The Three-dimensional Propagation of Tsunami-Generated Internal Waves in the Atmosphere

Yue Wu, Stefan G. Llewellyn Smith, James W. Rottman, Dave Broutman and Jean-Bernard H. Minster

Department of Mechanical and Aerospace Engineering,
University of California San Diego
wuyue@ucsd.edu

Abstract

Tsunami-generated internal waves have been observed to propagate in the atmosphere up to the ionosphere where they have an impact on the total electron content. We simulate numerically the three-dimensional propagation of linear internal waves in an atmosphere with vertically varying stratification and horizontal background winds. Our goal is to investigate the effect of wind jets with different magnitude, width and direction on wave transmission and reflection, and to quantify how much energy is transmitted through the atmosphere up to the ionosphere in a few idealized cases and one realistic case.

1 Introduction

Tsunami-generated internal waves can propagate through the atmosphere up to the ionosphere, where their impact on the total electron content can be detected by ionospheric sounding techniques using the Global Positioning System (GPS). The information carried by atmospheric waves could be used for tsunami alerts (Hines, 1972). Artru et al. (2005) first observed from the GEONET network in Japan a short-scale ionospheric perturbation produced by the tsunami associated with a magnitude 8.2 earthquake in Peru. Numerical models, e.g. Occhipinti et al. (2006, 2008, 2011), Hickey et al. (2009) and Mai and Kiang (2009) have simulated the three-dimensional propagation of tsunami-generated internal waves through the atmosphere.

Background winds are an important feature of the real atmosphere. Background winds and the vertically varying stratification in the atmosphere affect the transmission and reflection of waves. Broutman et al. (2014) examined the two-dimensional propagation of tsunami-generated gravity waves through a realistic atmosphere with vertically varying stratification. Wei et al. (2015) examined the two-dimensional propagation of internal waves with time evolution in an atmosphere with piecewise background winds.

We simulate numerically the three-dimensional propagation of linear internal waves in the atmosphere with vertically varying stratification and horizontal background winds. We extend the calculation of Broutman et al. (2014) to three dimensions to allow two-dimensional variations in the tsunami-perturbed sea surface height and variations of the direction of background winds with height. We consider the cases when the wind is not perfectly aligned in the direction of tsunami propagation and has vertically varying directions. Results show that the three-dimensional effects of background winds have a non-negligible influence on wave transmission and reflection, as well as on the distribution of the propagating and evanescent regions.

2 Formulation

In the tsunami reference frame, we follow Wu et al. (2016) in formulating the problem in terms of $\tilde{w}$, the Fourier transform of the perturbed vertical velocity scaled by a factor of $[\rho_0(z)/\rho_0(0)]^{1/2}$ that accounts for the altitude-dependent density. Then $\tilde{w}$ satisfies the Taylor-Goldstein equation expressed as

$$\tilde{w}_{zz} + m^2\tilde{w} = 0, \quad (1)$$

where m is the vertical wavenumber, the symbol tilde denotes Fourier transform in the horizontal coordinates x and y, and the subscripts z and zz denote the first and second derivatives in the vertical coordinate z, respectively.

The vertical wavenumber m is found as a function of buoyancy frequency N, background wind velocity with horizontal components (U, V), density scale height H (expressed by $S = 1/H$) and the horizontal wavenumbers (k, l):

$$m^2 = \frac{N^2}{\frac{(Uk+Vl)^2}{k^2+l^2}} + \frac{S(U_z k + V_z l)}{Uk + Vl} - \frac{U_{zz}k + V_{zz}l}{Uk + Vl} + \frac{S_z}{2} - \frac{S^2}{4} - k^2 - l^2. \quad (2)$$

We obtain m^2 for each Fourier component using (2) and select initially propagating waves with $m^2 > 0$ at $z = 0$. We integrate (1) from top to bottom with a radiation upper boundary condition representing energy leaving the domain without entering from above. We write the numerical solution to (1) in the form of a WKB approximation in order to separate the upgoing and downgoing waves. At the non-reflecting sea surface, upgoing waves are rescaled to satisfy the lower boundary conditions while the downgoing waves and their effects on the tsunami itself is neglected. We calculate the transmission and reflection coefficients $T(k, l)$ and $R(k, l)$ for each Fourier component and the total energy transmission coefficient. Finally we perform the inverse Fourier transform and plot the upgoing and downgoing waves in the spatial domain separately, but do not show the solutions in regions where the WKB approximation is not valid.

