Mizzi et al. (2016) derived CPSRs for assimilation of full retrieval profiles, i.e. all elements of the retrieval profile, even bad observations were assimilated. Such bad observations should be discarded prior to the assimilation step to reduce: (i) computational costs; and (ii) analysis errors. Mizzi et al. (2017a) extend CPSRs to the assimilation of truncated retrieval profiles. Mizzi et al. (2017a) derive CPSRs the same way as Mizzi et al. (2016) except they discard known bad observations prior to application of the compression and diagonalization transforms. Their derivation is as follows:

\[ y_v = (I - A) y_n = z - A y_n \]  

where \( y_v \) is the retrieval profile (dimension \( n \)), \( A \) is the averaging kernel (dimension \( n \times n \)), \( y_n \) is the true atmospheric profile (unknown; dimension \( n \)), \( y_v \) is the retrieval profile prior (dimension \( n \)), and \( A \) is the measurement error in retrieval space (dimension \( n \times n \)) with error covariance \( R_w \). We begin by discarding the good elements of \( y_n \) that are bad observations. The resulting dimension of the truncated retrieval profile \( y_v = n - m \). We also discard the corresponding rows of \( A \) and the corresponding rows and columns of \( R_w \). The resulting dimensions are \( n - m \) and \( n - m \). Due to Mizzi et al. (2016)’s use of singular value decompositions (SVDs) for the compression and diagonalization transforms, the rest of the derivation is unchanged. This approach reduces the computational cost of assimilating CPSRs beyond that obtained in Mizzi et al. (2016) by a factor of \((n - m)/n\).