Analyzing Constellation Performance for the Radio Occultation Tomography of Internal Gravity Waves

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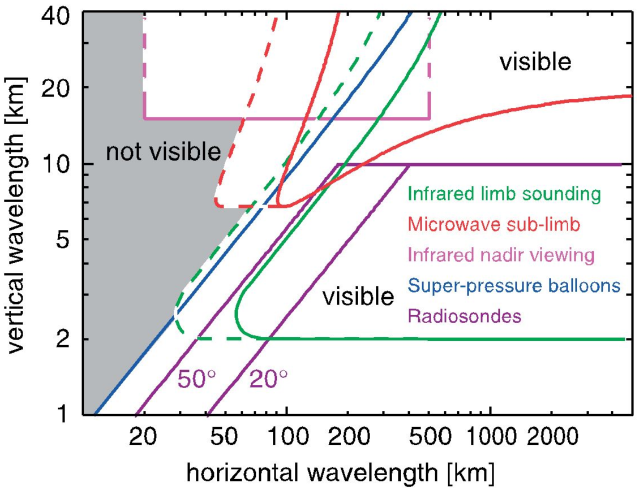
Outline

- Formation Flying for Clustered Occultations
- Occultation Cluster Quality and Distribution
- Examining Trends in Cluster Quality
- Conclusions

Formation Flight & Constellations

Previous Work: RO Tomography of Gravity Waves

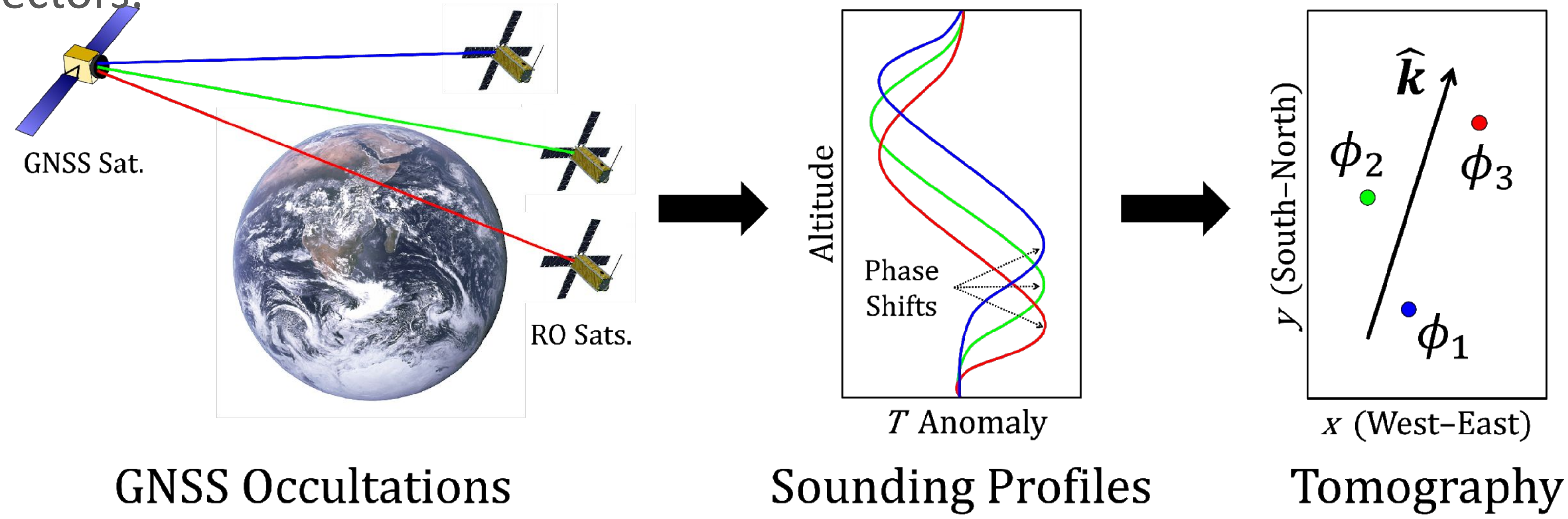
- Technique demonstrated with COSMIC-1 data, before satellites dispersed: Wang and Alexander, JGR (2010); Schmidt, Alexander, de la Torre (2016). Sensitive only to long wavelengths (~1000 km) and low frequencies (~1/day) in order to find any clusters.
- A dedicated mission can find short wavelength (~50 km), high frequency waves (~1/30 min). Otherwise ~10,000 randomly distributed LEO satellites would be needed to develop an RO momentum flux climatology of these waves.



Alexander et al., QJRMS (2010)

Formation Flight for Tomography

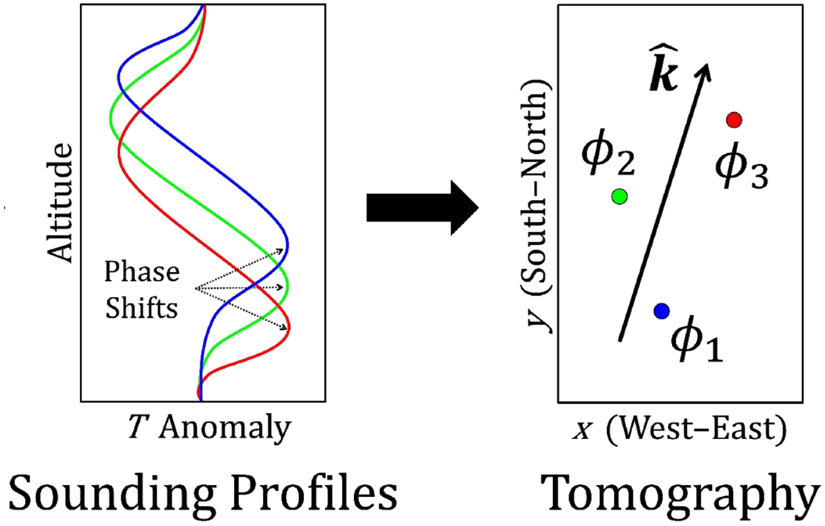
Flying small satellites in close formation yields RO sounding that are closely spaced on the ground. These can be used to infer internal gravity wave vectors.