3 Results

We add two-dimensional variations to the tsunami-perturbed sea surface such that it satisfies the same model profile as in Peltier and Hines (1976) in the x direction and curves in the y direction to simulate a spreading profile in a more realistic fashion (Figure 1). We examine the three-dimensional propagation through an idealized wind jet at a fixed direction and through the realistic atmospheric profile corresponding to the 2004 Sumatra tsunami (Figure 2).

3.1 The case of an idealized wind jet at a fixed direction

We add a horizontal wind jet to the background so that the wind and the tsunami are in opposite direction. The left two panels of Figures 3 show the normalized vertical velocity w/W_0 for the upgoing and downgoing waves, respectively, where W_0 is the maximum tsunami vertical velocity. The plots follow a similar pattern to that in the two-dimensional problem shown in Figure 5 of Broutman et al. (2014): waves generated by the tsunami propagate upward in the atmosphere until reach their turning points when encountering the wind jet at approximately 90 km altitude and partial energy is transmitted and the

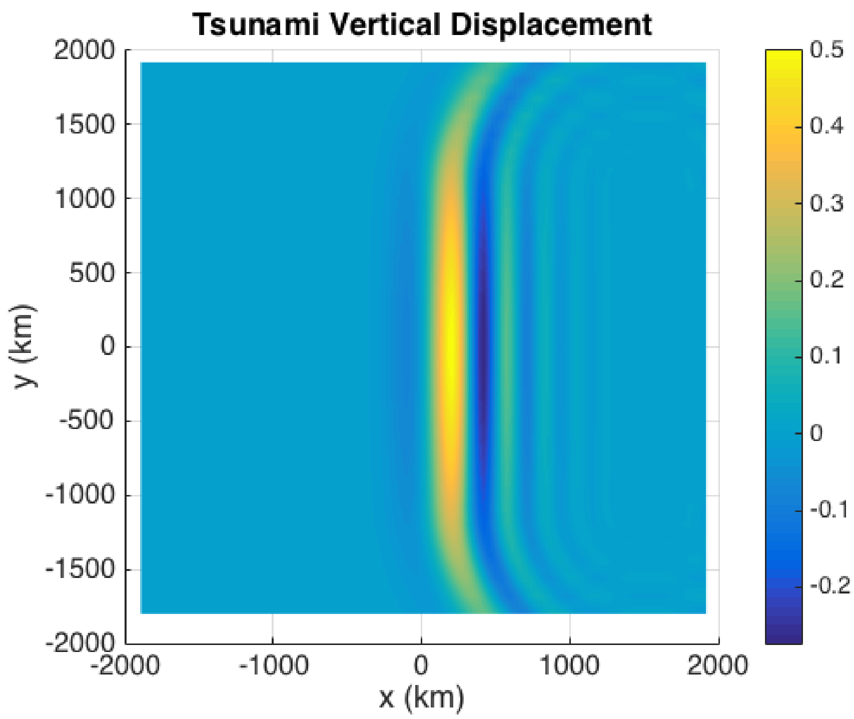

Figure 1: Tsunami vertical displacement.

rest is reflected. Turning points correspond to heights where $m = 0$ and evanescent regions correspond to regions where m is imaginary. At 90-120 km, waves are driven into evanescence then propagate again above 120 km. We observe a distribution in y where waves with maximum amplitudes appear in the slice $y = 0$ where the bottom boundary condition has no decay. The bottom right panel shows the transmission coefficient $T(k, l)$ where waves with small $|k|$ and large $|l|$ have greater transmission.

The total energy transmission coefficient depends on the amplitude, width and direction of the wind jet. The dependence is illustrated in Figures 4 showing that the faster and the wider the wind, the smaller the energy transmission. By varying the angle between the wind and the tsunami propagation, we find when the wind is against the tsunami, the energy transmission reaches its minimum, and when the wind is aligned in the same direction as the tsunami, there is no reflection and the energy transmission reaches its maximum which is 1.

3.2 The case of the realistic atmosphere

We investigate wave propagation through the realistic atmospheric profile in the 2004 Sumatra tsunami. The total energy transmission coefficient turns out to decrease by 9.61% compared to the two-dimensional results. One reason is that wavenumber l has a negative effect on (2) resulting in fewer propagating waves; another reason is that the wind velocity V in y is so strong at higher altitude making more waves evanescent, hence the energy transmission coefficient decreases. A clear cut-off wavenumber $k \approx -4K_0$ appears in the bottom right panel indicating that waves are either not initially propagating at the sea surface or are evanescent at the top of the domain.

