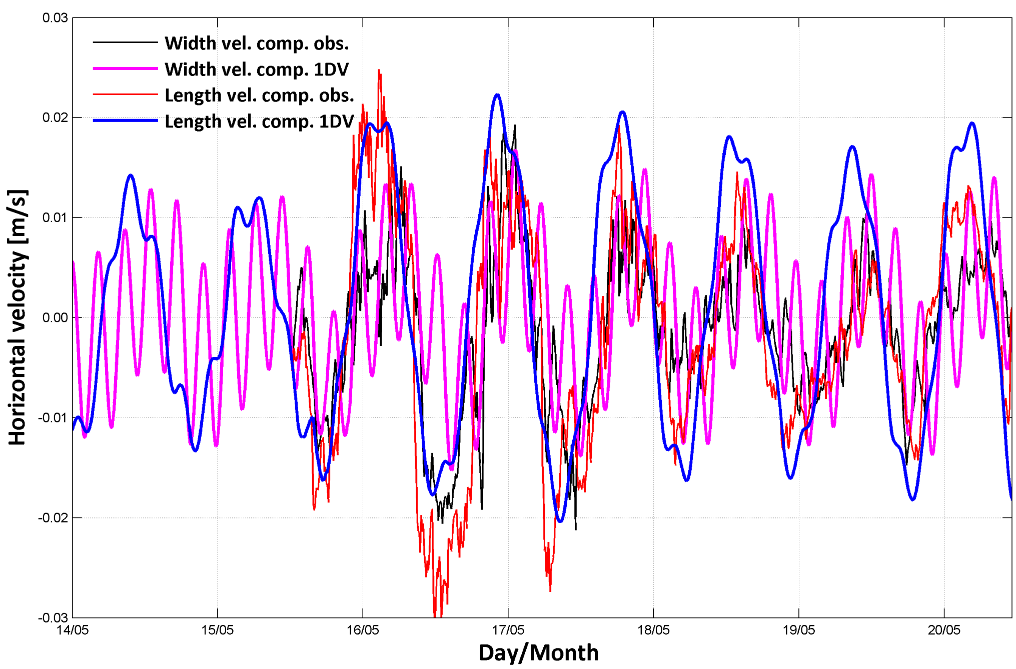


Figure 3: 1DV simulated versus the observed (Rezvani, 2016) horizontal velocity vector 1m above the bed at 47m depth of Lake Hallwil, see Figure 2.

4 Conclusions and recommendations

The essence of this paper is the addition of the temporal equation (5) for the baroclinic pressure gradient to existing 1DV model(s). This extension allows the simulation of internal seiche modes for given longitudinal wave length as demonstrated by a linear stratified case and velocity observations in the centre of a 47m deep thermally stratified lake. Further, this extension opens another avenue for exploring the energy transfer from wind to internal-seiche modes and subsequently to turbulence and mixing notably in benthic boundary layers in lakes.

Depending on the numerical method of an existing 1DV model further numerical research may be required to allow for a stable and accurate integration of the temporal equation for the baroclinic pressure gradient. In case of zero turbulence mixing our Euler-explicit integration and momentum-conserving discretisation introduced occasionally step-wise velocity profiles that were suppressed by solving (5) with a marginal diffusion of 1.10^{-6} m^2 s^{-1}.

Acknowledgements

The first author gratefully thanks Johny Wüest and Damien Bouffard for hosting him at EPFL/Lausanne where he conducted the research for this paper. Under supervision of Scott Socolofsky of Texas A&M the second author collected the field data in Lake Hallwil through work supported by the U.S. National Science Foundation under Grant No. CBET-1034112 to Texas A&M University and Virginia Tech.

References

Aparicio Medrano, E., Uittenbogaard, R.E., Dionisio Pires, L.M., Van de Wiel, B.J.H. and Clercx, H.J.H. (2013). Coupling hydrodynamics and buoyancy regulation in *Microcystis aeruginosa* for its vertical distribution in lakes. *Ecological Modelling*, 248, 41–56.

Cuypers, Y., Vinçon-Leite, B., Groleau, A., Tassin, B. and Humbert, J-F. (2011). Impact of internal waves on the spatial distribution of *Planktothrix rubescens* (cyanobacteria) in an alpine lake. *The International Society for Microbial Ecology Journal*, 5, 580–589.

Dijkstra, Y. M, Uittenbogaard, R.E., Van Kester, J.A.Th.M and Pietrzak, J.D. (2016). In search of improving the numerical accuracy of the k-ε model by a transformation to a k-τ model. *Ocean*

Modelling, vol. 104, Aug., p. 129-142

Hodges, B.R., Imberger, J, Saggio, An., and Winters, K.B. (2000). Modeling basin-scale internal waves in a stratified lake. *Limnol. Oceanogr*. 45(7), 1603-1620

Horn, D.A., Imberger, J. and Ivey, G.N. (2001). The degeneration of large-scale interfacial gravity waves in lakes. *J. Fluid Mech*., vol. 434, 181-207.

Lemonnier, B., Uittenbogaard, R.E. , Wüest, A., Stöckli, A. and Bouffard, D. (2016). Modelling vertical distribution of *Plantothrix rubescens* in Lake Hallwil. Paper in submission.

Lorke, A., Umlauf, L., Jonas, T. and Wüest, A. (2002). Dynamics of turbulence in low-speed oscillating bottom-boundary layers of stratified basins. *Env. Fluid Mech*., vol. 2, 291-313.

McGinnis, D.F., Lorke, A., Wüest, A., Stöckli, A. and Little, J.C. (2004). Interaction between a bubble plume and the near field in a stratified lake. *Water Res. Res*, vol. 40, W10206 (11 pages)

Perroud, M., Goyette, S., Martyniv, A., Beniston, M. and Anneville, O. (2009). Simulation of multiannual thermal profiles in deep Lake Geneva: A comparison of one-dimensional models. *Limnol. Oceanogr*. 54(5), 1574-1594.

Read, J.S., Winslow, L.A., Hansen, G.J.A., Van den Hoek, J., Hanson, P.C., Bruce, L.C. and Markfort, C.D. (2014). Simulating 2368 temperate lakes reveals weak coherence in stratification phenology. *Ecological Modelling*, 291, 142-150.

Rezvani, M. (2015). Bubble plumes in crossflow: laboratory and field measurements of their fluid dynamic properties with application to lake aeration and management. PhD. Dissertation, Texas A&M, May.

Wüest, A. and Lorke, A. (2003). Small-scale hydrodynamics in lakes. *Ann. Rev. Fluid Mech*., vol. 35, 373-412.