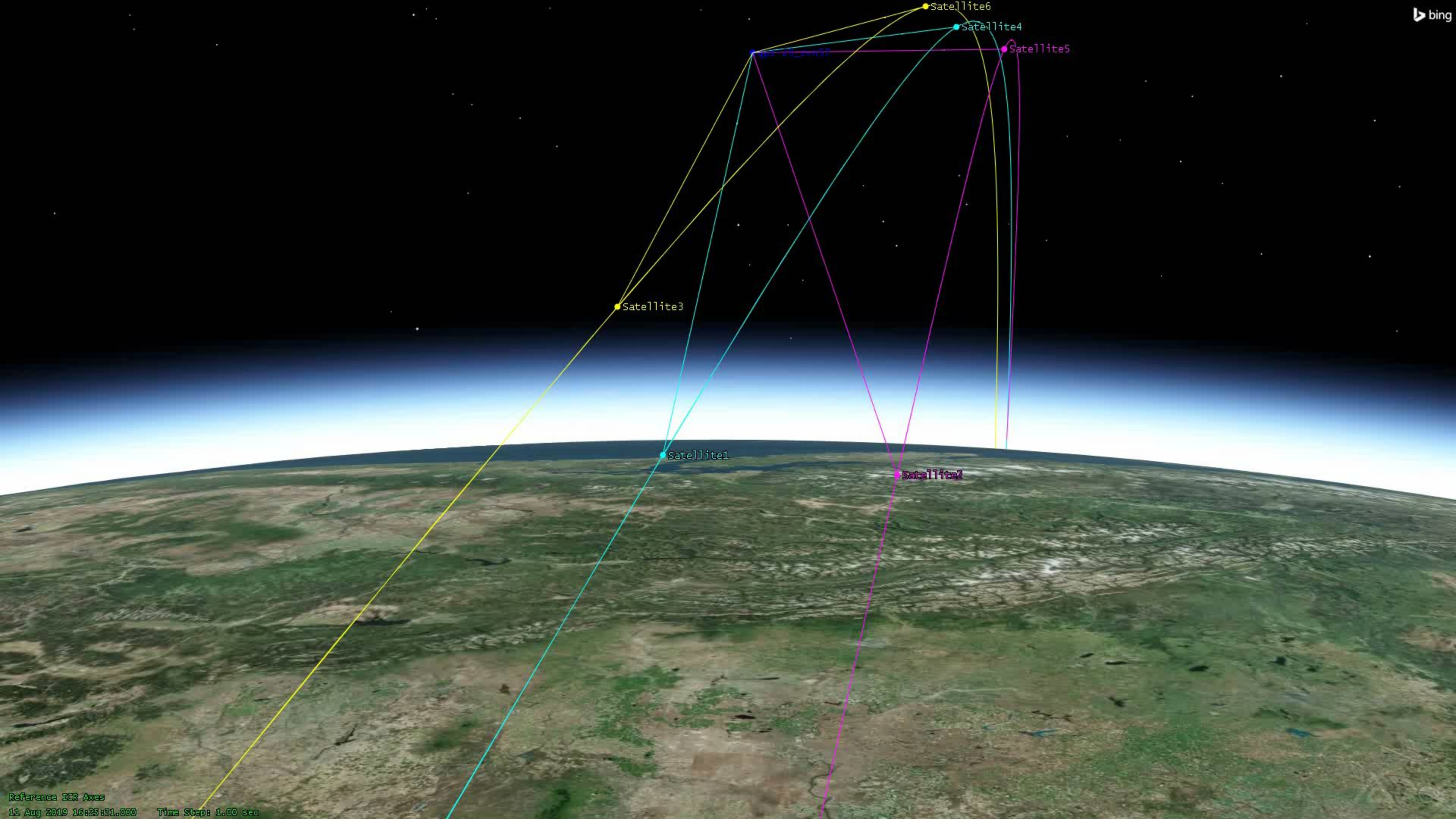


Formation Flight for Tomography

Flying small satellites in close formation yields RO sounding that are closely spaced on the ground. These can be used to infer internal gravity wave vectors.

- This image is idealized, simplifying
  - Frequency dependence
  - Sounding slant angle
  - Vertical wave vector component
  - 2π modularity
- Regardless of complications, wave tomography requires 2D spread of soundings; we focus on this effect.

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Satellite4
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bing
Reference ICR Axes
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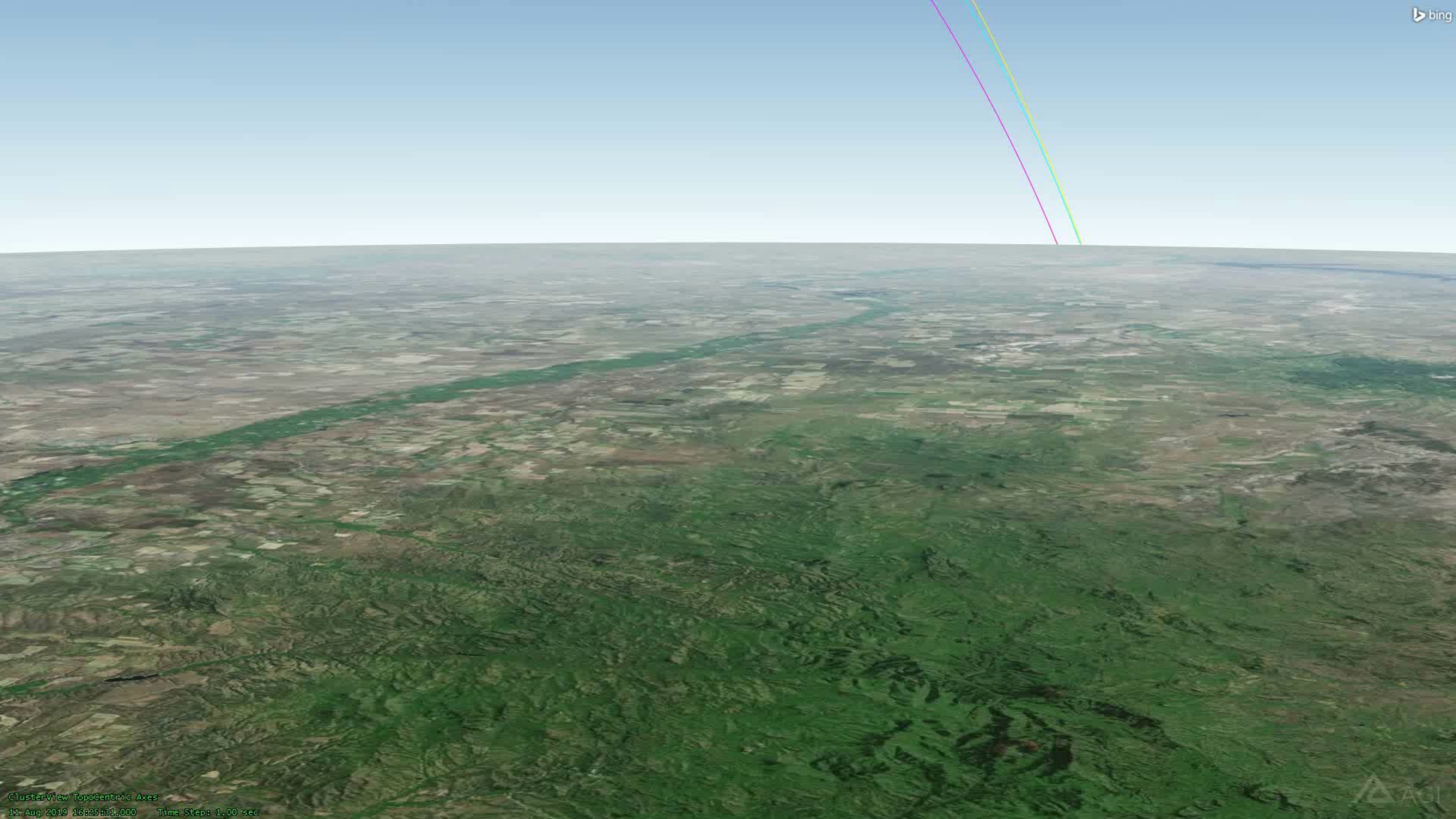
Time Step: 1.00 sec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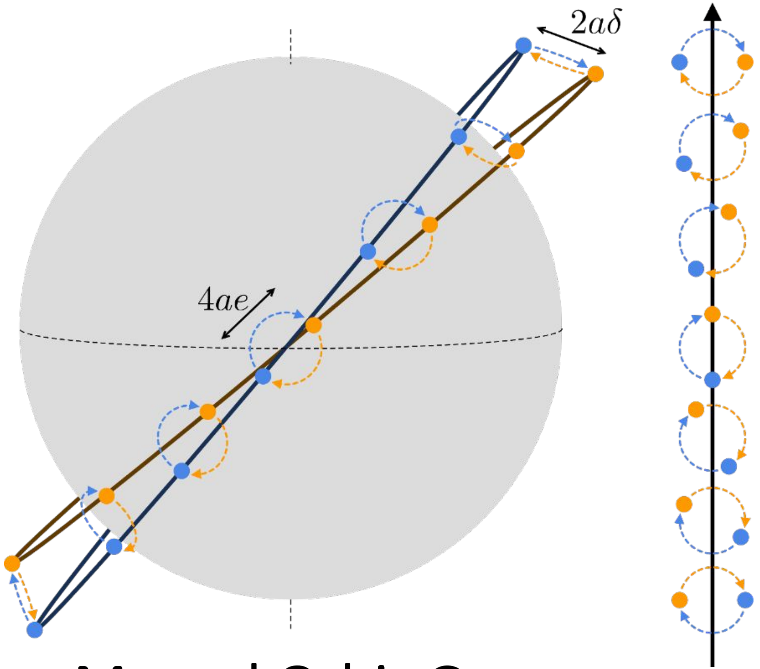
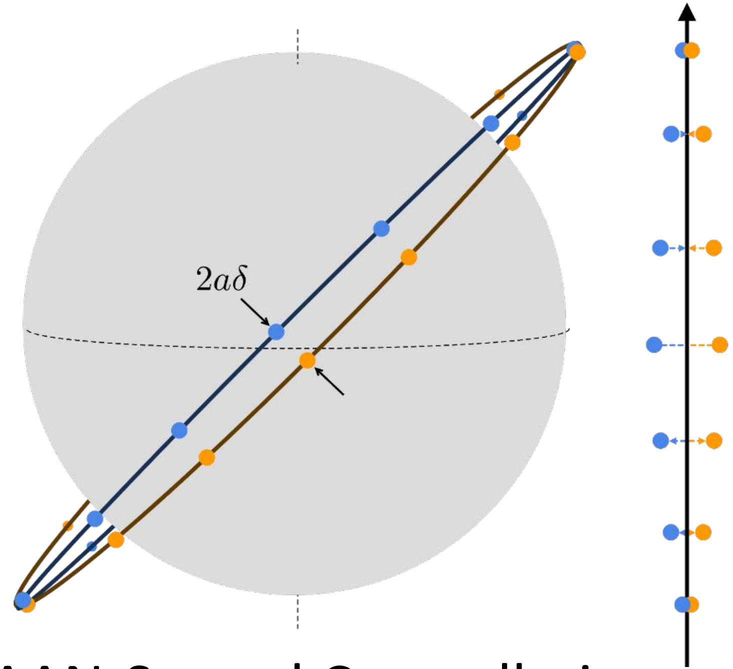
Mutual Orbit Group v. RAAN-Spread