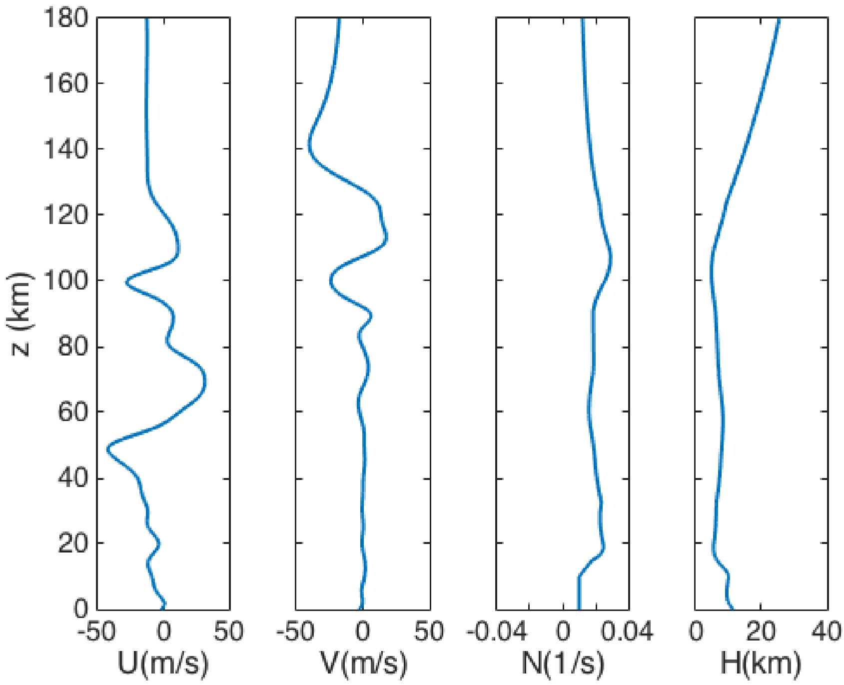

Figure 2: Atmospheric profiles in the 2004 Sumatra tsunami case.

4 Conclusion

Three-dimensional effects have a non-negligible influence on the number of initially propagating waves, as well as on wave propagation, including the number and height of turning points, the distribution of propagating and evanescent regions, and wave transmission and reflection. For future work, we plan to study the initial value problem to investigate the real-time propagation of tsunami-generated atmospheric waves and to quantify the first arrival of waves. We also plan to investigate the stochastic properties of the environment variables and their effects on wave propagation.

Acknowledgements

This work was supported by ONR Award N00014-13-1-0347.

References

Artru, J., Ducic, V., Kanamori, H., Lognonné, P., and Murakami, M. (2005). Ionospheric detection of gravity waves induced by tsunamis. *Geophys. J. Int.*, 160:840–848.

Broutman, D., Eckermann, S. D., and Drob, D. P. (2014). The partial reflection of tsunami-generated gravity waves. *J. Atmos. Sci.*, 71:3416–3426.

Hickey, M. P., Schubert, G., and Walterscheid, R. L. (2009). Propagation of tsunami-driven gravity waves into the thermosphere and ionosphere. *J. Geophys. Res.*, 114.

Hines, C. O. (1972). Gravity Waves in the Atmosphere. *Nature*, 239:73–78.