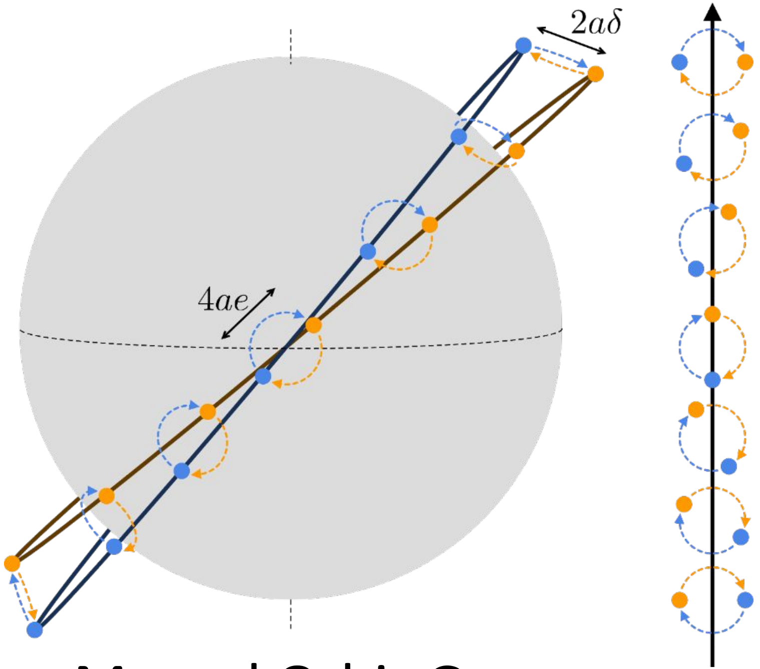


Mutual Orbit Groups

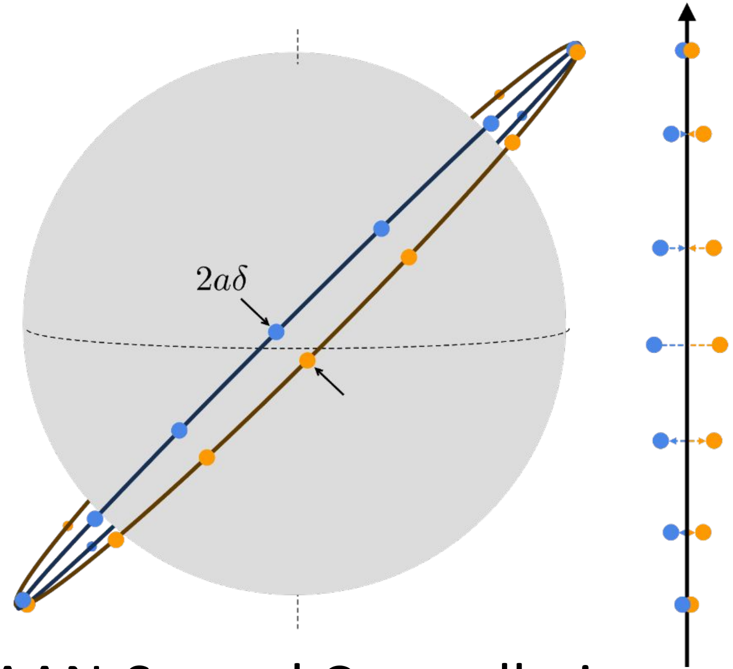


RAAN-Spread Constellation

See previous paper (in IEEE JSTARS) on MOG maintenance at DOI: 0.1109/JSTARS.2019.2961084



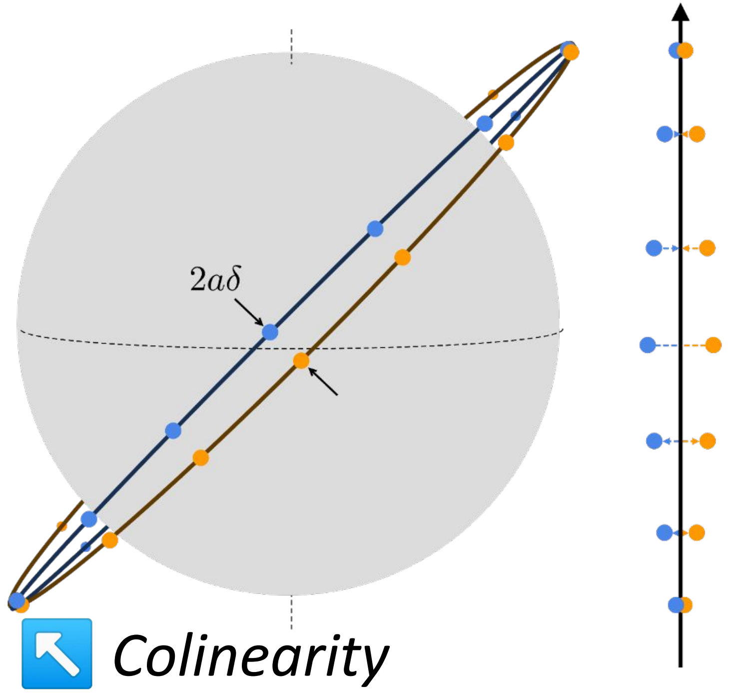
Mutual Orbit Groups



RAAN-Spread Constellation

MOG Alternative: RAAN-Spread

- RAAN-Spread constellations space satellites only in right ascension of the ascending node (RAAN)—no inclination difference is required.
- However, groups approach collinearity at min./max. elevation out of equatorial plane; the clusters produced tend to approach lines.



Quantifying the MOG/RS Tradeoff

- Main question: Can we quantify the tomographic performance tradeoff of the MOG v. RAAN-Spread architectures?
- What constellation designs are best for tomography, without requiring prohibitive constellation maintenance propulsion?

Occultation Cluster Quality

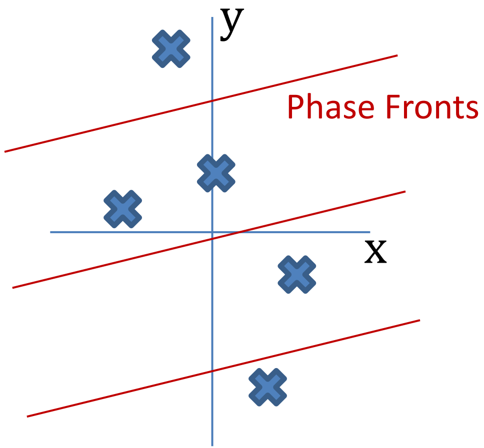
Occultation Cluster Quality Metrics

- χ^2 minimization relates sounding position to uncertainty in wave vector reconstruction.

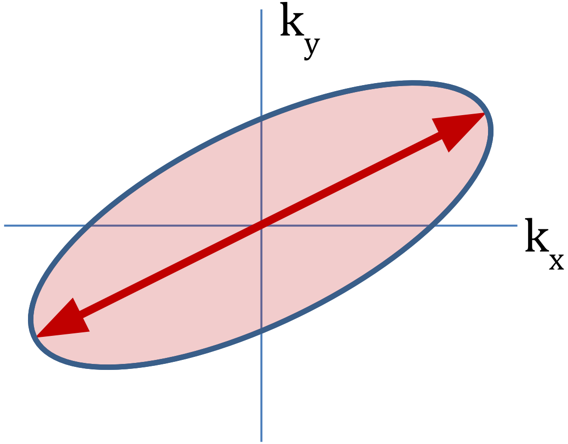
$$\chi^2 = \frac{1}{\sigma_\phi^2} \sum_{i=1}^{n} (\boldsymbol{k} \cdot \boldsymbol{r}_i + \phi_0 - \phi_i)^2$$

- $q_1 \propto$ uncertainty area
- $q_2 \propto$ least-certain axis

Occultation Cluster



Wave Vector Uncertainty



$$R = \begin{bmatrix} \delta x_1 & \delta y_1 \\ \vdots & \vdots \\ \delta x_n & \delta y_n \end{bmatrix}$$

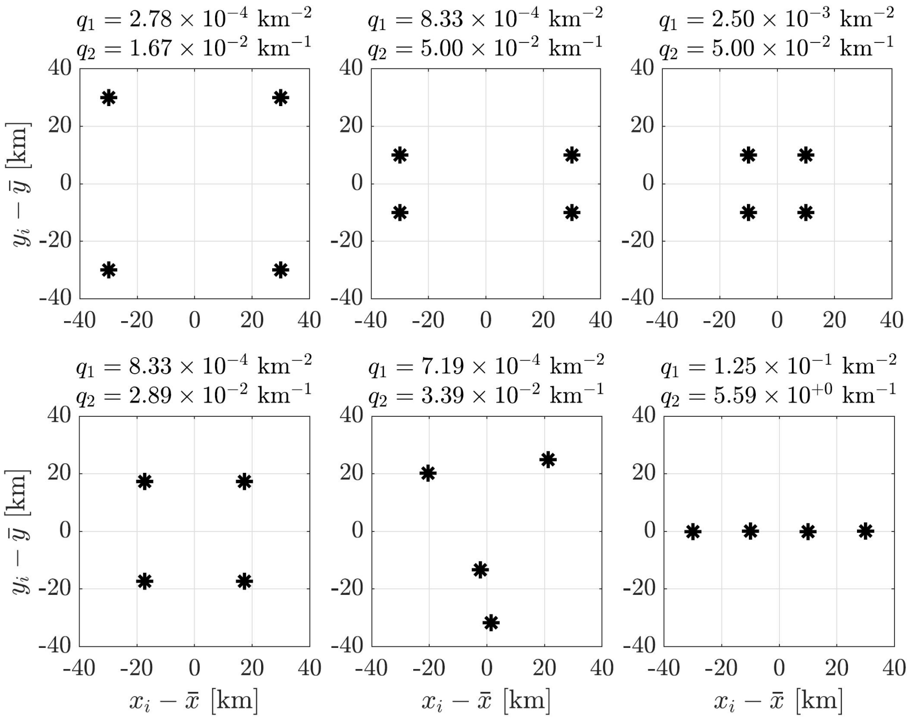
$$q_1 = \sigma_\phi^2 \left(\det R^\mathsf{T} R\right)^{-1/2}$$

$$q_2 = \sigma_\phi \left[\lambda_{\min}\left(R^\mathsf{T} R\right)\right]^{-1/2}$$

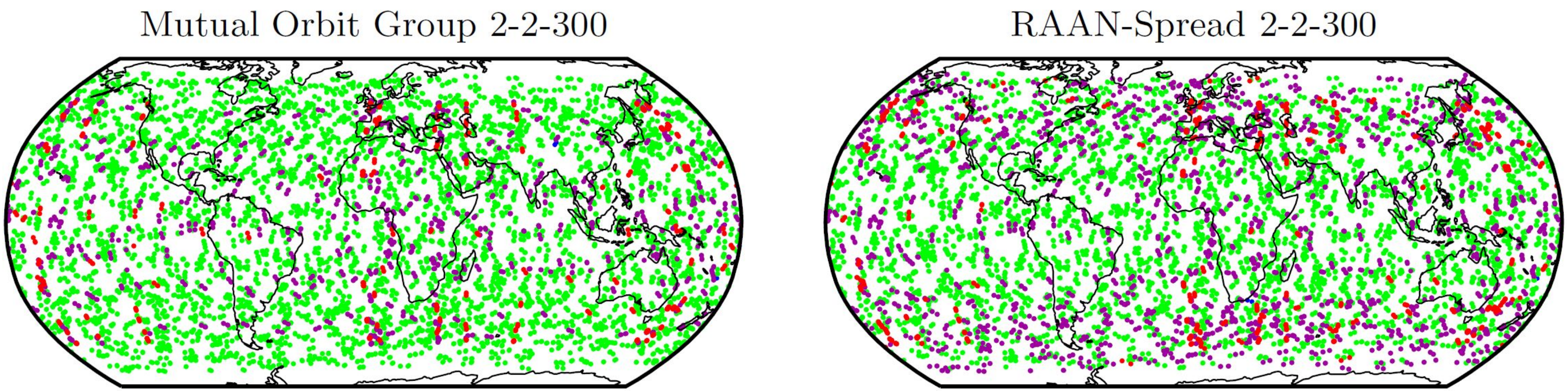
Example Clusters and Qualities

Lower $q_{1/2}$ is preferable:

1. Wide, even spread is best.
2. Decreasing small dimension lowers both q_1 and q_2.
3. Decreasing large dimension lowers only q_1; q_2 is constant.
4. Metric q_1 goes with area; q_2 prefers uniformity.
5. Roughly comparable to [4], but with random geometry.
6. Linearity is bad for both $q_{1/2}$.



Latitude Dependence of Quality

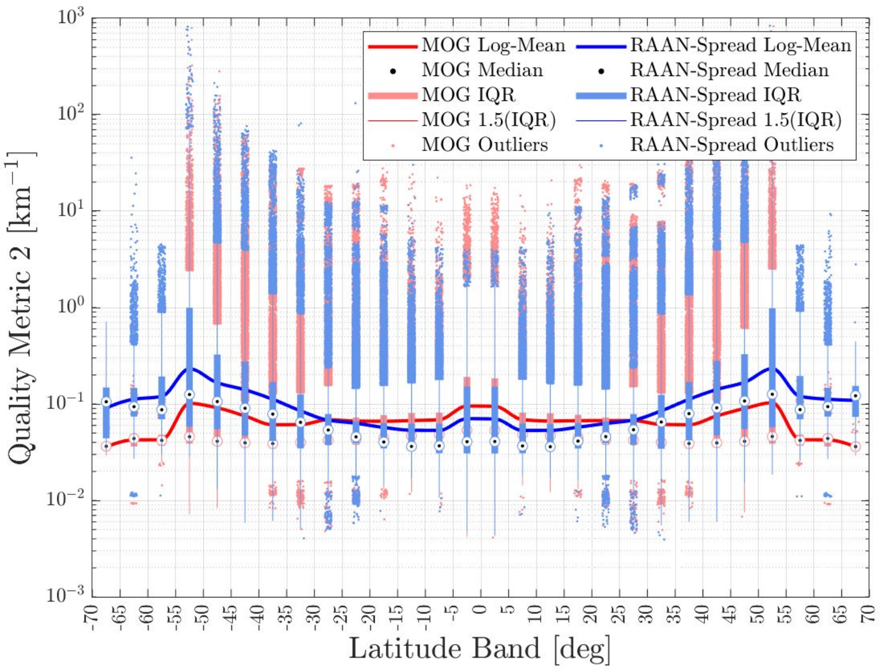


- Best, $q_2 \in [0, 10^{-2}]$ km^{-1}
- Good, $q_2 \in [10^{-2}, 10^{-1}]$ km^{-1}
- Fair, $q_2 \in [10^{-1}, 1]$ km^{-1}
- Poor, $q_2 \in [1, \infty]$ km^{-1}

Even distribution of good clusters for both; RAAN-Spread constellation has more fair/poor clusters at high latitudes.