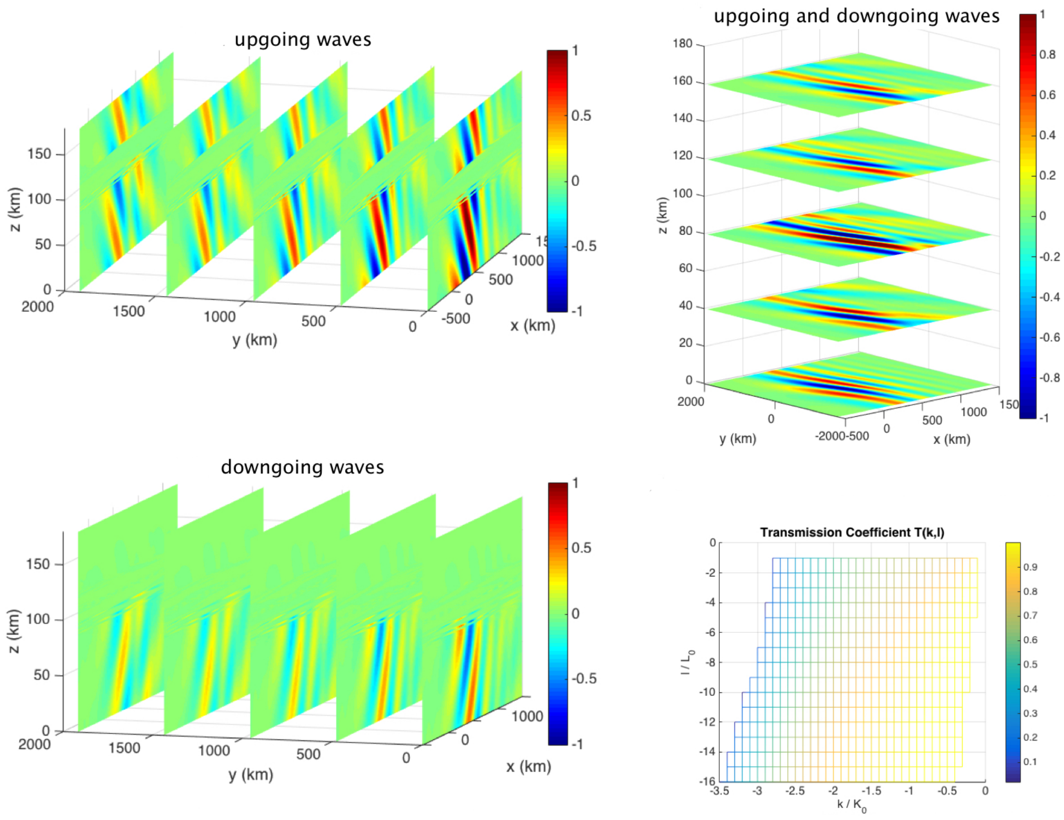

Figure 3: w/W_0 for upgoing and downgoing waves with $T(k,l)$ in the wind jet case.

Mai, C.-L. and Kiang, J.-F. (2009). Modeling of ionospheric perturbation by 2004 sumatra tsunami. *Radio Science*, 44.

Occhipinti, G., Coïsson, P., Makela, J. J., Allgeyer, S., Kherani, A., Hebert, H., and Lognonné, P. (2011). Three-dimensional numerical modeling of tsunami-related internal gravity waves in the hawaiian atmosphere. *Earth, Planets and Space*, 63:847–851.

Occhipinti, G., Kherani, E. A., and Lognonné, P. (2008). Geomagnetic dependence of ionospheric disturbances induced by tsunamigenic internal gravity waves. *Geophysical Journal International*, 173:753–765.

Occhipinti, G., Lognonné, P., Kherani, E. A., and Hébert, H. (2006). Three-dimensional waveform modeling of ionospheric signature induced by the 2004 sumatra tsunami. *Geophys. Res. Lett.*, 33.

Peltier, W. R. and Hines, C. O. (1976). On the possible detection of tsunamis by a monitoring of the ionosphere. *J. Geophys. Res.*, 81:1995–2000.

Wei, C., Bühler, O., and Tabak, E. G. (2015). Evolution of tsunami-induced internal acoustic–gravity waves. *J. Atmos. Sci.*, 72:2303–2317.

Wu, Y., Llewellyn Smith, S. G., Rottman, J. W., Broutman, D., and Minster, J.-B. H. (2016). The propagation of tsunami-generated acousticgravity waves in the atmosphere. *J. Atmos. Sci.* Manuscript was accepted in May 2016.