Latitude-Dependent Distributions

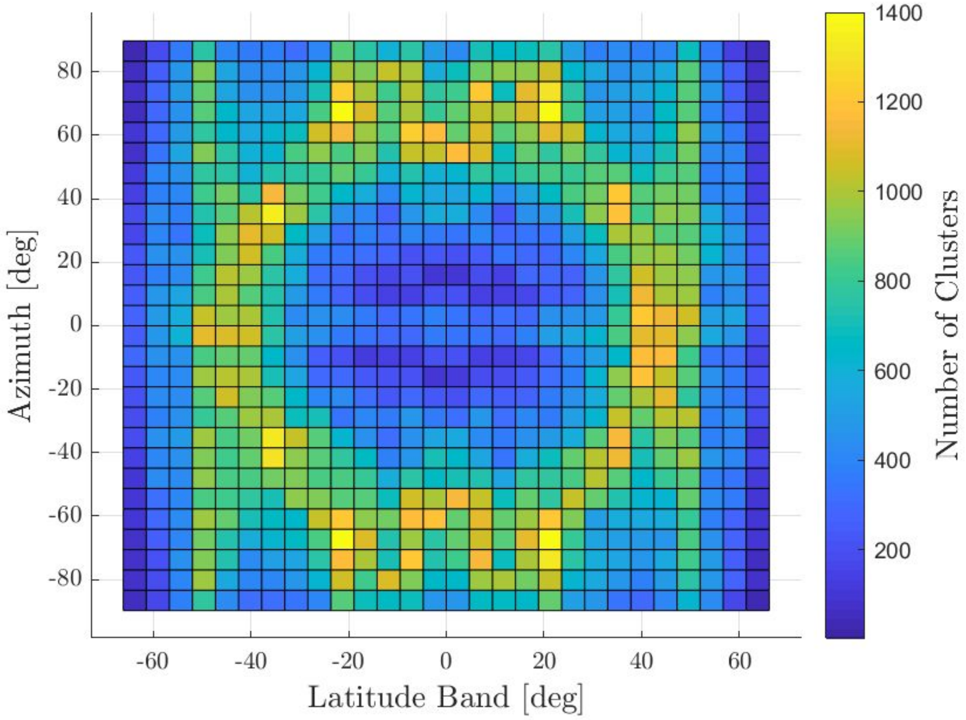
- Plot: MOG v. RAAN-Spread over all latitude bands.
- Curve shows the log-mean of all soundings over 6 mo.
  - All four GNSS constellations
  - Simulated using AGI STK
- RAAN-Spread better near equator, MOG better in high latitude (q_1 is comparable.)



Examining Trends in Quality

Latitude-Azimuth Distribution

- Shown: Distribution of all clusters in latitude and ray path azimuth angle.
- Equatorial soundings tend to be closer to east-west, high-latitude clusters tend to be aligned north-south.
- Distribution dependent on reference orbit, not constellation type.



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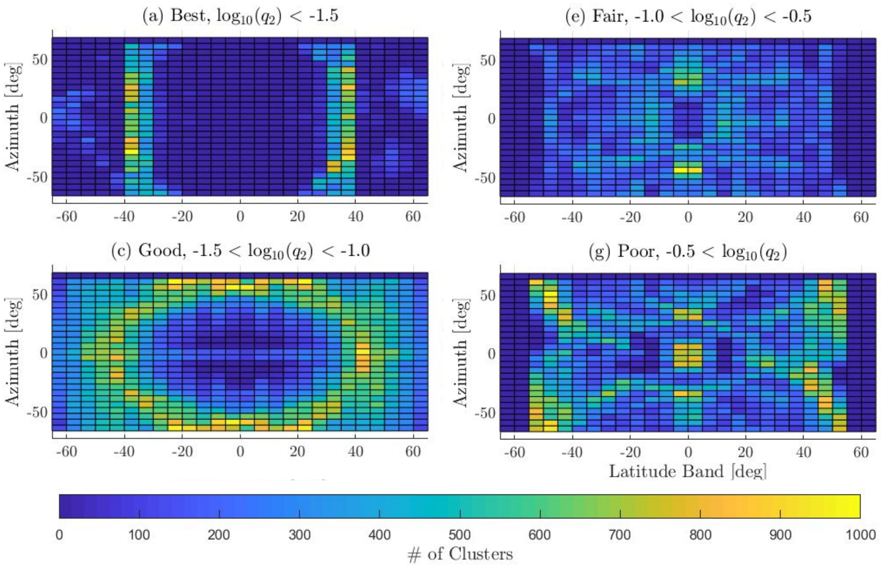
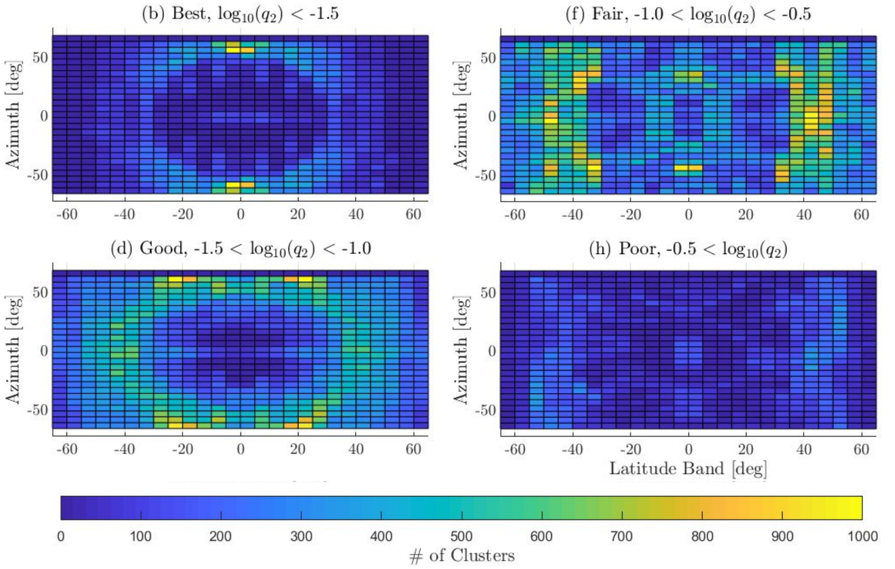


Mutual Orbit Groups: *Best* clusters occur at a specific latitude, but consistent *good* clusters matching total distribution.

RAAN-Spread: Fewer *best* clusters, and higher-latitude *good* clusters get cut down to *poor* (though fewer *poor*).



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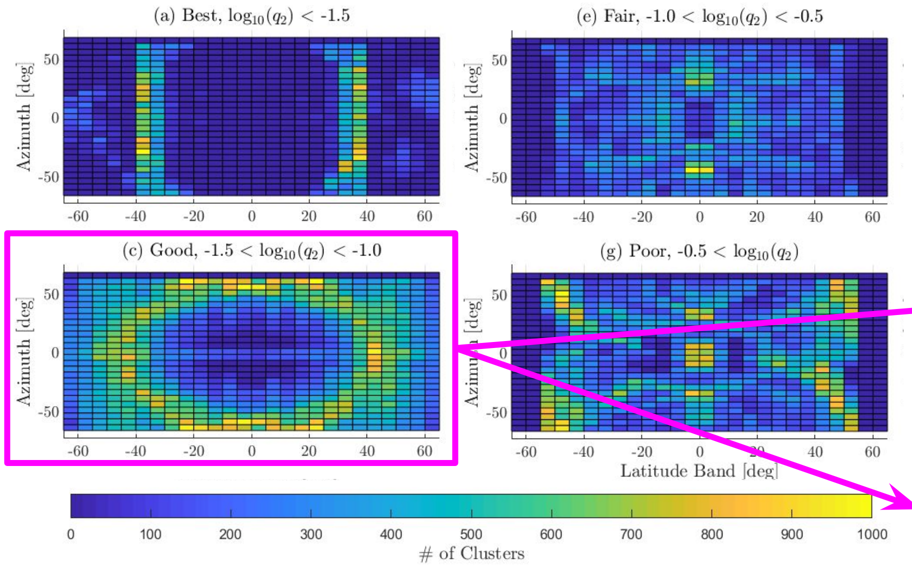

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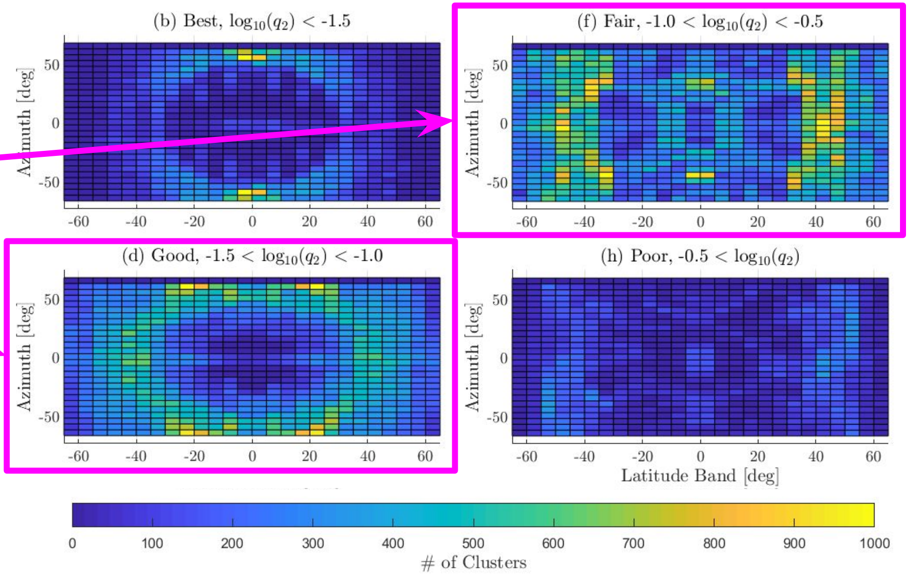


Causes of Cluster Quality

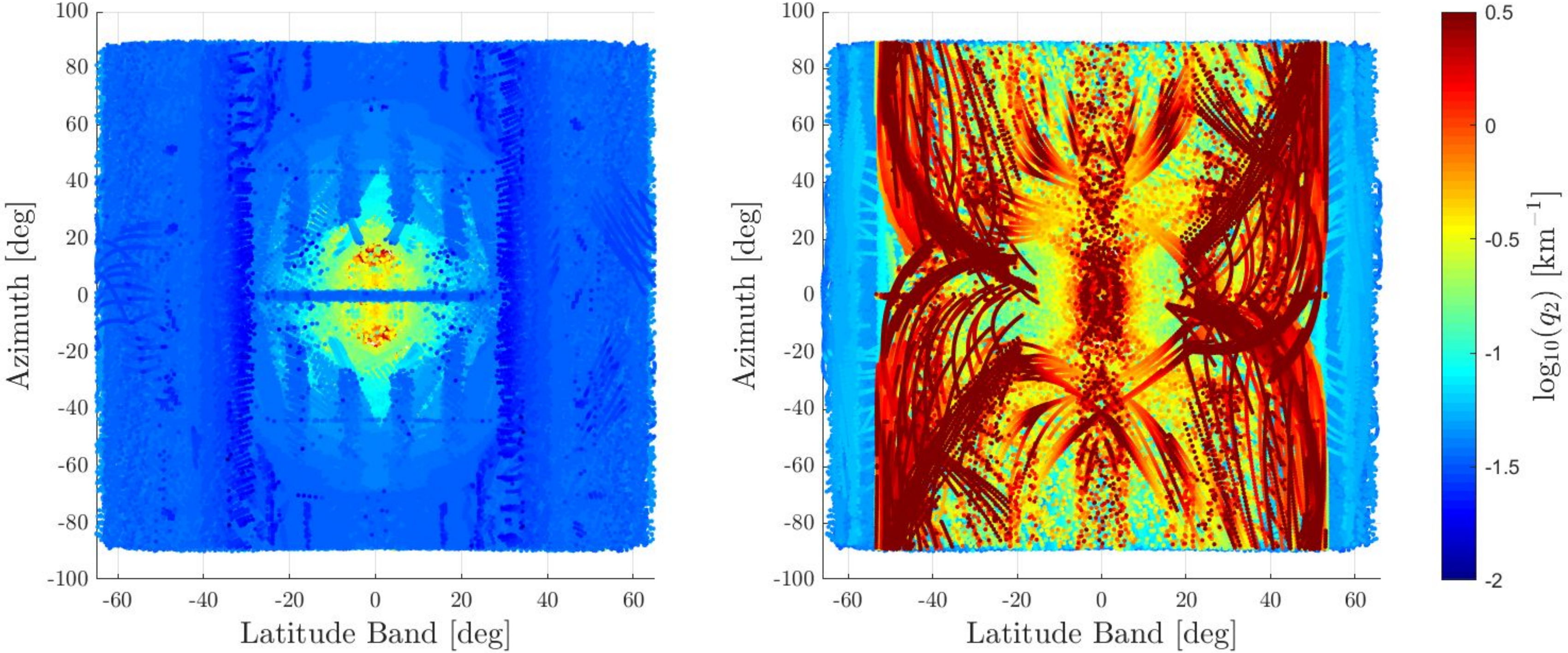
Mutual Orbit Groups

- Even distribution of good-quality clusters over all surveyed latitudes.
- Worst clusters due to geometric effects that stop at latitudes > inclination.
- Good clusters extend all the way up to high latitudes.

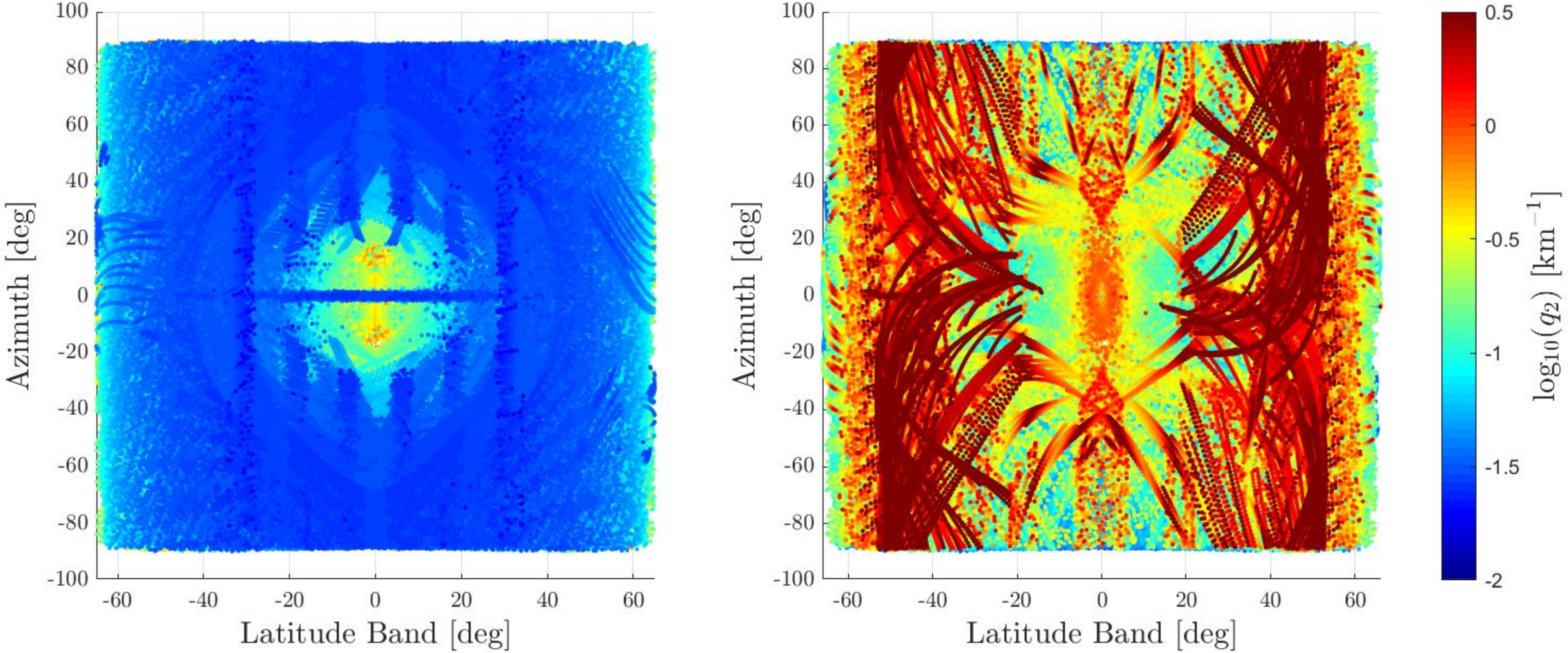
RAAN-Spread

- Good clusters tend to taper off into the high latitudes.
- Geometric effects still present, but bad clusters continue above inclination.
- Best clusters concentrated into equatorial regions.