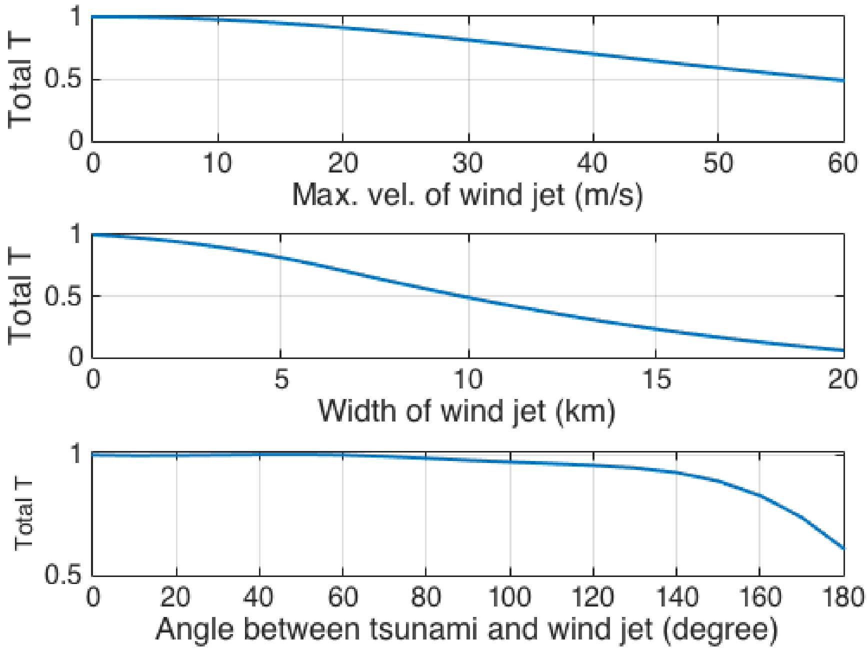

Figure 4: Total energy transmission for winds with different maximum velocities, widths and directions.

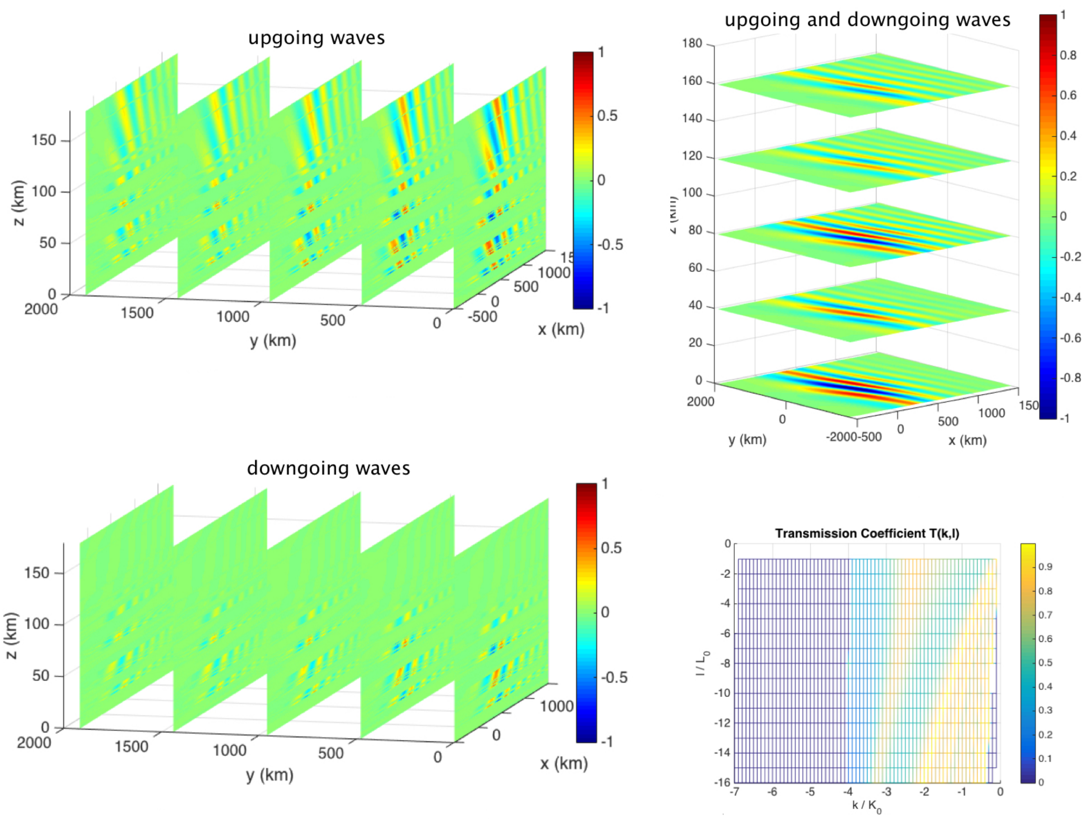

Figure 5: w/W_0 for upgoing and downgoing waves with $T(k,l)$ in the tsunami case.