Cluster Quality (MOGs)



Cluster Quality (RAAN-Spread)



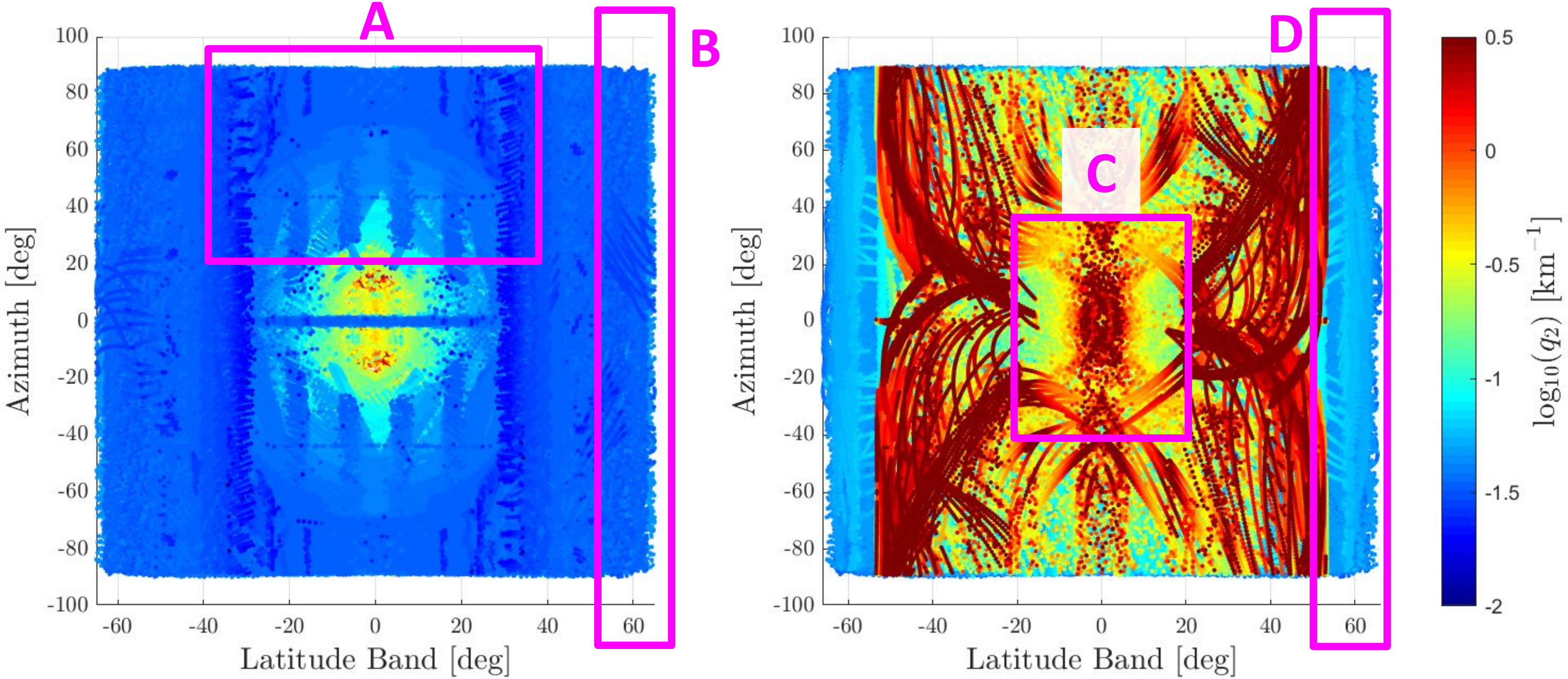
MOGs

A. RS improves east-west equatorial clusters.

B. Best high-latitude clusters are worse for RS.

C. North-south eq. clusters are worse for MOG.

D. MOG has no bad clusters above orbit inclination.



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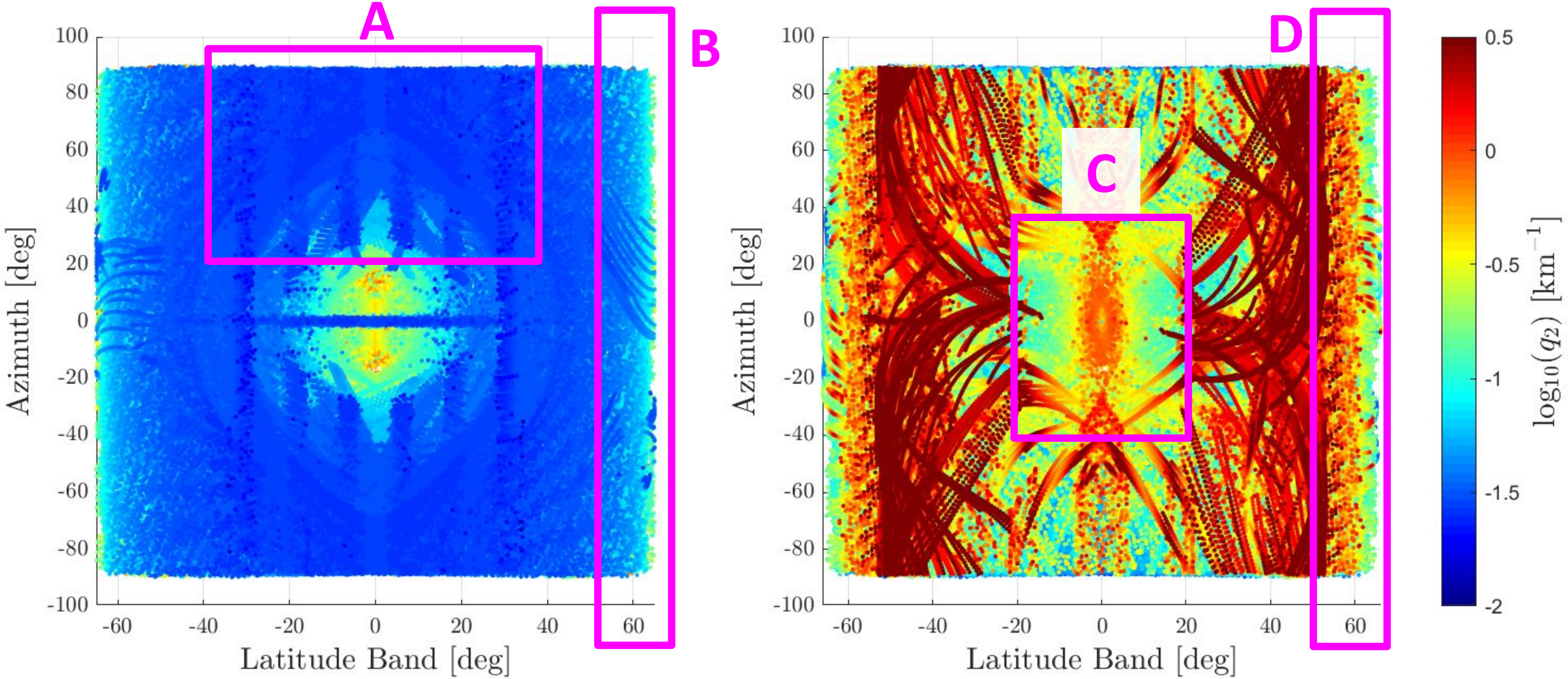


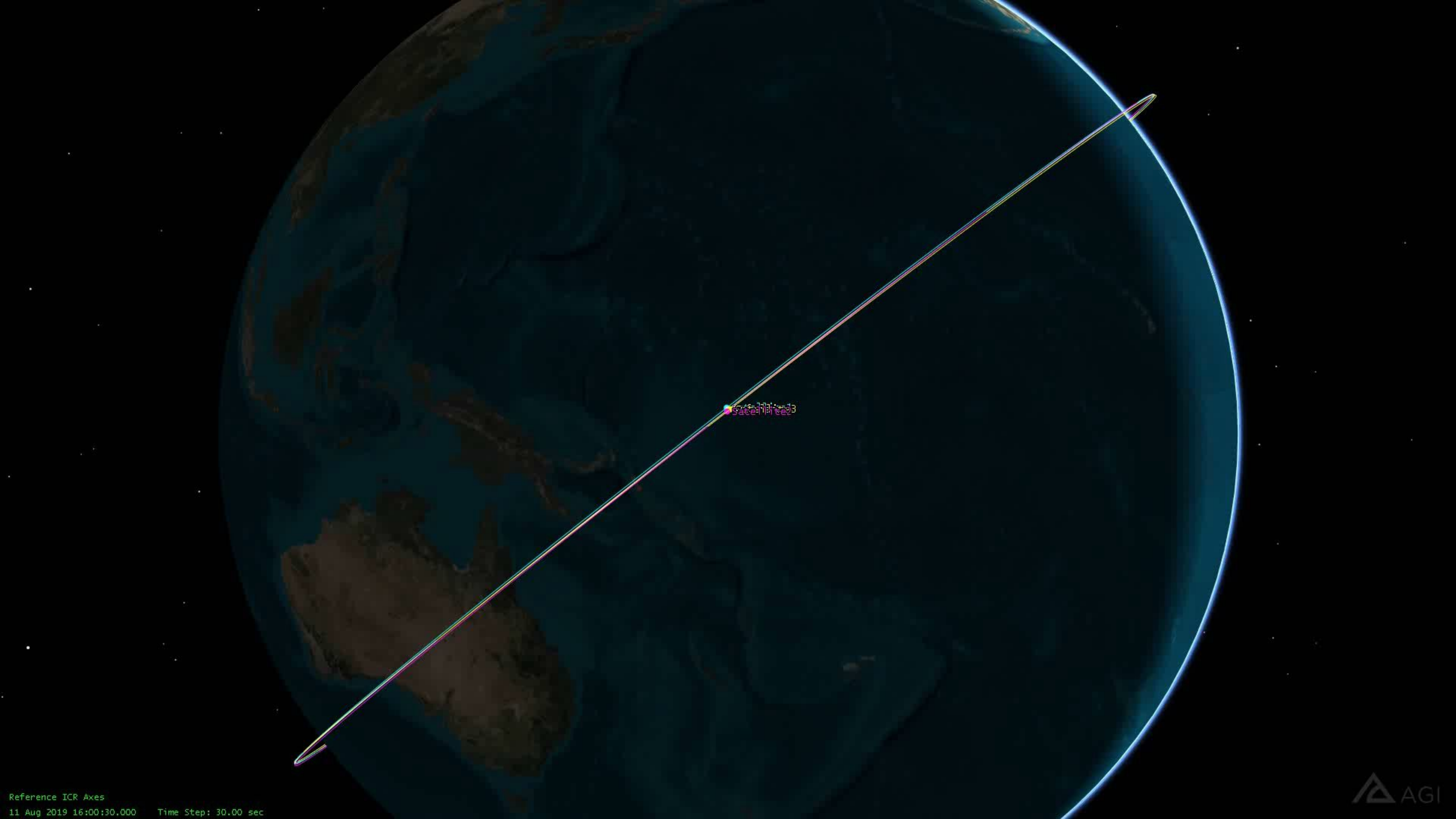
Conclusions

- Though Mutual Orbit Group architecture yields slightly-improved cluster geometry in higher latitudes, it requires significant propulsion to maintain constellation; this limits mission lifetime.
- RAAN-Spread constellations, which do not require RAAN maintenance, are a valid alternative to MOGs; they have slightly improved performance in equatorial regions, but more low-performance clusters in higher latitudes.
- This work is in second review with IEEE JSTARS.

Questions?

Thank you to the National Science Foundation
for funding this study via NSF Award 8150276.

Backup Slides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Reference ICR Axes
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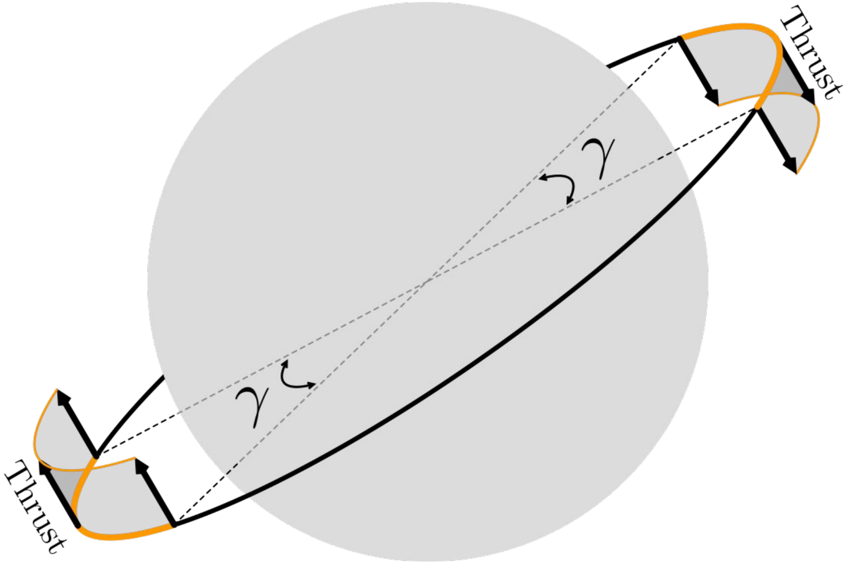
AGI

Out-of-Plane Thrusts

- Symmetric arcs of thrust at the maximum and minimum latitudes can counter J_2 torques.
- Angle γ gives required thrust arc length:
  - Impulsive propulsion lets γ approach zero.
  - Electric propulsion requires that γ be a substantial fraction of the orbit arc.



Burn Arc Length and Efficiency

- Rate of ΔV use is determined by ideal rate (α^*, from orbit geometry) and efficiency (η, from burn arc):

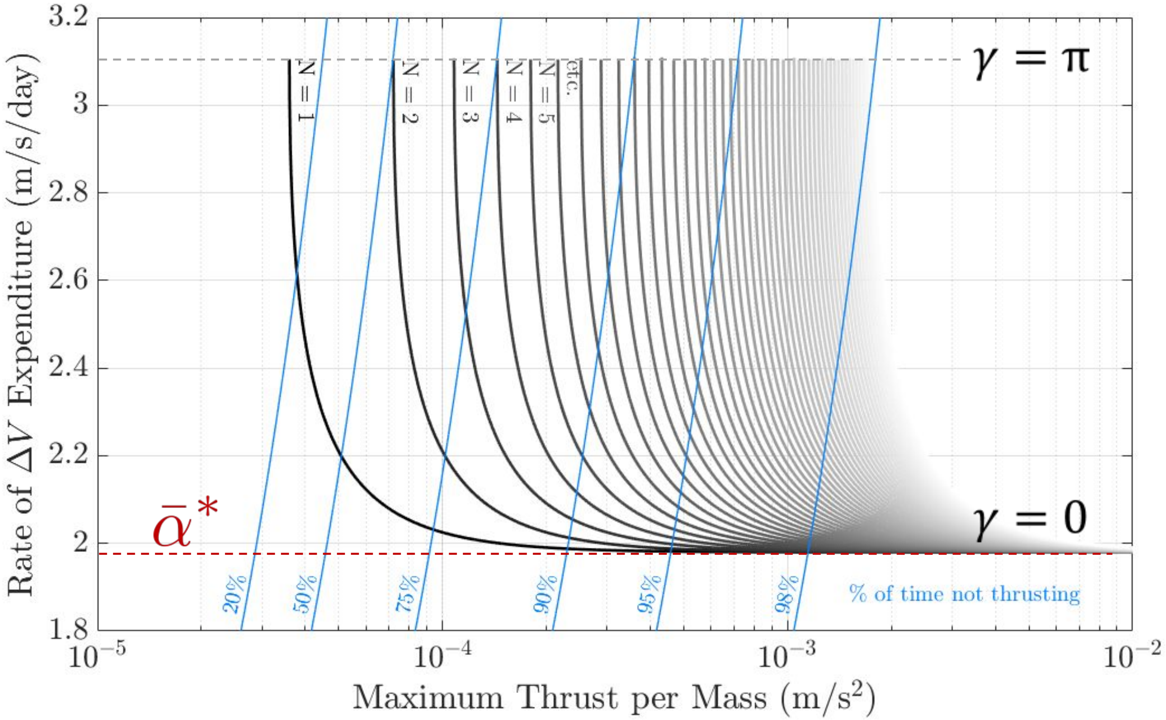
$$\bar{\alpha}_j \equiv \bar{\alpha}_j^* / \eta \qquad \eta = \operatorname{sinc} \gamma / 2$$

- For correction every N^{th} orbit with thrust/mass α_{max}:

$$\bar{\alpha}_j = \frac{2\alpha_{\max}}{N\pi} \sin^{-1}\left(\frac{N\pi}{2} \frac{\bar{\alpha}_j^*}{\alpha_{\max}} \right)$$

Rate of ΔV Expenditure

- Example (ISS):
  - $i = 51.4^0$
  - $\delta = 0.172^0$
  - $h = 400$ km
- Frequent burns or higher thrust per mass raises efficiency.



Latitude-Dependent Distributions